

# Terrestrial Magnetism and Atmospheric Electricity

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## THE ELECTRIC CURRENT-SYSTEMS OF MAGNETIC STORMS

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1. This paper continues my discussion of the average characteristics of the field of geomagnetic disturbances or storms, given in two previous papers<sup>1</sup> which will be referred to as *1* and *2*; the irregular part of magnetic disturbance is not considered. A condensed representation of the main average features of magnetic storms is given in *1* (pp. 61-72) and *2* (pp. 242-264), and evidence is described which supports the view that the type of the field remains fairly constant throughout a considerable range of intensity for the field as a whole.

In *2* it is shown<sup>2</sup> that certain electric current-systems in the Earth's atmosphere<sup>3</sup> could produce a disturbance-field of the observed type (this will be called the *D*- or disturbance-field; the letter *D* is also used in terrestrial magnetism for the declination but I think no confusion need arise from its use in this further sense). These current-systems are indeed somewhat too simple, because so far as their high-latitude portions are concerned they apply less to the actual Earth than to an ideal Earth for which the magnetic and geographic axes coincide; but I believe that they constitute a useful first approximation to the more complicated (atmospheric) current-system appropriate to the real Earth.

2. It is, of course, in principle impossible to infer uniquely, purely from observations of a magnetic field (of external origin) at the Earth's surface, the location of the current-system which is the source of the field. So long as no further considerations are taken into account, the problem has not only one but an infinity of solutions. This may be illustrated as follows: Imagine any current-system outside the Earth, whether linear, or distributed over a surface or throughout a volume of any form. Observation of its field at the Earth's surface enables the potential of the field to be determined and expressed as a series of spherical harmonic terms. Along any radius from the Earth's center *O* the variation of the term of degree *n* (*n*=1, 2, . . .) is proportional to  $r^n$ . This expression for the potential is valid up to the boundary of any sphere centered at *O*

<sup>1</sup>*1*—An outline of a theory of magnetic storms. Proc. R. Soc., A, 95, 61-83 (1918); *2*—On certain average characteristics of world-wide magnetic disturbance. Proc. R. Soc., A, 115, 242-267 (1927).

<sup>2</sup>The current-system diagram given in *1*, p. 76, took no account of the disturbance-fields in high latitudes, and is superseded by the diagrams of *2*, p. 263.

<sup>3</sup>These currents induce secondary electric-currents within the Earth which themselves contribute to the observed disturbance-field; cf. S. Chapman and T. T. Whitehead, Trans. Camb. Phil. Soc., 22, p. 463 (1922), and S. Chapman and A. T. Price, Phil. Trans. R. Soc., A, 229, pp. 427-460 (1930). The present paper deals mainly with the external part of the disturbance-field and the external current-systems, and references to the field and currents are to be read in this sense.

which does not intersect the external current-system. The same surface-field can be produced by an infinite variety of external current-systems differing from the actual one; in particular, it can be produced by current flowing in any sphere concentric with the Earth. The current-distribution in such a spherical sheet can be very simply deduced from the spherical harmonic series for the potential. The radius  $R$  may have any value whatsoever (of course  $R > a$ ,  $a$  being the Earth's radius). The greater the radius, the more prominent will be the spherical harmonic terms of higher degree in the current-distribution, since their relative magnitudes there will be proportional to  $(R/a)^n$ .

The current-distribution over a spherical sheet can easily be represented by a diagram using any projection of the sphere upon a plane. This is one method of representing the potential of the field graphically; it is of value in this way even if the field is produced by a non-spherical current-distribution. When used for this purpose, the current-distribution must be drawn for a definite radius, which may conveniently be  $a$  itself, or some radius  $R$  differing so slightly from  $a$  that the relative proportions of the main harmonic terms are not seriously different from those corresponding to  $a$ ; in other words,  $(R/a)^n$  must be small for the values of  $n$  which are important. Within the range of  $R$  satisfying this condition, the current-system may be considered independent of  $R$ ; of course this range is fairly small, but if the field is so simple that only the first three or four harmonics in it are important, the range may be considered to include values of  $R$  differing from  $a$  by not more than 300 or 400 km, which is the thickness of the atmosphere up to and including the main known ionized layers; this is because the ground value of the ratio of the  $n^{\text{th}}$  to the first harmonic is increased at 400 km by the factor  $(6770/6370)^{n-1}$ , and up to  $n=4$  this does not exceed 1.2.

Considerations of continuity indicate that if a field is produced by a current-sheet that departs from the spherical concentric form, but only by a moderate amount, the character of the current-distribution in the sheet can be inferred approximately from that of the spherical current-distribution of the same mean radius, which would give the same surface-field.

3. In attempting to decide between the infinite number of possible current-systems that could produce a given field, additional "non-magnetic" considerations of various kinds may be employed.

One of these concerns the space-distribution of electric conductivity, or, what is equivalent in the case of a rare medium such as alone comes into question for the space surrounding the Earth, the distribution of ionized gas. Any information as to this may indicate a certain region as a likely location for the current-system of a particular field, or may show that it is an improbable or impossible situation for that current-system.

For example, the spherical current-distributions corresponding to the fields ( $S$  and  $L$ ) of the solar ( $S$ ) and lunar ( $L$ ) daily magnetic variations are much more intense over the sunlit than over the dark hemisphere, and even over the latter their intensity decreases perceptibly from sunset to dawn; (see Fig. 1 for the  $S$  current-system, as drawn by Bartels from my spherical harmonic analysis of  $S$ ). Now there are ionized regions in the atmosphere which share these characteristics, and it is natural to suppose, in the absence of evidence to the contrary, that the

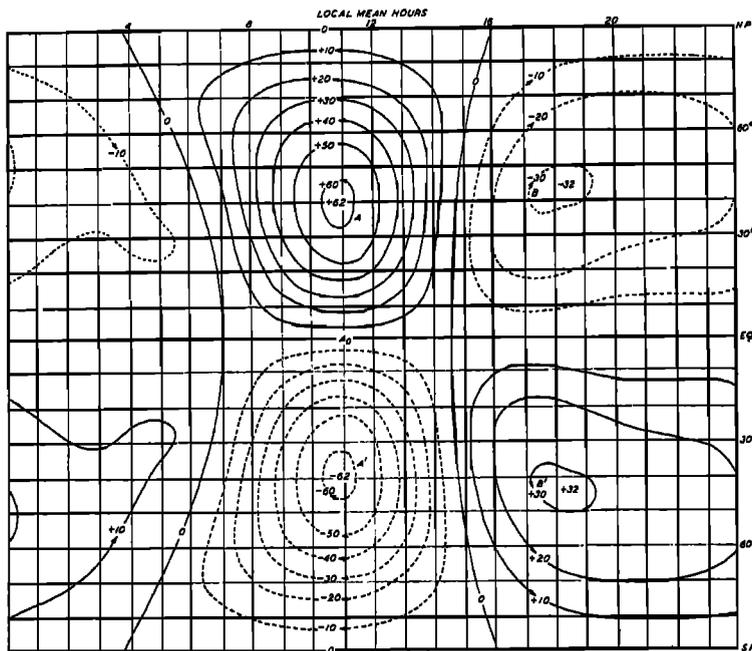


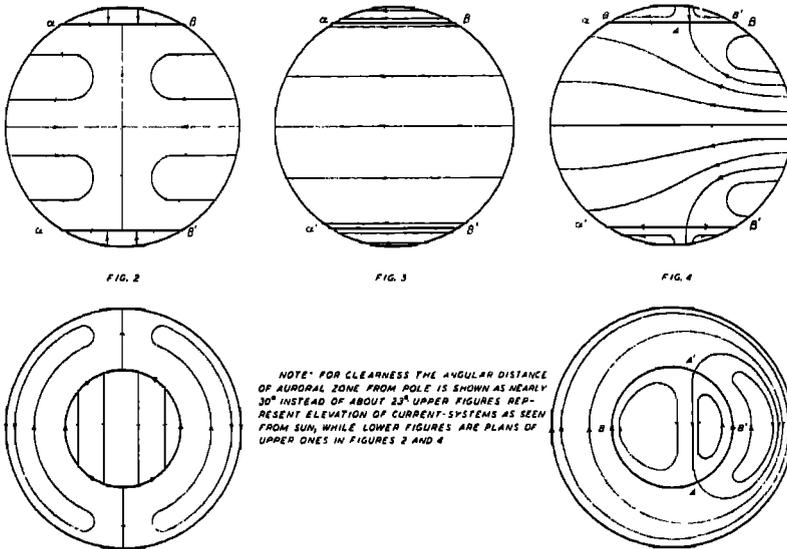
FIG. 1—CURRENT-SYSTEM,  $S_D$ , CORRESPONDING TO EXTERNAL PART OF SOLAR-DIURNAL MAGNETIC VARIATIONS ON QUIET DAYS, EQUINOX, SUNSPOT-MINIMUM (10000 AMPERES FLOW BETWEEN SUCCESSIVE CURRENT LINES)

$S$  and  $L$  current-systems flow in one or other of these layers. The conclusion is reasonable (though provisional) despite the dependence of the current-system upon the distribution of electromotive forces as well as upon the conductivity. The conclusion will gain strength if it is found that the  $S$  and  $L$  fields vary in intensity throughout the year and from year to year, and also irregularly, in unison with one or other of the ionized layers; in this way it may become possible to associate  $S$  with a particular ionized layer, and likewise  $L$  with another. There is, in any case, little or no reason to doubt that the  $S$  and  $L$  currents flow in our atmosphere in a layer which is very nearly spherical and concentric with the Earth.

4. The (spherical) current-system ( $D$ ) corresponding to the average field of magnetic disturbance is shown in Figures 2, 3, and 4 reproduced here from 2, p 263. The complete system, shown in Figure 4, may be analyzed into a part (Fig. 3) symmetrical about the Earth's axis, and a non-symmetrical part ( $S_D$ ) in which there is no resultant flow along the circles of latitude (Fig. 2); Figures 2 and 4 show the systems as viewed from the Sun, and also (below) as viewed from above the North Pole.

One of the outstanding features that distinguishes the  $D$  from the  $S$  and  $L$  current-systems is that the  $D$ -system is not more intense by day than by night; in the ("interzonal") belt between the two auroral zones the plane that divides the more intense from the less intense parts of the system passes through the Sun, the currents being stronger over the p. m. (post-meridiem) than over the a. m. hemisphere. In the auroral zones the order of intensity is reversed, but the dividing plane still passes

nearly through the Sun. Until recently no ionized layer of the atmosphere was known in which the ionization increased from a minimum at dawn to a maximum at sunset, hence it seemed doubtful whether the interzonal part of the  $D$  current-system flowed in the atmosphere. Now, however, it appears that there is a layer ( $F_2$ ) whose properties somewhat resemble this; it is the highest of the known ionized layers, its height at the equator being about 250 km. Moreover the general theory of the daily variation of atmospheric ionization indicates that the higher the layer, the later in the daytime should be its hour of maximum ion-content. It may be, therefore, that the  $F_2$ -layer, or possibly a layer somewhat higher, still unknown to us because screened by the  $F_2$ -layer, is the seat of this part of the  $D$  current-system. At least it no longer seems *necessary* to exclude the atmosphere as a possible situation for these currents, and consequently to locate them in the free space beyond.



FIGS 2,3,4 - CURRENT-SYSTEMS, & CORRESPONDING TO EXTERNAL PART OF AVERAGE FIELD OF MAGNETIC DISTURBANCE  
(FIG.2-NON-SYMMETRICAL PART  $S_D$ ; FIG.3-SYMMETRICAL PART ABOUT EARTH'S AXIS; FIG.4-COMPLETE SYSTEM)

5. On the other hand, the latter possibility requires consideration, particularly since it has often been suggested that a current-ring encircling the Earth, far outside the atmosphere, is responsible for the decrease in the horizontal magnetic force during and after magnetic storms. The reasons prompting this suggestion seem to have been of two kinds. One is the high degree of regularity, both in geographical distribution, and in azimuth at each station, shown by the average disturbing-force<sup>4</sup>, and also the slowness of its decline over a period of many days. It seems to have been thought that the force was too regular and perhaps too enduring to be produced by a current-system in our atmosphere. But there

<sup>4</sup>J. A. Broun, Trans. R. Soc. Edin., 22, Part 3 (1861); W. van Bemmelen, Met. Zeits., 12 (1895); G. Angenheister, Gött. Nachr., Math. Phys. Kl. (1924); Ad. Schmidt, Zs. Geoph., I, pp. 3-13 (1925). The force referred to occurs during what I have called the main phase and the phase of recovery of a magnetic storm; Angenheister has used the terms "impetus" and "after-disturbance" for these two phases; van Bemmelen calls the "after-disturbance" the "post-perturbation."

seems no obvious reason why a ring-current outside should be more regular than a current-system in our atmosphere nor, indeed, why the decay of the latter should be too short, in view of our present knowledge that the decline of ionization in any atmospheric layer will be slower, the higher the layer; certainly the slow decrease of the symmetrical part of the *D*-field would require the currents, if atmospheric, to be located high up.

The second reason for suggesting the existence of a ring-current round the Earth was given by Störmer,<sup>5</sup> who showed that a certain difficulty in his auroral theory might be overcome by postulating such a current; he assigned to it a very large radius (placing it beyond the Moon's orbit), and suggested that the ring might consist of electrons sweeping round the Earth, as their path from the Sun becomes subject to the deflecting influence of the Earth's magnetic field. The hypothesis of a ring-current with so large a radius seems, however, to create more difficulties than it solves. The chief argument against it is that no such ring-current could hold together against the mutual electrostatic repulsion of its parts if composed of charges of one sign only, particularly if these are electrons, as Störmer proposed. If, however, there are charges of both signs present, the suggested mode of formation of the ring breaks down, because the heavier ions would not follow the path calculated for the easily deflected electrons. What kind of compromise path would be arrived at by the two sets of particles moving together is not easy to determine. Ferraro<sup>6</sup> and I have attempted to solve the problem, and conclude that the ring, if it exists, is likely to be far smaller than Störmer supposed. The current in it will depend on the relative motion of the ions and electrons, and would disappear if they had the same velocity. Collisions between them will tend to equalize their speed, but Ferraro showed that a ring in which there was a moderate difference between the two speeds could carry the necessary current and endure for days in equilibrium, so far as concerns the decay of current due to collisions, at a distance of a few Earth-radii from the Earth's center. The calculation did not, however, take into account the daily rotation of the Earth's magnetic axis, which describes a cone of 22°-angle about the geographic axis; it may be that this would disrupt the ring, though only calculation can decide this question. A relatively small ring of this size would require far less energy<sup>6</sup> than a ring of the size suggested by Störmer, and it would be free from the objection against the latter, that it far more than nullifies the Earth's field in the region where it is set up, thus destroying and reversing the forces that are supposed to call it into being. (Störmer's large ring-current would modify the Earth's field appreciably throughout an immense volume of surrounding space, and would seem likely during a magnetic storm to alter considerably the normal paths of cosmic rays.)

While there seems to be no good reason for maintaining the hypothesis of so large a ring as Störmer's, the question as to a possible smaller ring is somewhat less easy to dismiss. It must be confessed, however, that we have no clear indication whether or how such a ring could be formed. Ferraro and I made some tentative suggestions on this point, but in their present form these have little compelling force, being proposed merely as

<sup>5</sup>C. Störmer, *C.-R. Acad. sci.*, **151**, 736-739 (1910).

<sup>6</sup>S. Chapman and V. C. A. Ferraro, *Terr. Mag.*, **36**, 77-97, 171-186 (1931); **37**, 147-156, 421-429 (1932); **38**, 79-96 (1933); also *Terr. Mag.*, **37**, 269-272 (1932).

the least improbable mechanism to account for such a ring, if on other grounds it is believed to exist. Of themselves, even when combined with the demonstration (under certain limited conditions) that the ring if formed might endure for several days, these suggestions hardly suffice to weigh the balance in favor of this hypothesis to explain the force-reduction during storms, as against the view that the currents responsible are located in the atmosphere, should a layer possessing appropriate properties be found to exist.

6. Added strength is given to the latter view by the consideration that the symmetrical and  $S_D$  parts of the  $D$ -field result only from a convenient analysis of what appears to be a unitary phenomenon. One mechanism should explain both together. This is somewhat difficult with any closed and long-enduring current-system circulating freely in the space round the Earth. The horizontal-intensity reduction on the equator during the moderate storms considered in 1 and 2 is about 58 gammas at 6 p. m., and about 30 gammas at 6 a. m.; a ring-current would have to be either very eccentric, or very elliptical, to explain such a difference. The atmospheric theory of the currents, on the other hand, can account for this difference by a divergence of the current-lines flowing from the 6 p.-m. to the 6 a.-m. meridian, outwards towards the auroral zones, as shown in Figure 4; such a divergence offers no such difficulty in a spherical current-sheet, as it does in regard to currents in free space. For the present, therefore, it seems proper to consider the interzonal currents of Figures 2 to 4 as real; at least the probability of this justifies the discussion of the magnitude of these currents if real.

7. We next consider the concentrated currents along the auroral zones represented in Figures 2 to 4 by the heavy lines  $\alpha\beta$  and  $\alpha'\beta'$ . The case for locating these in the atmosphere is here very much stronger and clearer. In the first place, we have evidence from radio measures that during magnetic disturbances the ionization along the zone is enhanced and intense at levels from about 90 km upwards; moreover, the aurorae themselves afford visual indication of increased ionization and excitation in the rather narrowly concentrated auroral zone at times when the  $D$ -field is most intense; this is shown by the negative bands in the auroral spectrum, which are due to nitrogen ions. This creates a presumption that the currents producing the notable magnetic changes near this zone are situated in this ionized region of the atmosphere.

Here another type of "non-magnetic" consideration (cf. paragraph 3) comes into play; it is an argument from *a priori* probability on the grounds of simplicity. The  $D$ -field near the zone is highly differentiated locally, and is explicable as due to a current far more concentrated than that suggested by the interzonal part of the  $D$ -field; the latter can best be accounted for by a current-sheet in which the current-density varies only gradually from point to point. In the auroral region, on the other hand, a current so limited laterally that it can, on a large-scale view, be regarded as a linear current, is compatible with the local  $D$ -field. Its latitude, height, and total intensity can be calculated (cf. paragraphs 16 and 17) as was done by Birkeland<sup>7</sup> and, more recently, by Goldie<sup>8</sup>. Such calculations do not establish a proof that the cause of the local  $D$ -field is a current

<sup>7</sup>K. Birkeland, Kristiania, Skr. Vid. selesk., Math-naturv. Kl., No. 1 (1901) [especially pp. 13-38]; Norwegian Aurora Polaris Expedition 1902-1903, 1, section 1 (1908) [especially chapter 2], and section 2, Part 2, chapter 2, Part 3, chapter 1 (1913).

<sup>8</sup>A. H. R. Goldie, Trans. Roy. Soc. Edin., 57, 143-177 (1931).

so placed and of such strength; alternate current-distributions remain possible (cf. paragraph 2), but the further their supposed location is from that calculated for the linear current, the more complicated their character becomes; if placed at a considerable distance away, they may involve an improbable distribution of current-bands of opposite sign.

This can be illustrated by means of the considerations described in paragraph 2. For simplicity we will consider only the symmetrical part of the current in the auroral zone, which flows continuously all round the zone. Such a concentration of current in the system of Figure 3 requires the presence, in the spherical harmonic expression of the field and current-system, of a cluster of terms of high degree, whose effects are additive near the auroral zone, while elsewhere they nearly nullify one another. If the current-system is located at a considerable distance above the Earth, outside our atmosphere, these terms gain greatly in importance relative to those of lower order, and their own relative magnitudes become modified so that they no longer combine so exclusively in auroral latitudes, and neutralize one another elsewhere; hence they will then represent a highly banded distribution of current round the parallels of latitude. This must certainly be regarded as a less probable type of current-system than the approximately linear current along the auroral zone in the atmosphere.

8. The intensity of the current along the auroral zone is not uniform; it increases from zero at two points  $A, A'$  which in Figure 4 are at about 2 p. m. and 10 p. m., to maxima at two points  $B, B'$  which in Figure 4 are at 6 a. m. and 6 p. m. (on the actual Earth these times may be displaced by a few hours). Thus, all along the zone, except at the maxima, current must be entering or leaving. It is important to know whence comes this current and whither it goes.

Birkeland believed that current came into the zone from outer space, approximately along the Earth's lines of magnetic force, and that it left the Earth again, along the lines of force, after flowing along the zone for some distance. The inflow, whencesoever it comes, must be continuous over the portion  $BAB'$ , while there must be outflow over the remaining part of the zone,  $B'A'B$ . Birkeland's view would require that the closed circuit of any elementary tube of this current should be completed by a union, somewhere outside the Earth, of the further ends of the inflowing and outgoing current, possibly greatly diffused and far away; only the nearer parts of the circuit would contribute much to the  $D$ -field.

This system of current intake and outflow for the auroral zones may be contrasted with that corresponding to the spherical current-sheet that could also account for the observed  $D$ -field. Figures 2 and 4 represent the auroral zone as forming an intense and concentrated part of current-circuits, some of which are interzonal, while in others the flow is across the polar caps within the zones. Of course there are other possible current-systems intermediate between the spherical one and that envisaged by Birkeland; for example, part of the zonal currents might be supplied as he suggested, while the remaining part flows in the Earth's atmosphere, as in Figures 2 to 4. The details of the current-distributions that correspond to the various theoretically possible situations for the current are not clear without examination; for example, it is perhaps doubtful whether a current-system of the kind suggested by Birkeland,

including concentrated currents in the atmosphere along the zone, could explain the observed  $D$ -field, particularly in the center of the polar caps, without involving some improbable requirements as to the currents at a distance from the Earth. We may imagine the current-sheets of Figure 4 gradually distorted so that while their concentrated zonal currents remain *in situ*, the two polar sheets, or the interzonal sheet, or all three of these current-sheets, are moved outwards, each polar-cap sheet being stretched upwards over the diameter  $BB'$  so as to form two sheets, with downward inflow over the part  $BB'A$ , and upward outflow over  $B'A'B$ . It seems to me likely that the current-distribution in these sheets would have to be somewhat singular in order to yield the smooth distribution of the  $D$ -field that appears to exist near the center of the polar caps. On the ground of *a priori* probability of simplicity, therefore, the spherical current-sheet distribution over the polar cap seems preferable to the Birkeland theory; but perhaps this argument should not be too strongly pressed.

A more important question is whether free currents of the Birkeland type can occur in nature, or whether they could endure sufficiently long if they were started, at a reasonable distance from the Earth. This question is one that I hope to discuss in a separate paper. If we assume, however, that the  $D$ -field within the polar cap and in the interzonal region, not too near the auroral zone in either case, is due to an atmospheric current-sheet only a few hundred km above the ground, it is still possible that the total current which their currents could supply to the auroral zone would not account for the observed currents there; in that case we should gain a definite indication of the necessity for some outside supply to the zone; similarly, if the supply from the current-sheets outside the zone was excessive, that would imply the entry of opposed currents from outside. So far as I know, no attempt has hitherto been made to examine this point, or indeed, to estimate the strength of the  $D$ -currents outside the zone.

The known spherical-sheet current-distributions responsible for  $S$  and  $L$  have been derived from the observations of the  $S$  and  $L$  surface-fields by a somewhat complicated mathematical process—spherical harmonic analysis. Their general nature and main circuits can however be divided by simple inspection from the daily variation-curves for the three magnetic elements from many stations; the same applies to the  $D$  spherical-sheet also, which cannot be represented by any simple series of spherical harmonic terms (cf. 2, paragraphs 10-15). Fortunately it is possible also to make an approximate *quantitative* determination of the total current in the various circuits of the  $D$ -field in a manner which, though quite simple, does not seem to have been described hitherto. The method will be illustrated, and its accuracy tested, by first applying it to the  $S$ -field, and comparing its results with those derived from the spherical harmonic analysis.

9. Consider the daytime current-systems at the equinoxes (Fig. 1). They encircle points ( $A, A'$ ) in each hemisphere on the meridian 11 a. m., in latitude  $40^\circ$ . On this meridian the horizontal-force  $S$ -variation attains its maximum (between latitudes  $40^\circ$  north and south) or minimum (beyond these latitudes); at latitude  $40^\circ$  the horizontal-force variation is reversed. The east-force daily-variation passes through one of its zeros at the same hour, 11 a. m. The maxima or minima of horizontal force

occur, for each circle of latitude, at the points where the current-flow is wholly eastward or westward, as is the case on the 11<sup>h</sup> meridian  $AA'$ . From  $A$  to  $A'$  the current-flow is eastward, and its intensity  $i$  rises from zero at  $A$  or  $A'$  to a maximum  $i_0$  at  $A_0$  on the equator; near this maximum the current-intensity  $i$  varies slowly from point to point in the sheet, and for points outside the sheet, at a small distance above or below  $A_0$ , the field of the sheet will be nearly the same as that of an infinite plane easterly current-sheet of intensity  $i_0$ —that is, it is horizontal, northerly, and of amount  $2\pi i_0$  (independent of the distance from the sheet if this is truly an infinite plane). This, however, is not the observed intensity  $F_0$  at the point on the ground immediately below  $A_0$ , because  $F_0$  contains a contribution due to the secondary currents induced in the Earth by the "primary" field of the atmospheric sheet. Suppose that a fraction  $f$  of  $F_0$  is due to the primary field, then  $fF_0 = 2\pi i_0$ , or

$$i_0 = fF_0/2\pi$$

The value of  $f$  depends on the conductivity of the Earth and the degree of the main spherical harmonics in the series-representation of the field. The conductivity has been determined from the analysis of the  $S$ -field, so that in using the value of  $f$  derived from this knowledge we are partly depending on the exact analysis; but this knowledge can be applied with fair accuracy to the  $D$ -field without, in that case, having analyzed the field mathematically (though with T. T. Whitehead and A. T. Price I have made such an analysis of part of the  $D$ -field). In the case of extensive current-circuits fairly uniformly distributed and covering a considerable fraction of the Earth's area (from 1/5 to 1/10, say), the value 0.6 may be considered a fair approximation to  $f$ .

Substituting this value of  $f$  in the above equation, and also  $F_0 = 20$  gammas, the value appropriate for  $S$  in the horizontal force at the equinoxes at the equator at 11 a. m., we find  $i_0 = 1.9 \times 10^{-5}$  c. g. s.  $= 1.9 \times 10^{-4}$  amp/cm or 21,000 amp per  $10^\circ$ -range in latitude. This may be compared with the value of about 23,000 indicated in Bartels' diagram (Fig. 1). To get the total current ( $I_{day}$ ) flowing round the daytime current-circuit, we require to know the position of the center  $A$  of this system, and the mean current-density  $\bar{i}$  between that point and the equator. The latter may be estimated by assuming that the current varies "parabolically" from its zero-values at the two circuit-centers to its maximum at the equator; this implies  $\bar{i} = (2/3)i_0$ . If  $\bar{i}$  were supposed to vary between these two points like  $\sin x$  between  $x=0$  and  $x=\pi$ , the factor  $\bar{i}/i_0$  would be  $2/\pi$ , which is not materially different. The latitude  $l$  of  $A$  is that of reversal of type of the horizontal-force daily-variation, namely, about  $40^\circ$  at the equinoxes ( $35^\circ$  in summer). Thus,  $I_{day} = \bar{i}l$ , where  $\bar{i}$  is reckoned in amperes per degree of latitude, or  $I_{day} = (2/3)i_0 l = (2/3) 2100 \times 40 = 56,000$  amperes. This is a reasonably good approximation to the value of 62,000 amperes derived from the spherical harmonic analysis.

The same method may be applied to the night current-circuits, whose centers  $B, B'$  lie in approximately the same latitude ( $40^\circ$ ). Taking the night maximum value of  $F$  at the equator as 12 gammas, we have  $I_{night} = (12/20) I_{day} = 33,600$  amperes (Fig. 1 shows 32,000).

The longitude of  $B$  and  $B'$  is 18<sup>h</sup>, as is best shown by the vanishing of the variation of the east force from the mean at that hour in latitude.

Between  $A$  and  $B$ , in latitude  $l$ , there flows northward (between  $11^h$  and  $18^h$ ) the combined current of the two circuits, namely, 89,600 amperes according to our approximate calculations, or 94,000 according to Figure 1. This combined value can be inferred independently from the east-force variation in latitude  $l$ . The distance in cm between  $A$  and  $B$ ,

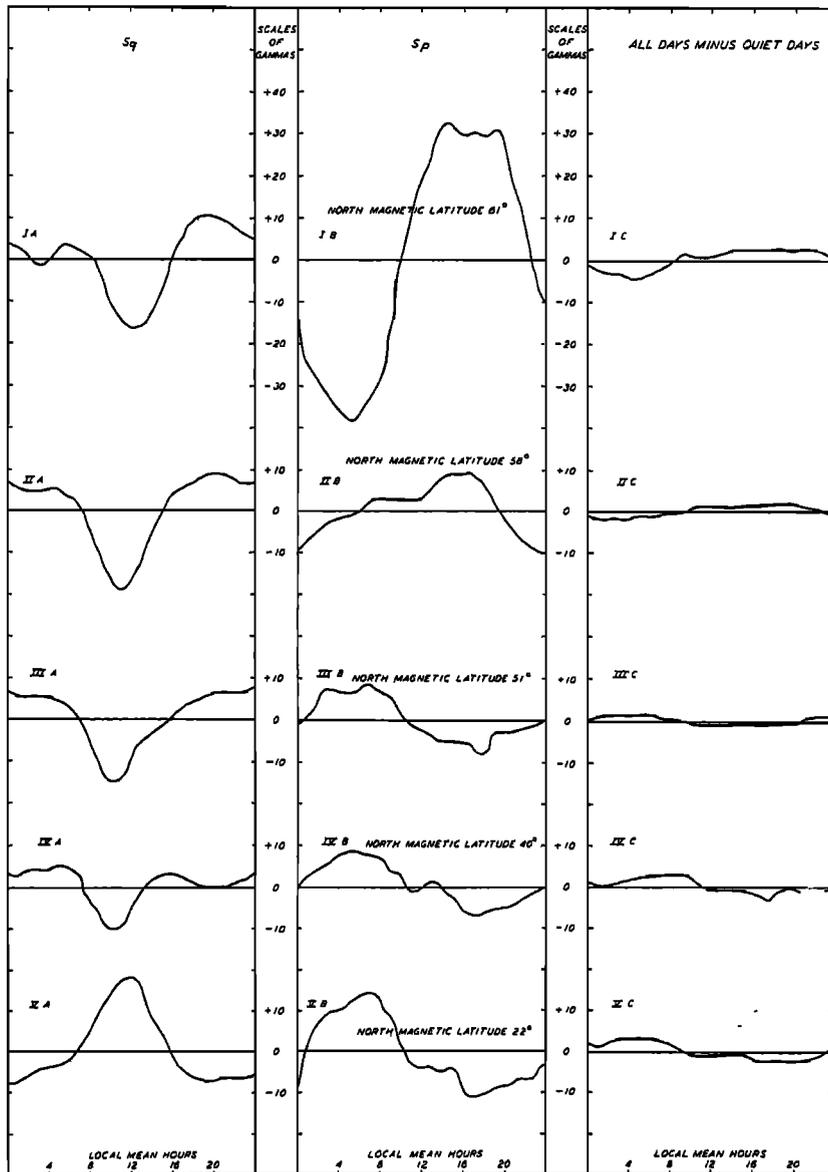


FIG. 5—MAGNETIC DIURNAL VARIATION OF HORIZONTAL FORCE AT FIVE GROUPS, I TO V, OF OBSERVATORIES IN NORTHERN HEMISPHERE; SECTION A FOR QUIET DAYS ( $S_q$ ), SECTION B FOR ADDITIONAL VARIATION ON STORM-DAYS ( $S_p$ ), SECTION C FOR (ALL DAYS MINUS QUIET DAYS)

in latitude  $40^\circ$  and  $7^h$  apart in longitude, is  $8.9 \times 10^8$ . The value  $F_0$  (now eastwards), from the east-force daily-variation, is about 16 gammas, giving  $i_0 = 1.5 \times 10^{-4}$  amp/cm, and  $\bar{i} = 1.0 \times 10^{-4}$  amp/cm. Hence,  $I = 1.0 \times 10^{-4} \times 8.9 \times 10^8 = 89,000$  amperes, in reasonable accord with the value of 89,600 based on horizontal-force data alone.

The northward current between  $B$  and  $A$  is smaller in intensity and covers a far greater range in longitude, namely, 17 hours. Its distribution, with the intensity fading away towards the dawn meridian, can be approximately inferred from the east-force variation during the night. The departure of east force from the daily mean is almost zero in the 10 1/2-hour interval between  $17\ 1/2^h$  and  $4^h$ , and the current-density must be very small during this interval, as shown by Fig. 1. Over the 7-hour interval  $4^h$  to  $11^h$ , the maximum divergence of east force from the mean is 14 gammas, giving a total southward current of  $(14/16) \times 89,000 = 78,000$  amperes (by comparison with the northward current between  $11^h$  and  $18^h$ , covering an equal distance in longitude but with a larger  $F_0$ ); the remaining 11,000 amperes must be spread over the long interval  $17\ 1/2^h$  to  $4^h$ . This is in reasonably good agreement with Figure 1.

The present method of approximately determining the intensity of a spherical current-system corresponding to a given surface-field should not supersede the much more accurate method of spherical harmonic analysis, where this is practicable, as is the case for  $S$  and  $L$ . Even for these, however, it may usefully supplement the analytical method; not much is gained by taking more than a few terms in the harmonic expression for the potential, yet the few main harmonic terms fit the derived data only moderately well. It would be worth while to consider the part of the daily variations, at a number of observatories, which remains after subtraction of the part represented by the harmonic series, and from this residual part to determine the corresponding current-system, in the above manner. This could then be combined with the current-system corresponding to the harmonic series, to give the whole  $S$  current-system. It would, of course, vary throughout the day. The same method might usefully be applied to  $L$ .

10 The above method will now be applied to determine the current-flow in the  $S_D$  current-system (Fig. 2). The observational material on which the following considerations are based is shown in Figures 5 to 7, reproduced here from paper 2. Five groups of observatories, here referred to as I to V, have been represented. In each of the figures, the left-hand section, marked  $a$ , shows the curves for diurnal variation  $S_q$  on quiet days (five per month); the center section, marked  $b$ , shows the curves for the additional diurnal variation  $S_D$  during the first days of 40 selected magnetic storms [that means, the total diurnal variation on these storm-days was  $(S_q + S_D)$ ]; the right-hand section, marked  $c$ , shows, in the average for a number of years, the difference of the diurnal variations on all days minus that on quiet days. All curves correspond to the average conditions throughout the year.

The centers of the two (a.-m. and p.-m.) circuits between the auroral zones are at local times  $6^h$  and  $18^h$  approximately. Their latitude  $l$  can be inferred from the  $S_D$ -curves given in Figure 5, section  $b$ , for the first day of the series of moderate magnetic storms considered in 1. The lati-

tude of reversal of the  $S_D$ -variation in horizontal force occurs between Curves II*b* and III*b*; the former is for Pavlovsk (magnetic latitude  $58^\circ$ ) and the latter for the mean of Pola, Potsdam, and Greenwich (mean magnetic latitude  $51^\circ$ ); thus  $l$  may be taken as  $55^\circ$ , the distance from the equator being  $6.1 \times 10^8$  cm. Curve V*b* of Figure 5 gives the maximum horizontal-force divergence from the mean at magnetic latitude  $22^\circ$  as

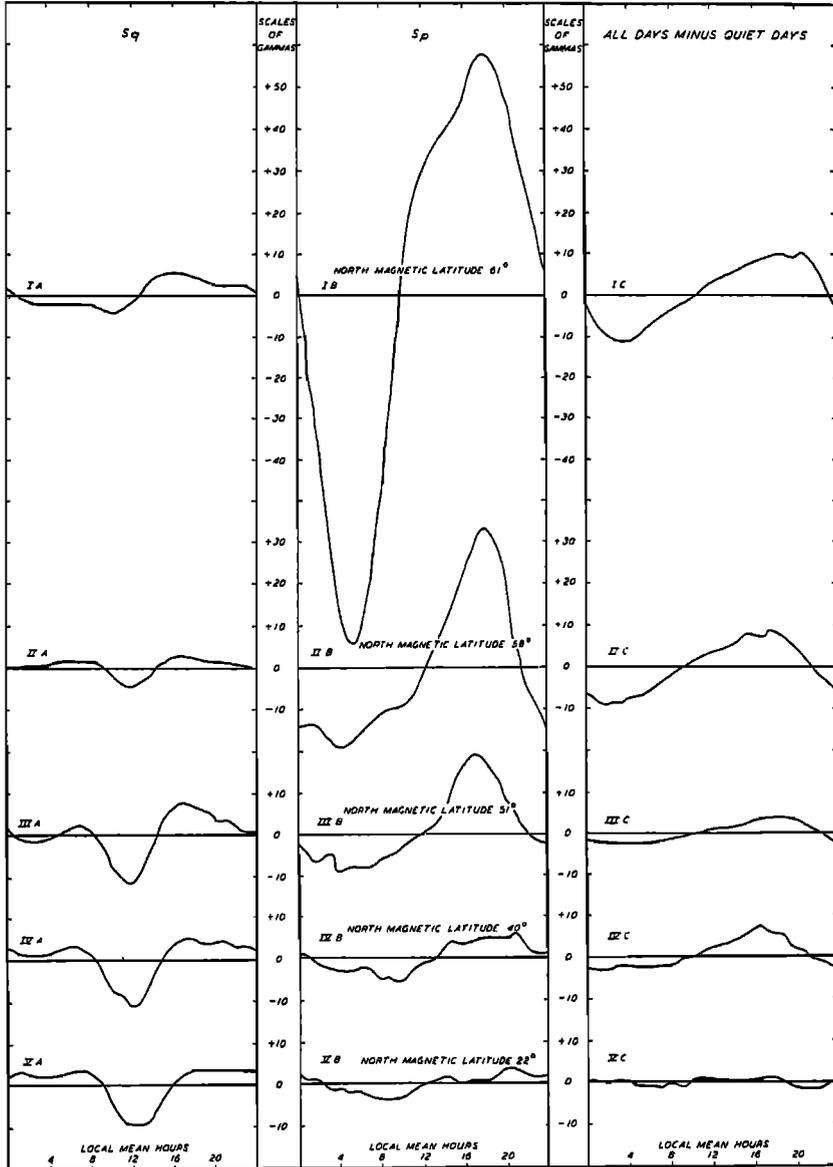


FIG. 6—MAGNETIC DIURNAL VARIATION OF VERTICAL FORCE FOR SAME GROUPS AS IN FIGURE 5

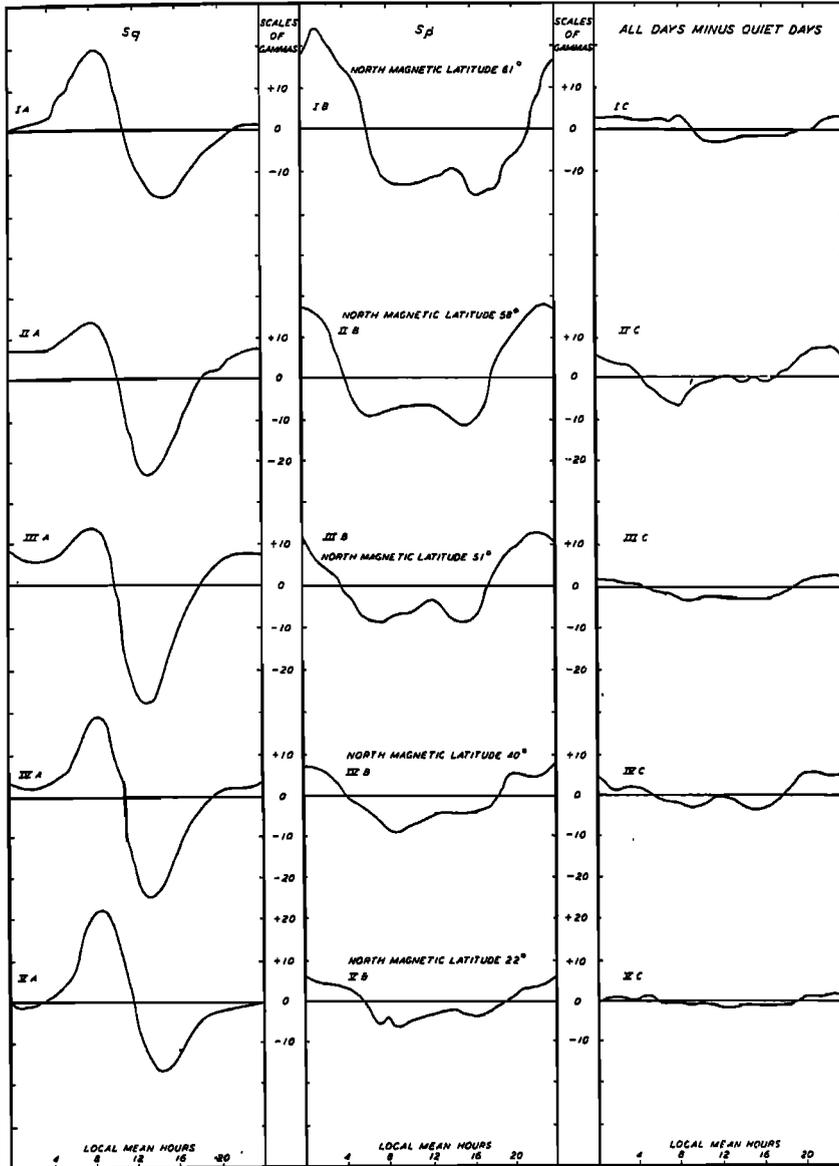


FIG. 7—MAGNETIC DIURNAL VARIATION OF EAST DECLINATION FOR SAME GROUPS AS IN FIGURE 5

12 gammas, and the equatorial value may be estimated as 13 gammas. This gives  $i_0 = 1.2 \times 10^{-4}$  amp/cm,  $\bar{i} = 8.3 \times 10^{-5}$  amp/cm, and  $I = 51,000$  amperes, for any one of the four current-circuits between the two auroral zones.

This total current can also be estimated from the  $S_D$ -variation in the east force at latitude  $l$  ( $55^\circ$ ); estimating  $F_0$  as 13 gammas, from curves IIb and IIIb of Figure 7, we have  $i_0 = 1.2 \times 10^{-4}$  amp/cm,  $\bar{i} = 8.3 \times 10^{-5}$

amp/cm, and  $I=48,000$  amperes, the width of the northward or southward current-flow being  $5.8 \times 10^8$  cm (namely,  $90^\circ$  of longitude on the  $55^\circ$  latitude-circle). This is in reasonably good agreement with the quite independent estimate from the horizontal-force variations. In round figures, the total current in each  $S_D$  current-circuit may be taken as 50,000 amperes.

As might be expected from a comparison between the magnitudes of the  $S_q$  and  $S_D$  daily variation-curves ( $a$  and  $b$  respectively in Figs. 5 to 7), this is about equal to the average of the currents in the night and day circuits of the  $S_q$ -system [namely, half of  $(62,000+32,000)$  or 47,000 amperes; cf. Fig. 1] in a sunspot-minimum year. In a sunspot-maximum year the  $S_q$ -currents are stronger, just as the  $S_D$ -currents are stronger during more intense storms.

The curves ( $c$ ) in Figures 5 to 7 refer to  $S_D$  on ordinary (all minus quiet) days, that is, to the average amount of disturbance present on five out of six days (one-sixth of all days being selected as quiet) over a period of years. The ratio of the amplitudes of corresponding curves ( $b$ ) and ( $c$ ) differs somewhat from one curve to another, presumably because of accidental irregularities in the data, but the average ratio seems to be about five, implying that these four  $S_D$  current-circuits on ordinary days each carry a total current of about 10,000 amperes. This is only a small fraction of the  $S_q$  current-strength; it is comparable with the strength of the main (daytime)  $L$  current-circuit in summer.

11. The symmetrical (or storm-time) part of the disturbance-field between the auroral zones will next be considered. This is due to westerly currents flowing round the parallels of latitude, as illustrated in Figure 3. The reduction of the horizontal force at maximum-phase in the moderate magnetic storms of  $I$ , below the initial value of the horizontal force, is shown in Figure 8 to be  $[+9 - (-29)] = 38$  gammas at magnetic latitude  $22^\circ$  (left-hand curve). The reduction decreases, with increasing latitude, to about  $[+7 - (-15)] = 22$  gammas at latitude  $53^\circ$  (right-hand curve). Near this latitude the reduction reaches a minimum and further north it appears to increase rapidly towards the auroral zone. We will estimate the total westward current between the equator and  $55^\circ$ , the latitude of the center of the  $S_D$  current-circuits. Using a simple graph showing the horizontal-force reduction as a function of latitude, from the three curves of Figure 8, we find its value to be 20 gammas at  $55^\circ$ , 41 gammas at the equator, with a mean value of 34 gammas over this  $55^\circ$ -range. This gives  $\bar{i} = 3.25 \times 10^{-4}$  amp/cm, flowing across a section of width  $6.1 \times 10^8$  cm; hence  $I = 198,000$  amperes, or, in round figures, 200,000 amperes. This great current is four times as strong as the current in each  $S_D$ -circuit.

The combination of the symmetrical current-flow with the  $S_D$  current-circuits gives the combined current-flow, between  $\approx 55^\circ$  latitude, shown in Figure 4. On the  $18^h$ -meridian its total amount is 250,000 amperes, on the  $6^h$ -meridian 150,000 amperes, between the equator and  $55^\circ$  latitude, or twice these values, 500,000 and 300,000 amperes, between  $\approx 55^\circ$  latitude. The difference, 100,000 amperes in each hemisphere, represents the decided asymmetry of the interzonal current-system, already referred to in paragraph 6; this current flows across the  $55^\circ$  latitude-circle towards the auroral zone.

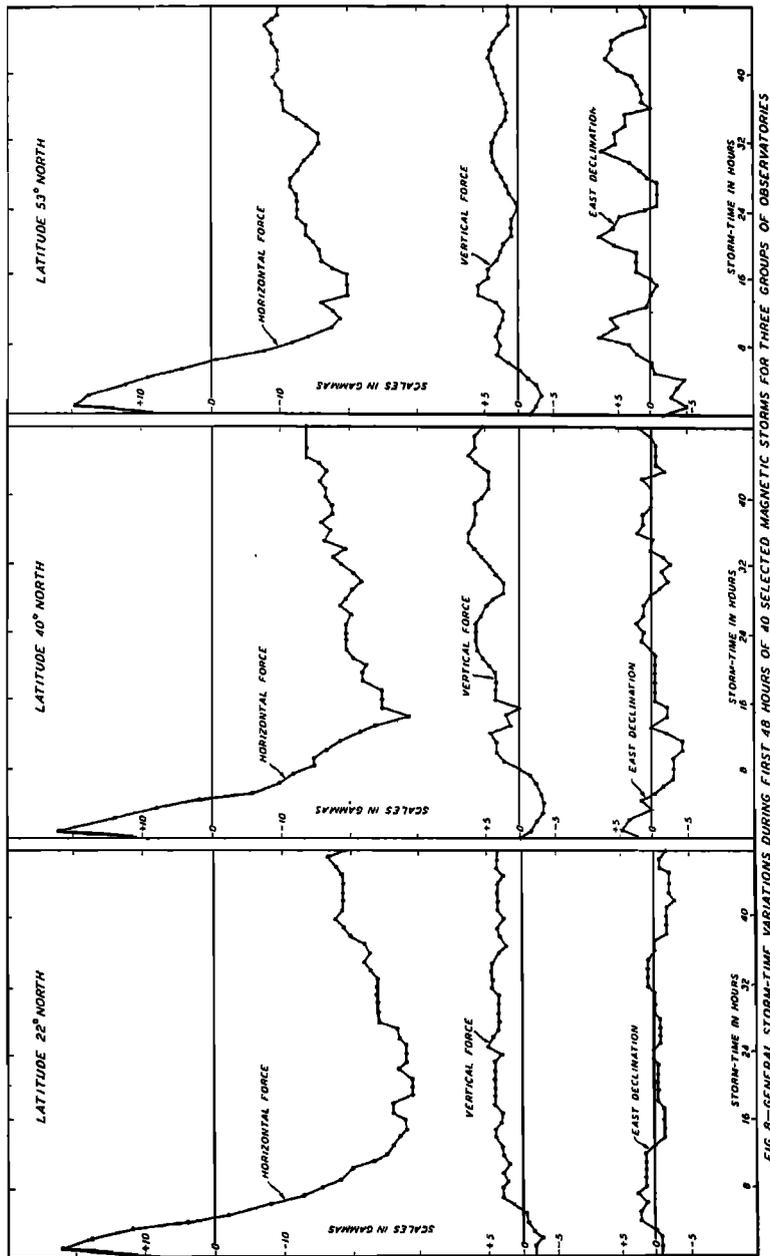


FIG. 8—GENERAL STORM-TIME VARIATIONS DURING FIRST 48 HOURS OF 40 SELECTED MAGNETIC STORMS FOR THREE GROUPS OF OBSERVATORIES

The average intensity of these currents on ordinary (all minus quiet) days is about one-fifth of the above values. After a prolonged period of magnetic calm the currents may decline to decidedly lower values.

12. We next consider the symmetrical part of the current-flow (Fig. 3)

from latitude  $55^\circ$  to  $70^\circ$ , and from this to the pole. The mean westward current-intensity probably has a sharp maximum between latitude  $55^\circ$  and  $70^\circ$ , from which it decreases towards the north, rapidly at first, to zero at the pole. Data for many periods of moderate magnetic storms not being available to me at the moment, the current-strengths on average (all minus quiet) days will be considered. The reduction of the daily mean horizontal force under the zone [(all minus quiet)-days] may perhaps be estimated as 20 gammas (cf. the Bossekop curve in Fig. 6 of paper 2); this may be contrasted with about four gammas at latitude  $55^\circ$ , and about 10 gammas at the equator. The maximum current-density of the westward (symmetrical) current in the zone  $55^\circ$  to  $70^\circ$  cannot on this account be estimated as just twice that at the equator, because near the auroral zone the current-density probably varies too rapidly with latitude for the current-sheet to be treated as if uniform and plane. But we are perhaps not likely to fall seriously into error if we estimate the total average westward current-intensity [for (all minus quiet)-days], between latitude  $55^\circ$  and  $70^\circ$ , as corresponding to that of a uniform plane current-sheet for which  $F_0 = 10$  gammas. This would give  $\bar{i} = 10^{-4}$  amp/cm, and  $I$ , across the  $15^\circ$ -belt of latitude between  $55^\circ$  and  $70^\circ$ , as 15,000 amperes.

If on days of moderate storms this current is magnified five times, as appears to be so for  $S_D$ , the corresponding value is 75,000 amperes.

Within the auroral zones, from latitude  $70^\circ$  to the pole, the storm-time current is still westward, presumably along the isochasms or parallels of magnetic latitude. The average reduction of horizontal force [(all minus quiet)-days] may be estimated as 10 gammas, giving  $I$ , for the  $20^\circ$ -belt, as about 20,000 amperes. On days of moderate storm it is perhaps five-fold, or 100,000 amperes. The estimates in the present section are less reliable than those made in paragraphs 10 and 11, because of the paucity of the data used, and because of the more rapid variation of intensity of the currents with latitude (which calls for a more refined treatment of the data). Unfortunately no check can be derived from the declination-data, as in paragraph 10. The vertical-force data should, however, aid in estimating these current-strengths.

13. We next consider the  $S_D$ -currents north of latitude  $70^\circ$ , for average (all minus quiet) days. Near the center of the polar cap the horizontal-force vector-diagrams are nearly circular, and uniformly described. Their radius is somewhat uncertain; note the difference between the curves for Kingua Fjord and Cape Evans in Figure 6 of paper 2, which, however, refer to quite different years. Here we shall assume that it is 30 gammas, which lies between the values for these two stations. As indicated in 2 (paragraph 14. 3), this type of diagram may be ascribed to a fairly uniform current-sheet in this region (cf. Fig. 7, plan, of paper 2). Its intensity  $i_0$  at the pole will be  $3 \times 10^{-4}$ ; this must decrease to zero near  $6^h$  and  $18^h$  in the auroral zones, where the current-direction is reversed; applying the usual factor to obtain  $\bar{i}$ , we find  $2 \times 10^{-4}$  amp/cm. Taking the limit of the sheet as latitude  $70^\circ$ , the whole (diametral) breadth of the cap is  $4.4 \times 10^8$  cm. Hence  $I$  across this breadth is 88,000 (or, say, 90,000) amperes.

On days of moderate magnetic storm this will be increased say five-fold, to 450,000 amperes; this estimate needs to be checked by reference to actual polar data for such days.

14. We have thus estimated that on average (all minus quiet) days there is a total current-flow northward into the latitude-zone  $55^{\circ}$  to  $70^{\circ}$  of amount 20,000 amperes (paragraph 10) and southward into this zone, of amount 90,000 amperes (paragraph 13); and that on days of moderate storm these estimates may be magnified five-fold.

Thus on average days 110,000 amperes flow into the zone, from the north and from the south, and this extra-zonal current-supply appears to divide and flow half eastward, half westward. These  $E$ - and  $W$ -currents will each carry 55,000 amperes, on which will be superposed the continuous westward current of 15,000 amperes. This will increase the westward current at  $6^{\text{h}}$  to 70,000 amperes and decrease that at  $18^{\text{h}}$  to 40,000 amperes. The estimated values on moderate storm-days are 350,000 ( $W$ ) and 200,000 ( $E$ ).

15. We have now arrived at an estimate of the extra-zonal current-supply to the auroral zone (or latitude-belt from  $55^{\circ}$  to  $70^{\circ}$ ). If this estimate was more reliable, and if reliable independent estimates of the actual  $E$ - and  $W$ -currents along the belt could be obtained from stations near the auroral zone, it would be possible to make a test of the hypotheses that the auroral zone does or does not receive current from free space (see the end of paragraph 8). This cannot be done here; the test must await a closer study of the magnetic data, especially those obtained during the Second International Polar Year—a task upon which E. H. Vestine is now engaged in cooperation with me. It is, however, worth while to make a very rough comparison of the extra-zonal current-supply with the estimates at present available for the currents along the zone.

16. Attempts to deduce the height and intensity of the currents along the auroral zone from their magnetic effects seem to have been first made by Birkeland<sup>9</sup>, who found intensities of the order of  $5 \times 10^5$  amperes at heights varying from 150 km to 600 km or more during magnetic storms not of outstanding intensity. Goldie also has made estimates of the zonal currents from the records of Eskdalemuir and Lerwick for ten of the greatest magnetic storms occurring in the year 1926. It is uncertain whether these were of greater or less average intensity than those considered in 1, but the fact that all the ten occurred in one year suggests that they were not much more than "moderate." The maximum current found from the mean of the ten storms was 595,000 amperes; this was a westward current, at height 290 km, and occurred at  $2^{\text{h}}$  local time. The maximum eastward current found was 480,000 amperes, at  $17^{\text{h}}$  local time; this current was nearly overhead at Lerwick<sup>10</sup> (magnetic latitude  $63^{\circ}$ ), and the estimate of its height was 370 km. These currents are of the same order as those mentioned at the end of paragraph 14, being, in fact, about twice as large; as there, the westward currents are the greater in magnitude, and occur in the morning hours, though at  $2^{\text{h}}$  instead of at  $6^{\text{h}}$  as in Figure 2; the eastward current found by Goldie occurs at  $17^{\text{h}}$ , or nearly at 6 p. m. as in Figure 2. This accordance is satisfactory as far as it goes, and if the storms considered by Goldie were on the average twice as intense as the average of the storms considered in 1, it would be natural that his currents should be about double those of paragraph 14;

<sup>9</sup>K. Birkeland, *The Norwegian Aurora Polaris Expedition, 1902-3*, vol. 1, pp. 306-311 (1908).

<sup>10</sup>The daily northward and southward motion of the zonal currents, apparently indicated by Goldie's results, may be due to the inclination of the magnetic to the geographic axis—a complication neglected in this paper.

this could be tested by finding the average storm-time variation for these storms at some low-latitude station. If, however, these storms were found to be less than twice as intense, there would appear to be a discrepancy between Goldie's zonal currents and the extra-zonal current-supply. It would be unsafe, however, to conclude from such a discrepancy that there was a supply of current from free space to the auroral zone. This is not only because of the rough nature of the results of paragraph 14, especially on account of the assumed value (five) of the ratio of the storm to the (all minus quiet)-day  $D$  current-system in polar regions; Goldie's estimates of the zonal currents are likewise uncertain, because they were derived without taking account of the influence of the induced internal currents.

17. There is need for further theoretical studies of the nature of the induced earth-currents, and of their magnetic field, in the polar regions. Until this is better understood, the determination of the zonal currents and their height will remain uncertain. It seems likely, however, that  $\Delta H_e$  and  $\Delta H_i$ , the external and internal parts of the (vector)  $D$ -change in horizontal force, will usually be similar in direction or sign, while  $\Delta V_e$  and  $\Delta V_i$ , for the vertical force, will have opposite signs. If so, the inclination of the resultant  $D$ -vector  $\Delta F$  to the horizontal plane will be less than that of its external component. The situation of the zonal current is found by drawing, from each of two stations  $A, A'$ , say (cf. Fig. 9), a line ( $l$  or  $l'$ ) in the vertical plane containing  $\Delta F$  at each station, perpendicular to  $\Delta F$ . Assuming that the disturbance is due to a hori-

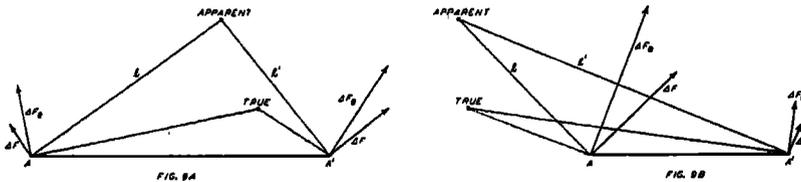


FIG. 9—ESTIMATING HEIGHT OF LINEAR CURRENT IN AURORAL ZONE FROM DRAWING PERPENDICULARS TO MAGNETIC DISTURBING FORCES OBSERVED AT TWO STATIONS ON OPPOSITE SIDES (FIG. 9A) OR ON THE SAME SIDE OF THE AURORAL ZONE (SECTION PERPENDICULAR TO AURORAL ZONE;  $\Delta F$ , TOTAL DISTURBING FORCE, GIVES APPARENT HEIGHT TOO GREAT;  $\Delta F_e$ , EXTERIOR PART OF DISTURBING FORCE, GIVES TRUE HEIGHT)

zonal linear atmospheric current between  $A$  and  $A'$ , this must lie along the common perpendicular to  $l$  and  $l'$ , if there is no induced field; the presence of the latter requires that  $l$  and  $l'$  be drawn normal to  $\Delta F_e$  instead of to  $\Delta F$ . The usual incorrect procedure gives too great a height for the current. The same is probably true in most cases when the situation of the current is deduced from two stations on the same side of the zone (Fig. 9) though it may not be desirable to use two such stations for the purpose.

Neglect of the internal field may also invalidate this estimate of the current-strength  $I$ , which is likely to be in excess. To take a simple illustration, suppose one station  $A$  is directly below the current:  $I$  will be equal to  $(1/2) h \Delta H_e$ , where  $h$  denotes the height of the current and  $\Delta H_e$  its magnetic effect at  $A$ . If  $\Delta H$  is used in this expression in place of  $\Delta H_e$ , which is less than  $\Delta H$ , then  $I$  will be over-estimated on this account;  $h$  also is over-estimated, so that  $I$  is still more an over-estimate. This may be partly the cause of the excess of Goldie's current-values over those mentioned at the end of paragraph 14.

18. The results obtained in paragraphs 10 to 14 are illustrated in Figures 10 to 15, which are improved quantitative versions of the Figures 2 to 4. The current-flow is indicated in each figure by lines, between each of which there flow 10,000 amperes, as in Figure 1. In the auroral

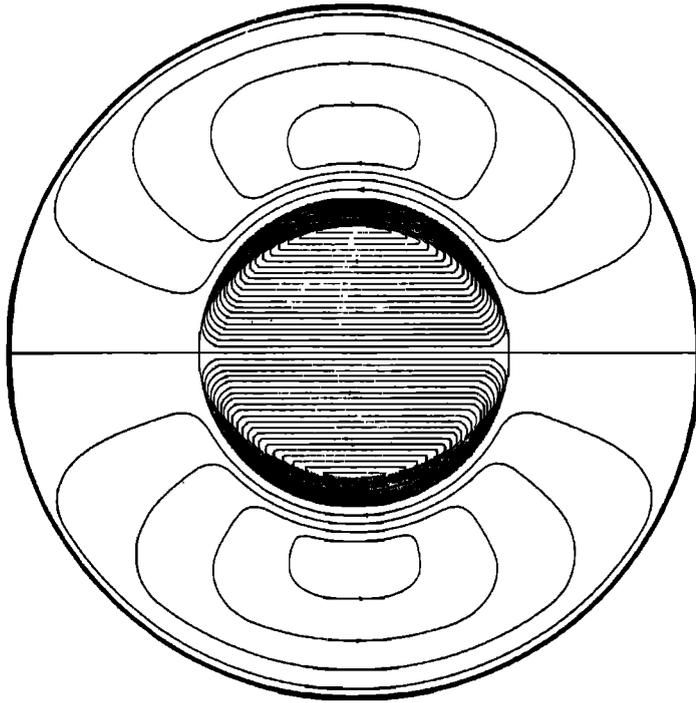


FIG. 11—VIEW FROM NORTH POLE

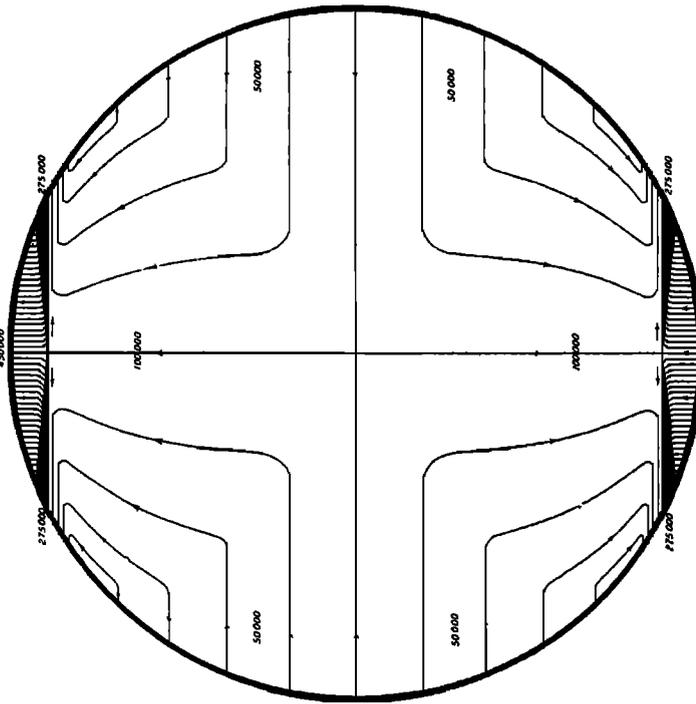


FIG. 10—VIEW FROM SUN

FIGS 10-11—QUANTITATIVE VERSION OF FIGURE 2—NON-SYMMETRICAL PART 5D  
(10,000 AMPERES FLOW BETWEEN SUCCESSIVE CURRENT-LINES IN FIGURES 10-11)

zone the lines are so crowded together that they cannot be distinguished individually; the increase of the zonal current from 0<sup>h</sup> or 12<sup>h</sup> to 6<sup>h</sup> or 18<sup>h</sup> is indicated by a crowding of lines, giving a wedge-like appearance to

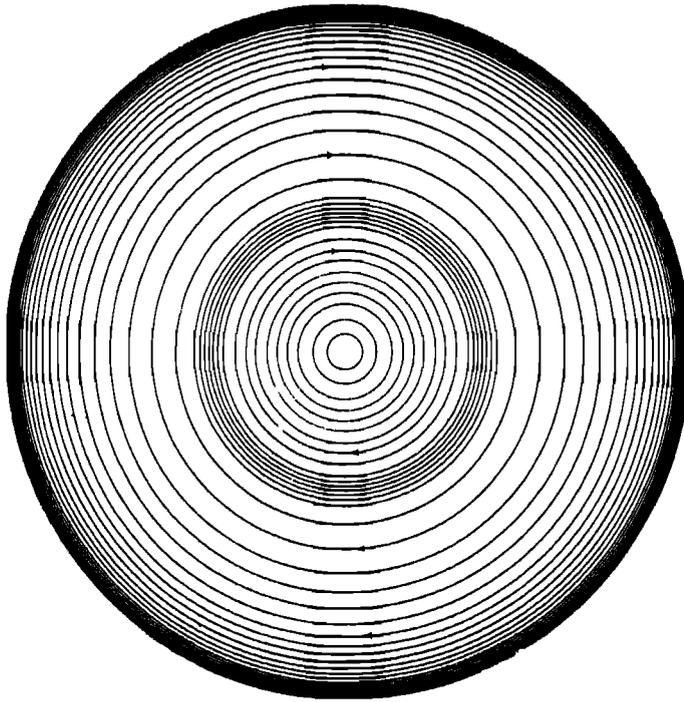


FIG. 13—VIEW FROM NORTH POLE

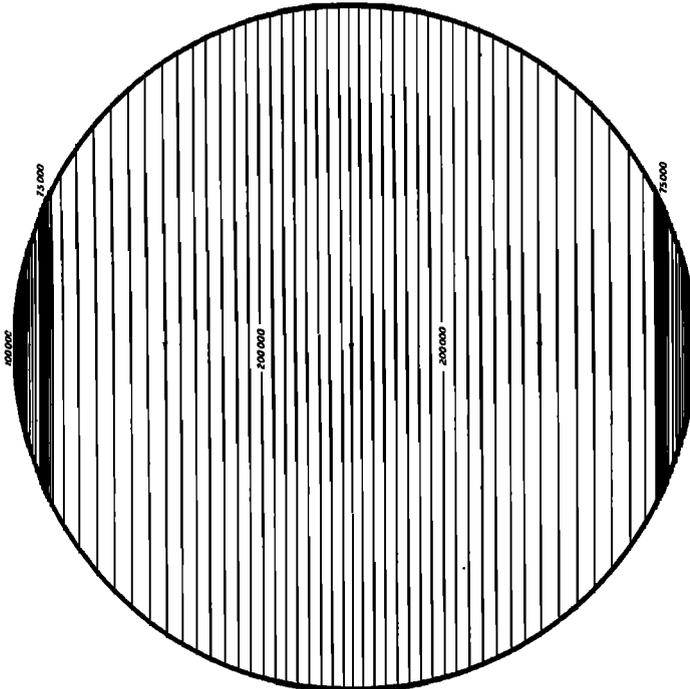


FIG. 12—VIEW FROM SUN

FIGS. 12-13—QUANTITATIVE VERSION OF FIGURE 5—SYMMETRICAL OR STORM-TIME PART

the zones which must not be interpreted as a real feature of them. The current-flow in each part of the system is indicated in the figures for days of moderate magnetic storm (at maximum storm-time phase); the

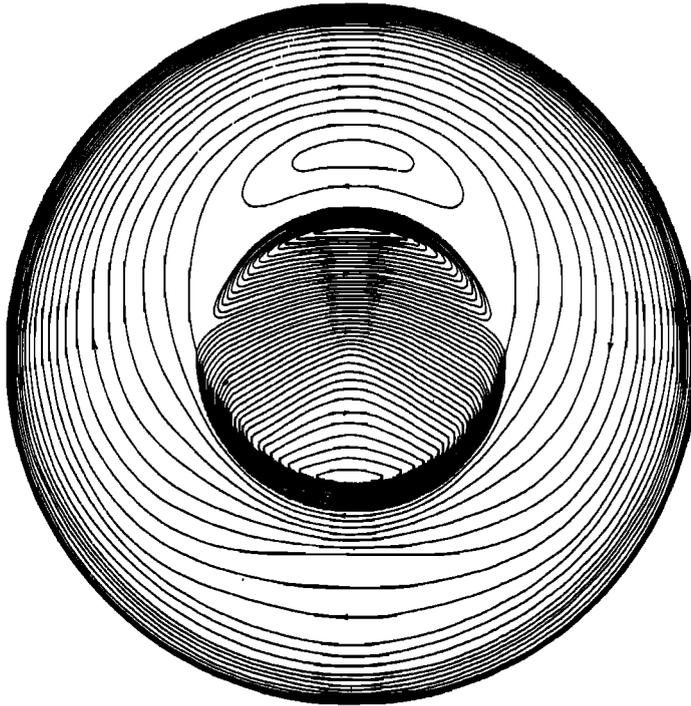


FIG. 15—VIEW FROM NORTH POLE

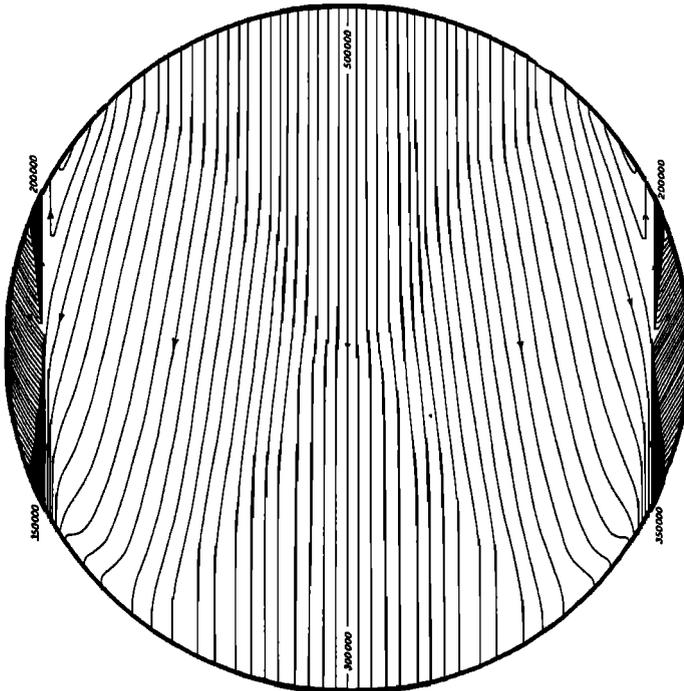


FIG. 14—VIEW FROM SUN

FIGS. 14-15—QUANTITATIVE VERSION OF FIGURE 4  
[COMPLETE D CURRENT-SYSTEM ABOUT 16 HOURS AFTER OUTBREAK OF MAGNETIC STORM]

corresponding figures for average (all minus quiet) days<sup>11</sup> may be taken as

<sup>11</sup>Average here includes the quiet days themselves, which (by convention) number five per month. If these are excluded from the average, the lower set of current-figures should be increased in the ratio 6/5. Magnetic storms are so relatively infrequent that they contribute only a small part to the 'average'-day disturbance and currents.

about one-fifth of these. In Figures 10, 11, 14, and 15 the zonal currents are given as calculated on the assumption that there is no current-supply to the zones from outside.

19. The currents shown in Figures 10 to 15, though amounting to 750,000 amperes from pole to pole in Figure 12, are at times very materially exceeded. One of the greatest storms on record occurred May 15, 1921; unfortunately most magnetic observatories lost a large part of their records on this remarkable day, but at Samoa a decrease of horizontal force by 800 gammas was observed during the first six hours of the storm (the rapid attainment of the maximum-phase is in accordance with the result found in paper 1 for great magnetic storms). At Samoa this maximum-phase occurred in the afternoon, so that part of the decrease was doubtless due to  $S_p$ . If, as seems reasonable, 600 gammas were due to the symmetrical current-system (Fig. 12), the intensity of the system would then have been 15 times that calculated here (assuming 41 gammas, see section 11) for moderate magnetic storms. It would be interesting to know to what degree on this occasion the relative proportions between the different parts of the current-system shown in Figures 10 to 15 were preserved. One striking fact about the storm of May 1921 was that the auroral zone, which expands equatorwards the more intense the disturbance, approached the zenith of Potsdam; consequently Potsdam experienced a very large vertical-force change, that is, a decrease of 400 gammas to a minimum at about 2<sup>h</sup> local time; this was undoubtedly of diurnal-variation type, such as is normally shown in the early morning hours by stations near to, and just outside, the auroral zone (Fig. 6, curve *IB*); a large part of the morning variation in horizontal intensity on May 15, 1921 was also of the  $S_D$ -type for such a station. In the afternoon at Potsdam the variations were much more like those usually experienced there during magnetic storms; the auroral zone would seem to have by then retreated far to the north of Potsdam.

It is a pleasure gratefully to acknowledge the skilled assistance of Mr. W. C. Hendrix in the preparation of the diagrams of this paper.

### Summary

Magnetic disturbance is due to electric current-systems above the Earth's surface, and to secondary induced currents within the Earth. The location of the external currents cannot be uniquely inferred from magnetic observations made at the Earth's surface. Additional considerations are required to resolve this difficulty; knowledge of the distribution of ionized gas above the Earth, such as radio observations afford, is one principal means, while another is the *a priori* probability that the current-system is fairly simple. The question whether or not the currents flow wholly in the atmosphere is discussed, and it is concluded, though not decisively, that the evidence favors this view. The strength of the various parts of the current-system is evaluated, and among other results it is shown that on days of moderate magnetic storm the total additional electric current flowing westward round the Globe, between the North and South poles, rises to 750,000 amperes.

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