GEOPHYSICAL RESEARCH LETTERS, VOL. 27, NO. 15, PAGES 2329-2332, AUGUST 1, 2000

The excitation of the Chandler wobble

Richard S. Gross

Jet Propulsion Laboratory, California Institute of Technology, Pasadena

Abstract. The Chandler wobble is an excited resonance of the Earth's rotation having a period of about 14 months. Although it has been under investigation for more than a century, its excitation mechanism has remained elusive. Here, the angular momentum of the atmosphere computed from the products of a numerical weather prediction analysis system and the angular momentum of the oceans computed from a global oceanic general circulation model driven by observed surface winds and fluxes are used to show that during 1985.0–1996.0 the Chandler wobble was excited by a combination of atmospheric and oceanic processes, with the dominant excitation mechanism being ocean-bottom pressure fluctuations.

Introduction

Any irregularly shaped solid body rotating about some axis that is not aligned with its figure axis will wobble as it rotates. For the Earth, this Eulerian free wobble is known as the Chandler wobble in honor of S. C. Chandler who first observed it in 1891 [*Chandler*, 1891]. From observations of the Chandler wobble taken since its discovery, its period and Q have been estimated [*Wilson and Vicente*, 1990] to be 433.0 ± 1.1 (1σ) days and 179 (with 1σ bounds of 74 and 789), respectively, giving an estimated e-folding amplitude decay time of 68 years. Because a damping time of 68 years is short on a geological time-scale, the amplitude of the Chandler wobble should quickly dampen to zero unless some mechanism or combination of mechanisms are exciting it. Since its discovery, many processes have been evaluated, without success, to determine whether or not they could be the excitation mechanism(s) of the Chandler wobble including atmospheric processes [*Wilson and Haubrich*, 1976; *Wahr*, 1983; *Furuya et al.*, 1996, 1997], continental water storage [*Chao et al.*, 1987; *Hinnov and Wilson*, 1987; *Kuehne and Wilson*, 1991], core-mantle interactions [*Rochester and Smylie*, 1965; *Gire and Le Mouël*, 1986; *Hinderer et al.*, 1987; *Jault and Le Mouël*, 1993], and earthquakes [*Souriau and Cazenave*, 1985; *Gross*, 1986].

Recently, *Celaya et al.* [1999], using the results of a coupled atmosphere-ocean-ice-land climate model, concluded that atmospheric processes alone, oceanic processes alone, or some combination of atmospheric and oceanic processes probably have enough power to excite the Chandler wobble. However, *Celaya et al.* [1999] were not able to discriminate between these possible excitation processes because, due to the nature of the climate model they used, they could not match time series of modeled excitation with that observed—their conclusions were based solely upon statistical analyses of the observed Chandler wobble and the output of the climate model. *Ponte and Stammer* [1999] have also recently investigated atmospheric and oceanic excitation of the Chandler wobble by fitting a sinusoid at the Chandler frequency to modeled and observed polar motion excitation series, finding better agreement with the observations when the sum of the modeled oceanic current and bottom pressure excitation is added to the sum of the modeled atmospheric wind and pressure excitation (the relative contribution of the individual oceanic and atmospheric processes to exciting the Chandler wobble was not reported). However, the excitation of the Chandler wobble is a broad-band process that occurs within a band of frequencies centered at the Chandler frequency. Investigating the excitation of the Chandler wobble therefore requires computing the excitation power that occurs within the Chandler band. Here, realistic atmospheric and oceanic general circulation models are used to obtain time series of modeled Chandler wobble excitation from which the power in the Chandler band is computed and compared to that observed to show that during 1985.0–1996.0 the Chandler wobble was primarily excited by ocean-bottom pressure fluctuations, with atmospheric pressure fluctuations being about half as effective. Oceanic currents had only a minor effect on the Chandler wobble during this time, while atmospheric winds, being out-of-phase with the pressure terms, acted to reduce the modeled Chandler excitation power to nearly that observed.

Polar Motion Excitation Functions

Measurements of the Earth's changing rotation are currently made by the space-geodetic techniques of satellite and lunar laser ranging, very long baseline interferometry, and global positioning system interferometry [*Lambeck*, 1980]. The Earth rotation series used in this study is a combination of these space-geodetic measurements known as SPACE97 [*Gross*, 1999] and consists of daily averaged values of Universal Time, polar motion, and their rates of change spanning 1976.7–1998.0. Strictly speaking, the observed polar motion parameters specify the location of the Celestial Ephemeris Pole (CEP) within the body-fixed terrestrial reference frame and will be so interpreted here. However, for periods long compared to a day, such as for the Chandler wobble, and to sufficient accuracy, the observed polar motion parameters can be interpreted as specifying the location of the rotation pole within the terrestrial reference frame [*Gross*, 1992].

Polar motion consists largely of: (1) a forced annual wobble having a nearly constant amplitude of about 100 milliarcseconds (mas), (2) the free Chandler wobble having a variable amplitude ranging between about 100 to 200 mas, (3) quasi-periodic variations on decadal time scales having amplitudes of about 30 mas known as the Markowitz wobble, (4) a linear trend having a rate of about 3.5 mas/yr, and (5) smaller amplitude variations occurring on all measurable time scales. This rich polar motion spectrum is caused by the rich variety of processes forcing polar motion. In the absence of external torques, the polar motion parameters x_p and y_p can be related to the processes forcing polar motion by linearizing the Liouville equation which expresses the conservation of angular momentum within a rotating, body-fixed reference frame [*Lambeck*, 1980]:

$$\mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{d\mathbf{p}(t)}{dt} = \chi(t) \qquad (1)$$

where σ_{cw} is the complex-valued frequency of the Chandler wobble and the complex-valued quantity $\mathbf{p} \equiv (x_p - i\, y_p)$ specifies

Paper number 2000GL011450.
0094-8276/00/2000GL011450$05.00

the x- and y-coordinates, x_p and y_p respectively, of the CEP with x_p being positive towards the Greenwich meridian and y_p being positive by convention towards 90°W longitude. Equation 1 is the expression for simple harmonic motion in the complex plane with the right-hand-side $\chi(t)$ being the forcing, or excitation, function which can be written as [*Wahr*, 1982]:

$$\chi(t) \;=\; \frac{1.61}{\Omega\,(C\text{–}A)}\left[\mathbf{h}(t) \;+\; \frac{\Omega\,\mathbf{c}(t)}{1.44}\right] \qquad (2)$$

where C and A are the greatest and least, respectively, principal moments of inertia of the Earth, the mean angular velocity of the Earth is Ω, the complex-valued quantity $\mathbf{c}(t) \equiv c_{13}(t) + i\ c_{23}(t)$ represents changes in the two indicated elements of the Earth's inertia tensor such as those due to atmospheric or oceanic mass redistribution, and $\mathbf{h}(t) \equiv h_1(t) + i\ h_2(t)$ represents relative angular momentum changes such as those due to changes in the atmospheric winds or oceanic currents. The factor of 1.61 includes the effect of core decoupling and the factor of 1.44 in the denominator accounts for the yielding of the solid Earth due to its changing load.

Because polar motion is resonant at the Chandler frequency (Equation 1), investigations of the excitation of the Chandler wobble are usually conducted by frequency-domain comparisons of the observed polar motion excitation functions with those computed from various geophysical processes. Here, the observed polar motion excitation functions are those determined from the SPACE97 polar motion values and rates using Equation 1 with the period and Q of the Chandler wobble set to 433.0 days and 179, respectively [*Wilson and Vicente*, 1990]. The observed excitation functions thus recovered are then compared to those caused by atmospheric wind and pressure changes and by oceanic current and ocean-bottom pressure changes. The atmospheric excitation functions used here are those computed from the National Centers for Environmental Prediction (NCEP) / National Center for Atmospheric Research (NCAR) reanalysis project and are available from the International Earth Rotation Service (IERS) Special Bureau for the Atmosphere [*Salstein et al.*, 1993]. The oceanic excitation functions used here are those computed by *Ponte et al.* [1998] and are available from the IERS Special Bureau for the Oceans.

Ponte et al. [1998] computed the polar motion excitation functions due to oceanic currents and, separately, ocean-bottom pressure changes from the products of a global oceanic general circulation model (OGCM) driven by 12-hour wind stress fields and daily surface heat and fresh water flux fields from NCEP. Atmospheric pressure was not used to force the OGCM. The ocean-bottom pressure excitation term was corrected by them for the effects of volume changes due to steric effects within the Boussinesq OGCM by adding a uniform sea level layer of fluctuating thickness to the sea surface height fields produced by the OGCM [*Greatbatch*, 1994]. The resulting oceanic current and ocean-bottom pressure excitation functions are 5-day-averaged values spanning January 1985 to April 1996.

The available atmospheric wind and pressure excitation functions computed from the NCEP/NCAR reanalysis project are 6-hour values spanning 1958.0 to the present. The pressure excitation term is available under two different assumptions for the response of the oceans to surface pressure changes: (1) the inverted barometer assumption wherein the oceans are assumed to respond isostatically to the imposed surface pressure variations, and (2) the rigid ocean assumption wherein the oceans are assumed to fully transmit without delay or attenuation the atmospheric pressure variations to the ocean-bottom. Since at periods long compared to a day the inverted barometer assumption should be valid [e.g., *Wunsch and Stammer*, 1997], the pressure term computed under this assumption is used here.

Since the oceanic current and ocean-bottom pressure excitation functions are given as 5-day-averaged values, 5-day-averaged values were also formed of the daily averaged observed excitation functions and of the 6 hourly atmospheric wind and pressure excitation functions. In order to reduce spectral leakage into the Chandler frequency band of annual excitation processes, a seasonal signal was removed from the 5-day-averaged observed, atmospheric, and oceanic excitation functions by least-squares fitting and removing a mean, a trend, and periodic terms at the annual and semiannual frequencies. This fit was done on that subset of the series spanning 1985.0–1996.0 in order to fit an integral number of annual and semiannual oscillations. All subsequent analysis will be done on this 11-year subset of the residual excitation series.

Chandler Wobble Excitation

Figure 1 shows power spectral density (psd) estimates of the observed (black curve), atmospheric (red curve), and sum of atmospheric and oceanic (green curve) excitation functions from which seasonal signals have been removed. A Hanning window was applied to each series prior to forming the spectral estimates, and the spectral estimates are of the time series including noise. In agreement with the conclusion of previous studies [*Wilson and Haubrich*, 1976; *Wahr*, 1983], it is seen that the sum of atmospheric wind and pressure fluctuations (red curve) does not have sufficient power to excite the Chandler wobble (the Chandler frequency of 0.8435 cycles/year (cpy) is indicated by the vertical dotted line). However, a good match to the observed Chandler wobble excitation power is obtained upon adding the excitation due to oceanic current and ocean-bottom pressure fluctuations to that due to atmospheric wind and pressure variations (green curve).

Since the time series whose spectra are displayed in Figure 1 consist of 800 5-day-averaged samples, the frequency resolution

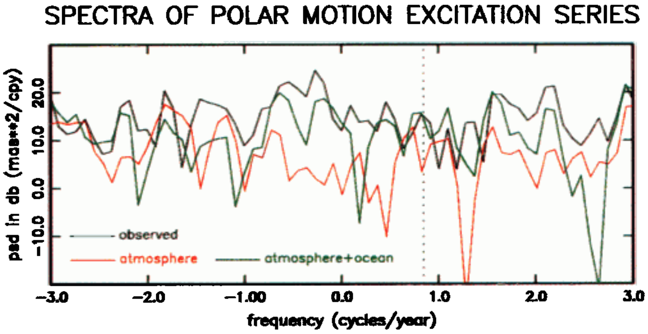

Figure 1. Power spectral density (psd) estimates in decibels (db) computed from time series of polar motion excitation functions $\chi(t)$ spanning 1985.0–1996.0 of: (a) the observed SPACE97 polar motion excitation function derived from space-geodetic Earth rotation measurements (black curve), (b) the sum of the excitation functions due to atmospheric wind and pressure changes (red curve) where the atmospheric pressure term is that computed assuming the inverted barometer approximation is valid, and (c) the sum of all atmospheric and oceanic excitation processes being studied here, namely, the sum of the excitation functions due to atmospheric winds, atmospheric pressure (inverted barometer), oceanic currents, and ocean-bottom pressure (green curve). A seasonal signal has been removed from all series prior to spectral estimation by least-squares fitting and removing a mean, a trend, and periodic terms at the annual and semiannual frequencies. The vertical dotted line indicates the Chandler frequency of 0.8435 cycles/year (cpy). The retrograde component of polar motion excitation is represented by negative frequencies, the prograde component by positive frequencies. The Chandler wobble is a strictly prograde oscillation.

of these time series is 0.0913 cpy which is just sufficient to resolve the Chandler frequency band. Near the Chandler frequency, the spectral estimates shown in Figure 1 are given at frequencies of 0.730 cpy, 0.822 cpy, and 0.913 cpy. Integrating the power spectral density estimates of Figure 1 across the Chandler frequency band, taken here to range between 0.730 cpy and 0.913 cpy, gives the power in the Chandler band shown in Table 1 for the various excitation mechanisms being studied here. The observed excitation power in this band is 4.87 mas^2 with the sum of the power due to atmospheric wind, atmospheric pressure, oceanic current, and ocean-bottom pressure excitation being slightly more than this at 5.44 mas^2. Ocean-bottom pressure fluctuations are seen to be the single most important mechanism exciting the Chandler wobble, containing about twice as much power in the Chandler band as that due to atmospheric pressure fluctuations. Oceanic current and atmospheric wind variations are minor contributors to the Chandler wobble excitation, having power in the Chandler band of only 0.12 mas^2 and 0.32 mas^2, respectively. Even so, destructive interference between the sum of atmospheric wind and oceanic current excitation and that due to the sum of atmospheric and ocean-bottom pressure excitation reduces the power in the Chandler band from 6.28 mas^2 to 5.44 mas^2.

Figure 2 shows the magnitude of the squared-coherence between the observed excitation functions and those due to atmospheric wind and pressure variations (red curve), the sum of ocean-bottom pressure and atmospheric pressure fluctuations (blue curve), and the total sum of atmospheric wind, atmospheric pressure, oceanic currents, and ocean-bottom pressure variations (green curve). The squared-coherence estimates were obtained by averaging over 5 frequency intervals and the 95% and 99% confidence limits on the magnitude of the squared-coherence estimates are indicated by the horizontal dashed lines. As can be seen, near the Chandler frequency (indicated by the vertical dotted line) atmospheric wind and pressure excitation is not coherent with the observed excitation, but that due to the sum of atmospheric wind, atmospheric pressure, oceanic currents, and ocean-bottom pressure is coherent with greater than 99% confidence. Since the sum of atmospheric pressure and ocean-bottom pressure is also coherent with the observed excitation near the Chandler frequency, the addition of atmospheric wind and oceanic current excitation reduces the power in the Chandler band to nearly that observed, but does not affect the coherence.

Table 1. Chandler band excitation power

Excitation process	Power (mas^2)
Observed	4.87
Atmospheric	
wind	0.32
pressure (i.b.)	1.87
wind plus pressure (i.b.)	1.44
Oceanic	
currents	0.12
ocean-bottom pressure	3.45
currents plus ocean-bottom pressure	3.69
Atmospheric plus oceanic	
wind plus currents	0.67
i.b. plus ocean-bottom pressure	6.28
Total of all atmospheric plus oceanic	5.44

i.b., inverted barometer

Discussion and Summary

The results reported here for the observed excitation power in the Chandler frequency band (4.87 mas^2) were obtained using those values of 433.0 days and 179 for the period and Q of the Chandler wobble, respectively, that were estimated by *Wilson and Vicente* [1990] from 86 years of optical astrometric polar motion observations. Other recent estimates for the period and Q

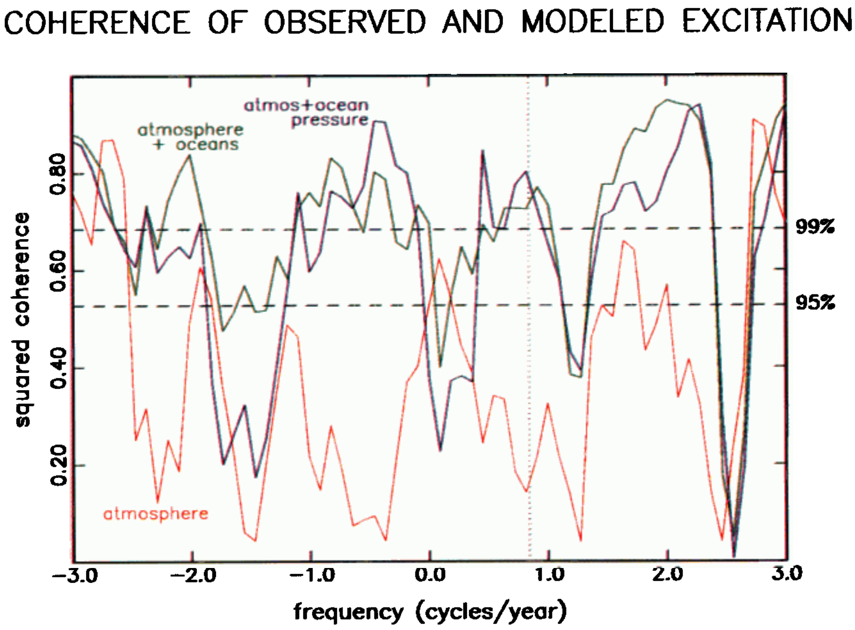

Figure 2. The magnitude of the squared-coherence between the observed polar motion excitation functions spanning 1985.0–1996.0 and the excitation functions due to: (a) the sum of atmospheric wind and pressure changes (red curve) where the pressure term is that computed under the inverted barometer approximation, (b) the sum of atmospheric (inverted barometer) and ocean-bottom pressure fluctuations (blue curve), and (c) the sum of all the atmospheric and oceanic excitation processes being studied here, namely, the sum of atmospheric wind, atmospheric pressure (inverted barometer), oceanic current, and ocean-bottom pressure variations. A seasonal signal has been removed from all series prior to coherence estimation by least-squares fitting and removing a mean, a trend, and periodic terms at the annual and semiannual frequencies. The vertical dotted line indicates the Chandler frequency of 0.8435 cycles/year (cpy) and the horizontal dashed lines indicate the 95% and 99% confidence levels of the magnitude of the squared-coherence.

of the Chandler wobble include those of *Kuehne et al.* [1996] who used just 8.6 years of modern space-geodetic polar motion observations to obtain a period of 439.5 ± 2.1 (1σ) days and a Q of 72 (with 1σ bounds of 30 and 500) and hence an estimated e-folding amplitude decay time of 28 years, and those of *Furuya and Chao* [1996] who used just 10.8 years of modern polar motion observations to obtain a period of 433.7 ± 1.8 (1σ) days and a Q of 49 (with 1σ bounds of 35 and 100) and hence an estimated e-folding amplitude decay time of 19 years. The estimates of *Wilson and Vicente* [1990] were preferred for this study since they are based upon the longest series of polar motion observations. In fact, of these three estimates, it is the only one that is based upon polar motion observations spanning a time interval (86 years) that is greater than its estimated Chandler wobble e-folding amplitude decay time (68 years).

Since the Chandler wobble is a resonance in the Earth's rotation, the observed excitation power near the resonance frequency, that is, in the Chandler frequency band, will be sensitive to the assumed value for the period and Q of the Chandler wobble (Equation 1). For example, if instead of using an assumed value of 433.0 days for its period and 179 for its Q, the period were to be kept unchanged at 433.0 days but the Q were to be changed to its 1σ lower bound of 74, then the observed excitation power in the Chandler frequency band would change from 4.87 mas^2 to 7.99 mas^2. In fact, an assumed period of 433.0 days and a Q of 138 yields an observed Chandler band excitation power of 5.45 mas^2, nearly matching that caused by the sum of atmospheric wind, atmospheric pressure, oceanic currents, and ocean-bottom pressure excitation (5.44 mas^2, see Table 1). Thus, the conclusion reached here that the Chandler wobble is being excited by a combination of atmospheric and oceanic processes is based upon assuming that the period of the Chandler wobble is nearly 433.0 days and that its Q is nearly 138. Given the scatter in the above cited estimates for the period and Q of the Chandler wobble, and their rather large 1σ uncertainties, more accurate estimates of the period and Q of the Chandler wobble are clearly required. Such improved estimates could perhaps be obtained when series of realistic oceanic angular momentum values become available that are as long as the 40-year-long series of atmospheric angular momentum values currently available from the NCEP/NCAR reanalysis project. This would then allow the Chandler wobble's period and Q to be determined from 40 years of polar motion observations and models of its atmospheric and oceanic excitation processes.

Numerous investigations have been conducted during the past century in attempts to elucidate the excitation mechanism of the Chandler wobble. Here it has been shown that during 1985.0–1996.0 the single most important mechanism exciting the Chandler wobble has been ocean-bottom pressure fluctuations, which contribute about twice as much excitation power in the Chandler frequency band as do atmospheric pressure fluctuations. Atmospheric winds and oceanic currents have been shown here to play only a minor role in exciting the Chandler wobble during this time. The ability to elucidate the role of atmospheric and ocean-bottom pressure fluctuations in exciting the Chandler wobble is a testament to the fidelity of the atmospheric and oceanic general circulation models that were used to compute the atmospheric and oceanic angular momentum estimates used in this study. The wide distribution of these atmospheric and oceanic angular momentum estimates by the IERS Special Bureaus for the Atmosphere and Oceans enables the type of interdisciplinary research whose results are reported here.

Acknowledgments. The work described in this paper was performed at the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration. Support for this work was provided by NASA's Office of Earth Science.

References

Celaya, M. A., J. M. Wahr, and F. O. Bryan, Climate-driven polar motion, *J. Geophys. Res.*, *104*, 12813–12829, 1999.

Chandler, S. C., On the variation of latitude, I, *Astron. J.*, *11*, 59–61, 1891.

Chao, B. F., W. P. O'Connor, A. T. C. Chang, D. K. Hall, and J. L. Foster, Snow load effect on the Earth's rotation and gravitational field, 1979–1985, *J. Geophys. Res.*, *92*, 9415–9422, 1987.

Furuya, M., and B. F. Chao, Estimation of period and Q of the Chandler wobble, *Geophys. J. Int.*, *127*, 693–702, 1996.

Furuya, M., Y. Hamano, and I. Naito, Quasi-periodic wind signal as a possible excitation of Chandler wobble, *J. Geophys. Res.*, *101*, 25537–25546, 1996.

Furuya, M., Y. Hamano, and I. Naito, Importance of wind for the excitation of Chandler wobble as inferred from wobble domain analysis, *J. Phys. Earth*, *45*, 177–188, 1997.

Gire, C., and J-L Le Mouël, Flow in the fluid core and Earth's rotation, in *Earth Rotation: Solved and Unsolved Problems*, edited by A. Cazenave, pp. 241–258, D. Reidel, Dordrecht, Holland, 1986.

Greatbatch, R. J., A note on the representation of steric sea level in models that conserve volume rather than mass, *J. Geophys. Res.*, **99**, 12767–12771, 1994.

Gross, R. S., The influence of earthquakes on the Chandler wobble during 1977-1983, *Geophys. J. Roy. astr. Soc.*, *85*, 161–177, 1986.

Gross, R. S., Correspondence between theory and observations of polar motion, *Geophys. J. Int.*, *109*, 162–170, 1992.

Gross, R. S., Combinations of Earth orientation measurements: SPACE97, COMB97, and POLE97, *J. Geodesy*, in press, 1999.

Hinderer, J., H. Legros, C. Gire, and J-L Le Mouël, Geomagnetic secular variation, core motions and implications for the Earth's wobbles, *Phys. Earth Planet. Int.*, *49*, 121–132, 1987.

Hinnov, L. A., and C. R. Wilson, An estimate of the water storage contribution to the excitation of polar motion, *Geophys. J. Roy. astr. Soc.*, *88*, 437–459, 1987.

Jault, D., and J.-L. Le Mouël, Circulation in the liquid core and coupling with the mantle, in *Observations of Earth from Space*, edited by R. P. Singh, M. Feissel, B. D. Tapley, and C. K. Shum, *Adv. Space Res.*, *13*, (11)221–(11)233, Pergamon, Oxford, 1993.

Kuehne, J., and C. R. Wilson, Terrestrial water storage and polar motion, *J. Geophys. Res.*, *96*, 4337–4345, 1991.

Kuehne, J., C. R. Wilson, and S. Johnson, Estimates of the Chandler wobble frequency and Q, *J. Geophys. Res.*, *101*, 13573–13579, 1996.

Lambeck, K., *The Earth's Variable Rotation: Geophysical Causes and Consequences*, 449 pp., Cambridge University Press, New York, 1980.

Ponte, R. M., and D. Stammer, Role of ocean currents and bottom pressure variability on seasonal polar motion, *J. Geophys. Res.*, *104*, 23393–23409, 1999.

Ponte, R. M., D. Stammer, and J. Marshall, Oceanic signals in observed motions of the Earth's pole of rotation, *Nature*, *391*, 476–479, 1998.

Rochester, M. G., and D. E. Smylie, Geomagnetic core-mantle coupling and the Chandler wobble, *Geophys. J. R. astr. Soc.*, *10*, 289–315, 1965.

Salstein, D. A., D. M. Kann, A. J. Miller, and R. D. Rosen, The Sub-Bureau for Atmospheric Angular Momentum of the International Earth Rotation Service: A meteorological data center with geodetic applications, *Bull. Amer. Meteorol. Soc.*, *74*, 67–80, 1993.

Souriau, A., and A. Cazenave, Re-evaluation of the seismic excitation of the Chandler wobble from recent data, *Earth planet. Sci. Lett.*, *75*, 410–416, 1985.

Wahr, J. M., The effects of the atmosphere and oceans on the Earth's wobble - I. Theory, *Geophys. J. Roy. astr. Soc.*, *70*, 349–372, 1982.

Wahr, J. M., The effects of the atmosphere and oceans on the Earth's wobble and on the seasonal variations in the length of day - II. Results, *Geophys. J. Roy. astr. Soc.*, *74*, 451–487, 1983.

Wilson, C. R., and R. A. Haubrich, Meteorological excitation of the Earth's wobble, *Geophys. J. Roy. astr. Soc.*, *46*, 707–743, 1976.

Wilson, C. R., and R. O. Vicente, Maximum likelihood estimates of polar motion parameters, in *Variations in Earth Rotation*, edited by D. D. McCarthy and W. E. Carter, pp. 151–155, American Geophysical Union Geophysical Monograph Series, Washington, DC, 1990.

Wunsch, C., and D. Stammer, Atmospheric loading and the oceanic "inverted barometer" effect, *Rev. Geophys*, *35*, 79–107, 1997.

R. S. Gross, Jet Propulsion Laboratory, Mail Stop 238-332, 4800 Oak Grove Drive, Pasadena, CA 91109 (e-mail: Richard.Gross@jpl.nasa.gov)

(Received February 1, 2000; revised April 14, 2000;
accepted May 12, 2000.)