

Reflections on Error Bars and New Data for IDV13

Leif Svalgaard, October 2013

In this note we reflect on the possible errors in IDV and on the impact of newly acquired or re-processed data. In assessing the error in HMF *B* derived from IDV we must deal with the following sources of errors in IDV itself:

1) The intrinsic error in the variation of H has varied with time. From about the Second International Polar Year in the 1930s on, the error has generally been less than 1 nT because of the introduction of unifilar magnetometers to replace the old bifilar instruments. This paper by Eschenhagen [Terrestrial Magnetism and Atmospheric Electricity. Volume 5, Issue 2, pages 59–62, June 1900] describes the progress making this possible:

“For the measurement of the variations of the Earth's magnetic force, advantage is taken of a comparison with a compensating force; e.g., the magnetism of a bar and the directive force of the bifilar suspension. Both forces mentioned are not free from changes in the course of time, and are greatly affected by temperature. The force of torsion of a unifilar suspension has only been applied with advantage since the invention of quartz fibers. The systems of magnets and suspensions, which weigh at least 20–30 grams usually employed with magnetometers are not suited for quartz fiber suspensions, as too strong fibers must be used. Magnetometers have, therefore, been built at my request by the firm of O. Toepfer, of Potsdam, which are provided with a light system of magnets (weighing about 1.5 g.), and possess several advantages over the instruments used hitherto. Among these are to be mentioned: easy accessibility to the system of magnets, clamping of the lower suspension, so that slight displacements of the mirrors can be made; moreover, a damping adjustable at pleasure; finally, a fixed mirror for photographic registration, which can be displaced with ease and certainty. The system of magnets carries two mirrors, inclined at a certain angle to each other, so that the range of registration is increased twofold.”

The care exercised in measurements, data reduction, and publication obviously also plays a role and some observatories have a poor track record in that respect, e.g. Tokyo [TOK, 1897-1912], such as to render them unsuitable for IDV. But, by and large, intrinsic uncertainty is not an issue for data later than ~1933 and for some stations even back to 1900 and beyond. In a later section we shall address the situation before 1900.

2) The statistical error arising from using only one [preferably midnight] hour out of the 24 hours of a day. As the global ring current activity is sampled at different times at different stations, additional [extraneous] variability is introduced. Use of all 24 hours, as for the *u*-measure, could ameliorate this somewhat, but introduces problems of its own, such as the variability of the ‘regular’ diurnal variation. In addition, many 19th century stations did not observe at all 24 hours. A distinct advantage of using only one hour is that all stations will have the same ‘sampling’ error, regardless of how many hours [which itself varies over time even for the same station] were observed during the day. When for a year there are several stations contributing to IDV we can compute the Standard Deviation of the values going into IDV for that year. Figure 1 shows the result back to the 1880s [where we have more than 5 stations per year]:

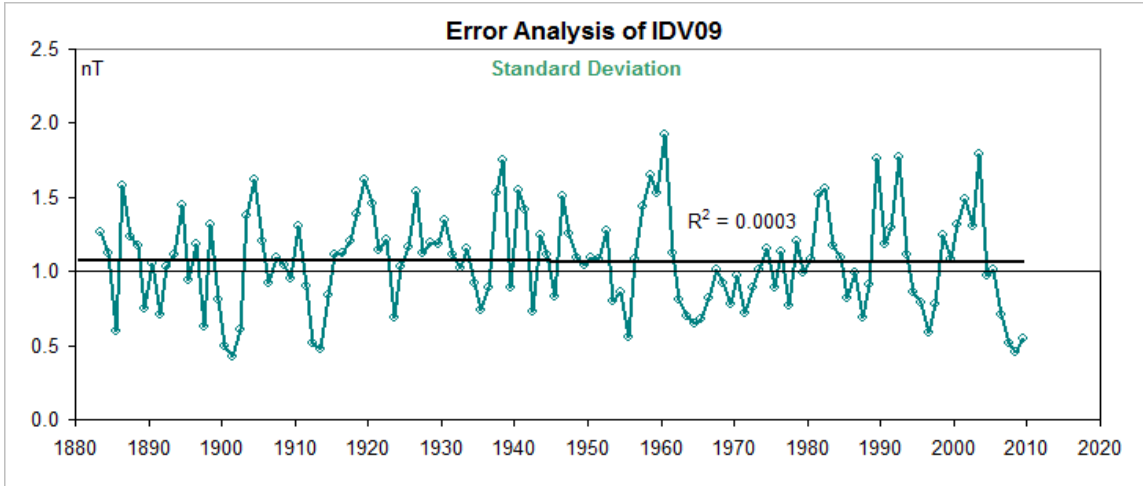


Figure 1: Standard Deviation of IDV09.

The average SD is 1.07 nT and there is no systematic variation with time, but, clearly, with IDV itself as evidenced by the solar cycle-like variation (made explicit in Figure 2). Both these properties are what we would fully expect:

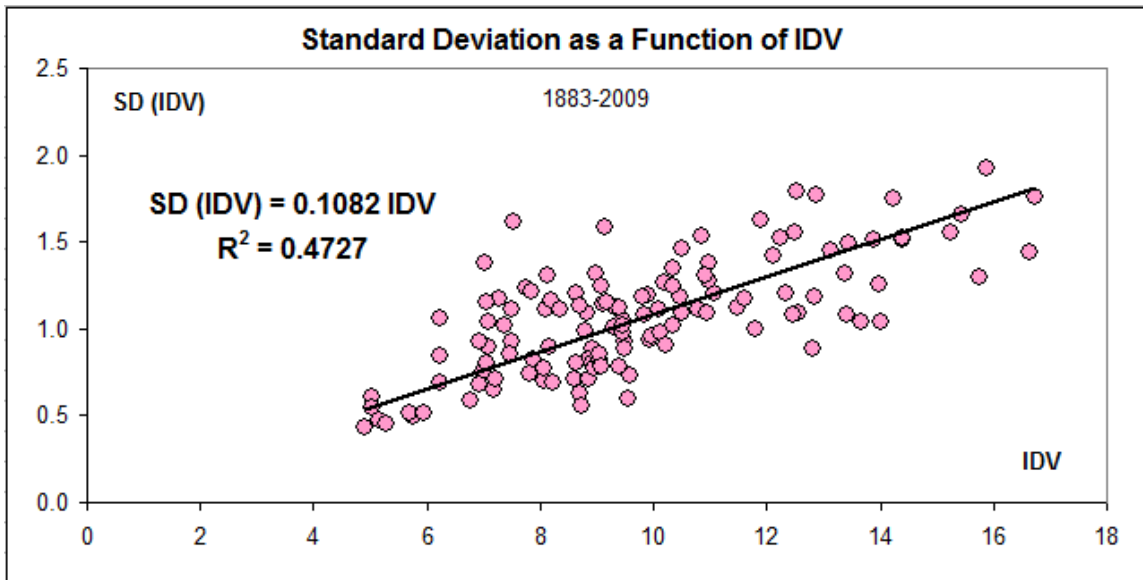


Figure 2: Standard deviation of observed values of IDV for each year of 1883-2009 plotted against average IDV for that year.

We shall posit that a conservative $2\text{-}\sigma$ error band ($\pm 1\text{-}\sigma$) is on average $\text{IDV} \pm 0.1082 \text{ IDV}$, i.e. that there is a 68% change that a given station in a given year would yield a value for IDV within the band. For an average IDV of 10 nT, the 68% error-band is 2.2 nT wide, the 95% band is 4.3 nT wide. This can also be discerned directly from Figures 7 and 17 from SC10 and from Figures 3 and 4 from SC05.

Figure 3 shows average IDV [blue curve] surrounded by the $2\text{-}\sigma$ error band [pink curves]. An intuitively obvious, and trivial, but important observation is that the error band is narrow for low values of IDV and broader for high values of IDV:

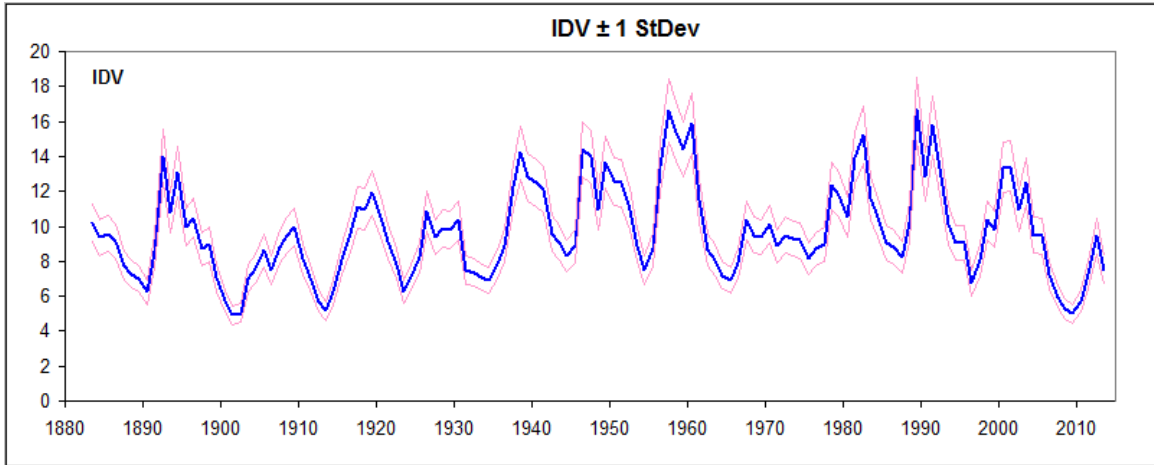


Figure 3: IDV and error band [pink curves] for each year of 1883-2013

If we assume that there exists an underlying actual physical quantity [the ‘real’ IDV] the average IDV should approach that ‘real’ IDV as the number of stations over which the average is taken increases. The Standard Error on the average [‘true’] IDV would be estimated by $SE = SD / \sqrt{N}$, where N is the number of stations. N ranges from 5 in 1883 to 60 in 1999. Since N is so large, the standard error of the mean is negligible [Figure 4]:

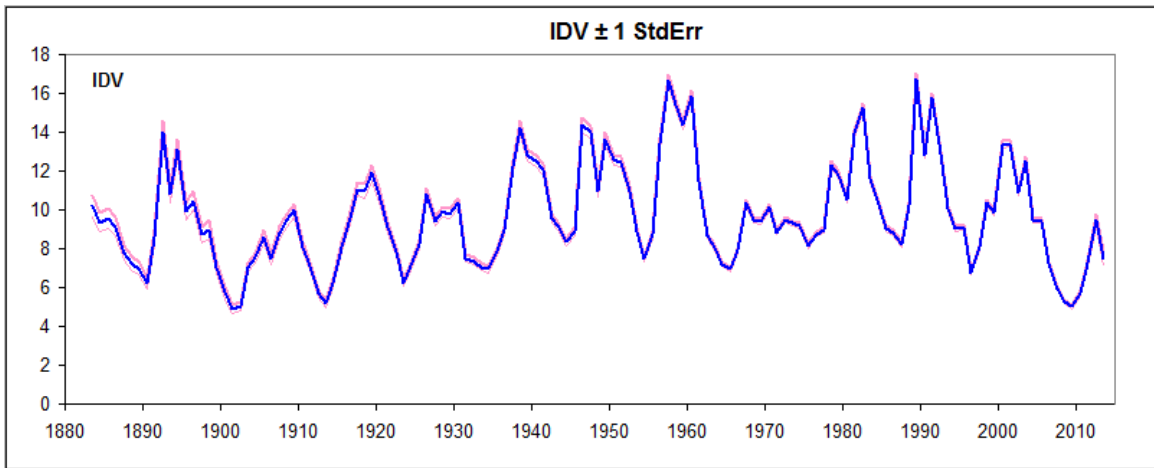
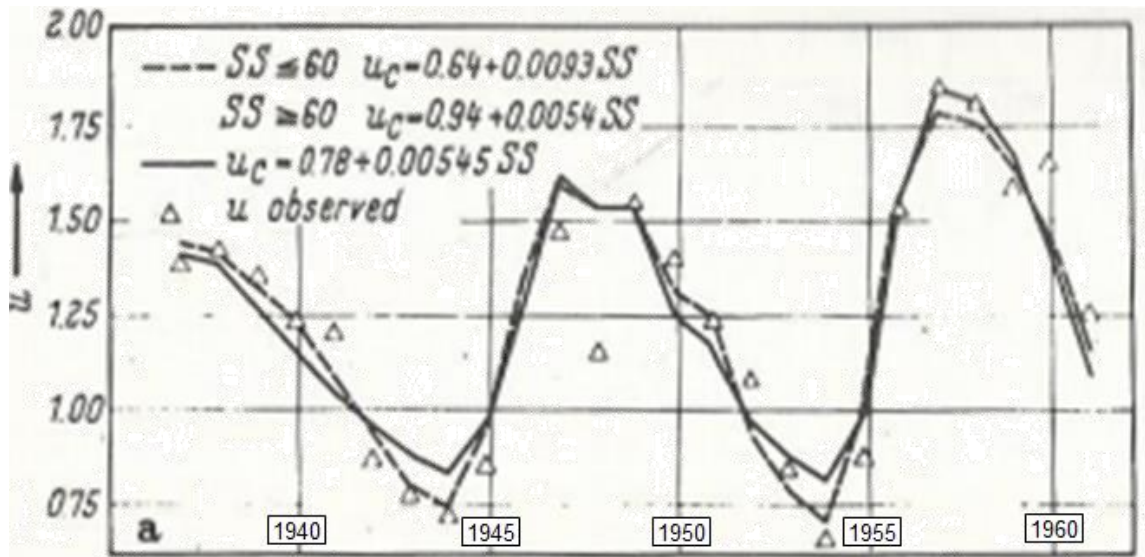


Figure 4: IDV and standard error band [pink curves] for each year of 1883-2013

From 1872-1961 the u -measure is available based on differences between daily means. The daily mean is contaminated by the day-to-day variation of the ‘regular’ diurnal variation, but the contamination is small [Mayaud was ‘astonished’ by how small the contamination was] at low latitudes, so the 1-day differences are a fair approximation to the proper IDV. This means that for the interval 1872-1961 the u -measure series forms a suitable IDV proxy.

Bartels computed u to 1936. Scott E. Forbush continued the series until 1961 using the method “as described by Bartels” and published a graph in his justly famous article in *Handbuch der Physics*, vol XLIX/1, e.d Flügge, Springer, Berlin, p 159-247, (1966):



geomagnetic activity index u was computed from the inter-diurnal variability of daily means of horizontal magnetic intensity as described by BARTELS [80].

Figure 5: Forbush's values for the u -measure. We have digitized the data points 1937-1961 from this Figure. There were two half-year points for 1937 and none for the year 1946.

It is of interest to see how these values compare with IDV. As the u -measure by definition is reduced to the latitude of Potsdam [Niemegk] no scaling is needed and we can directly compare IDV with $10u$ [remembering that u is expressed in units of 10 nT]:

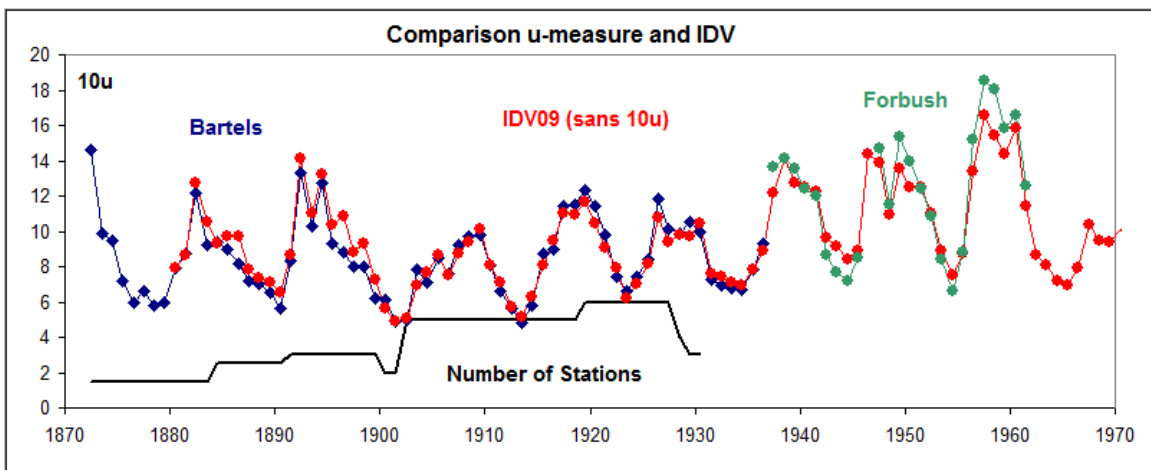


Figure 6: The u -measure as determined by Bartels [blue] and Forbush [green] compared to IDV calculated without including the u -measure. The number of stations, N , used by Bartels is indicated by the black ‘stairs’ curve. It is not known how many stations Forbush used, but it is reasonable to assume that he used about the same stations as Bartels, so we assume $N = 4$.

The difference between the u -measure and IDV is, assuringly, within the width of the standard deviation band. A simple way of reducing the issue with [real] u to irrelevance is to remove $10u$ from the IDV. This has almost no effect after 1883 so we have only 1872-1882 to worry about, and to that we turn next.

3) The error introduced by using a [slightly inappropriate] proxy for IDV. Bartels sought to extend the u -measure to before 1872 using the ‘summed ranges’ from Colaba [1847-1872] and the ranges of declination reported by Wolf. The latter only having half weight so the main data are summed ranges, s . First, let us see how s is defined. Moos writes [1910b, page 296]: “The variations in the range of an inequality are ordinarily accepted to represent the conditions of change in the inequality; but cannot always be depended upon to give the true indications of the progression”. Is it clear to you what Moos meant?

We no longer use the standard nomenclature of a century ago and few people today would know what ‘inequality’ meant in the context of magnetic variation. It meant ‘variation about the mean’. Specifically here ‘the inequality’ meant simply the diurnal variation [about the mean] of the magnetic element [H or X] in question. Figure 7 shows the diurnal variation, the inequalities, [on UT days] of X at Alibag [replacement station for Colaba (Bombay)] for the first half of each month of [the quiet year] 2009:

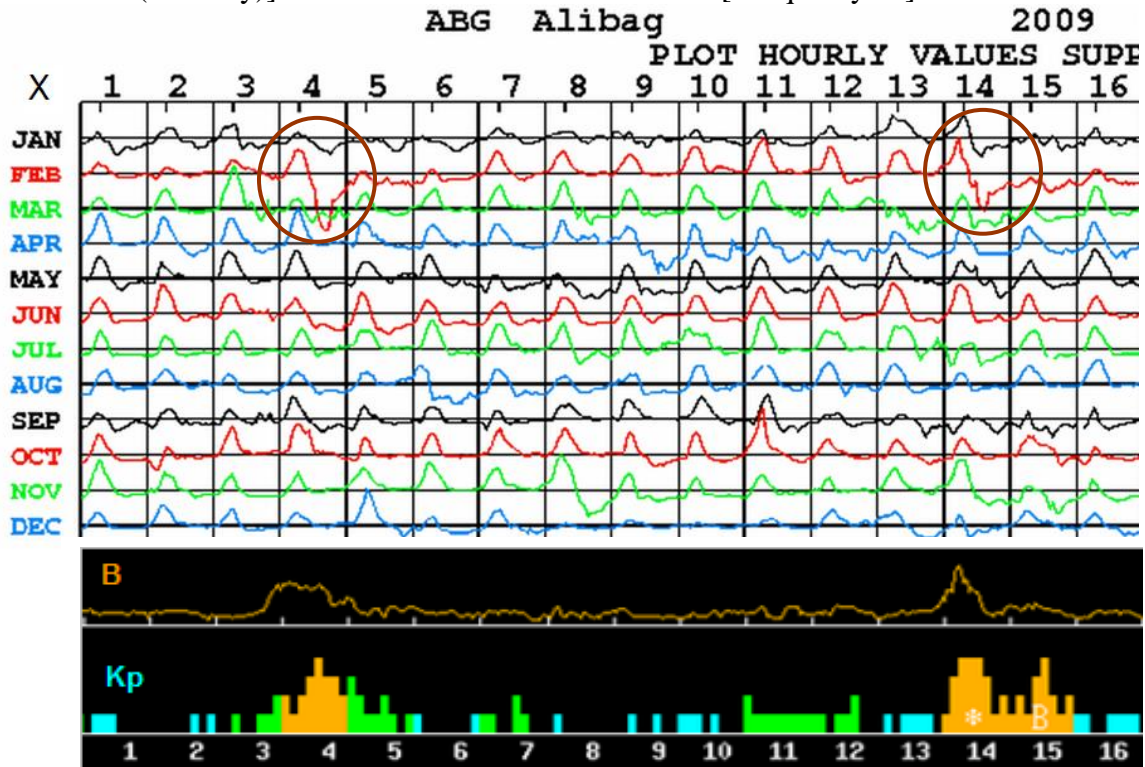


Figure 7: Diurnal Variation of X at Alibag and corresponding HMF B and Kp

The message of Figure 7 is that the diurnal variation is quite regular [at this near-tropical station] and that when it is not [e.g. the circled days in February] HMF B is large as shown in the lower panel. This is the simple basis for IDV [and u].

Continuing with Moos: “Hence perhaps the sum of all ordinates of the inequality without regard to signs which gives the average ordinate in 24 hours, may be considered as a more appropriate factor representing the variation due to disturbing effects”. Now, does he mean the ordinates of the daily inequalities or of the monthly [or yearly] inequality? Bartels’ interpretation [Bartels, 1932] was: “ s is derived from the mean diurnal variation of H at Bombay for each single month, expressed in departures from the average, and is the sum of these departures, summed without regard to sign. For the annual means 1872-1901, the values of u and s have a high correlation-coefficient +0.94 and stand in the linear relation (derived by least-square adjustment) $10u = 0.040 (s - 82)$ ”.

Continuing with Moos describing his effort of classifying days as quiet or disturbed (page 421): “To those familiar with such work of classification the great difficulty of correctly assigning a day to its proper division [Calm or Storm], is well known. Considerable judgment has to be exercised in several details, for there are movements and there are movements... In any case [for] a list of the kind ... involving a large personal equation, some additional data are clearly essential in order to make the classification more mathematically definite. The daily range, or preferably the summed ranges, figures of the diurnal inequality of each day would probably serve as the most appropriate data for this purpose; but as this is not possible on account of the heavy labor involved in their derivation, such of the ready available data as appear to best suit the requirements, have been utilized for the purpose.”

[Moos, describing the rationale for what later became the u -measure:] “It is noted ... that quiet days are almost always associated with days of high daily means, the disturbed conditions with days of comparatively low daily means, while the stormy days have invariably the lowest value of the daily means recorded in the month. As the daily means of the uncorrected tabulated values of the magnetograph ordinates were ready available, they were made use of, and the difference of the values of two consecutive days’ means (it may have suited the purpose better to take the departures of each day’s mean from the mean value of the month, but this would have involved large instrumental corrections and was not thereof attempted...) which were duly converted into force, was secured without much labor. The series furnishes the *crude* changes in the mean value of the absolute force from one day to the next, *uncorrected* for either temperature or instrumental errors. As however these errors (which, though, have been corrected for 1894-1905) are small for one day they may be neglected for the purposes of this chapter (to describe disturbances), and the figures so collected are given as a measure of the disturbance factor.”

But back to the summed ranges. Moos provides a table [page 724] with s -values back to 1841, and the Alibag Observatory Reports from 1906-1915 supply s through 1915. We can regress $10u$ against s and find $10u = (0.038 \pm 0.003) (s - (68 \pm 23))$ in good agreement with Bartels’ result (albeit with a slightly smaller correlation coefficient, +0.89), Figure 8. The rather large offset is, of course, due to the fact that a significant part of the variation of s is caused by variation of the FUV flux rather than by the solar wind, but the variation above the offset contains a large contribution from the solar wind.

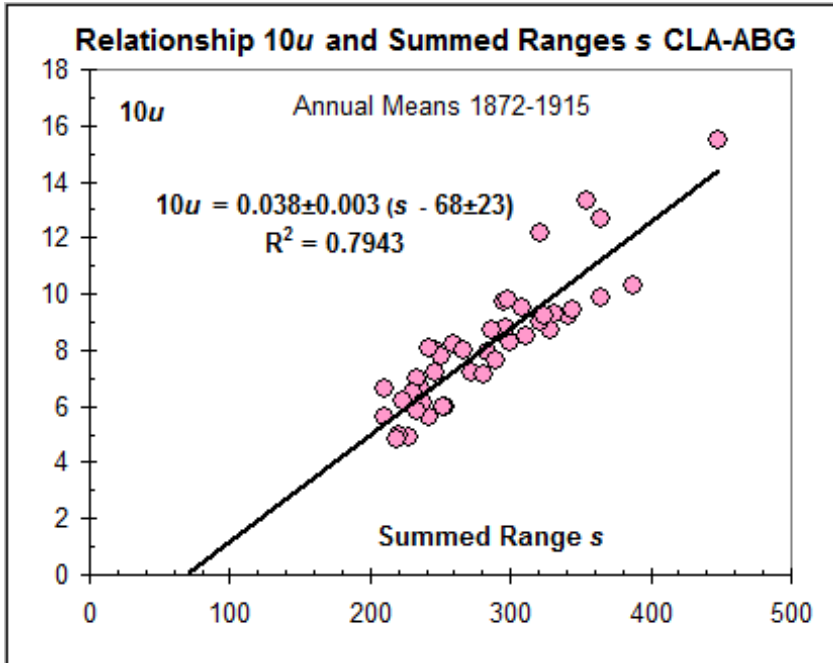


Figure 8: Linear regression $10u$ and summed ranges, s , over a month, from Colaba in 1872-1905 and Alibag in 1906-1915.

Using the regression formula we can get a calculated IDV from s [red curve in Figure 9]. For comparison, IDV derived without using s from the year 1872 and onwards is shown with large blue dots:

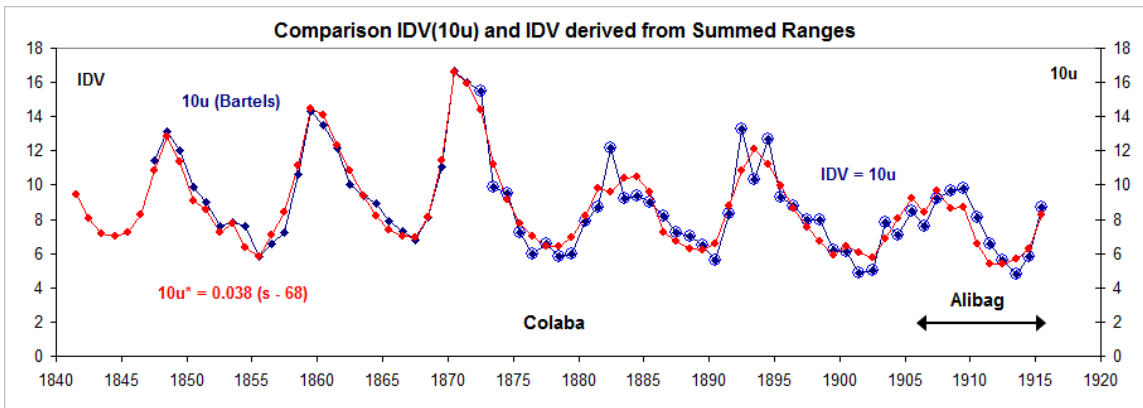


Figure 9: Synthetic $10u$ [red curve] calculated from summed ranges, s , at Colaba and Alibag compared to $10u$ [large blue dots] derived from differences of daily means as determined by Bartels [1932]. For illustration, the values of $10u$ given by Bartels derived from s before 1872 are shown by small blue diamonds in the left-hand part of the Figure.

The RMS difference between $10u$ and calculated $10u^*$ for 1872-1915 is 0.884 nT, which is 0.101 times the average $10u$, comparable to the fractional standard deviation of observed values of IDV. We therefore conclude that $10u^*$ [$10u$ calculated from summed ranges] has the same random uncertainty as any other actual station used in deriving IDV. This is comforting, showing that $10u$ derived from s is a useful proxy for $10u$ derived from the differences between consecutive daily means. There are some systematic differences, such as smoothing out the extremes [both high and low] because of the monthly means. The issue now is whether there are systematic *errors* and whether those can be detected and corrected. To this we turn next.

We can calculate our own version of s from Alibag. Digital hourly means are available 1923-2011. We calculate $s(\text{month})$ using the monthly ‘inequalities’ to be compatible with Bartels, regress IDV09 against s and compute IDV(s) with the result shown in Figure 10.

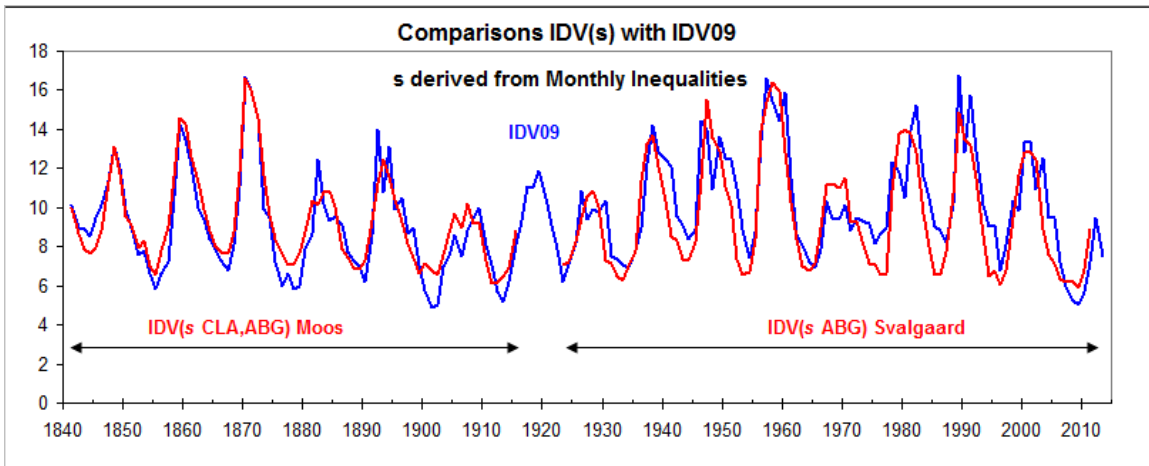


Figure 10: IDV derived from summed ranges, s (based on monthly means) [red curves] calculated from s , at Colaba and Alibag, compared to IDV09 [blue curve]; the left-hand portion as given by Moos and the right-hand portion as calculated by us.

The result is illustrative only and shows that the general level of IDV is reasonably well approximated by IDV(s), while the finer details are likely contaminated by the regular daily variation, because over a month S_R begins to dominate over D_{st} . See discussion later.

Remember that Moos [with his remarkable knowledge of the data that ensued from decades of observing the phenomenon of geomagnetic variations] suggested that “The daily range, or preferably the summed ranges, figures of the diurnal inequality of **each day** would probably serve as the most appropriate data for this purpose; but ... this is not possible on account of the heavy labor involved in their derivation”. Today we are not hampered by lack of computing power, so we can take up his suggestion and compute $s(\text{day})$, scaled to NGK, Figure 11:

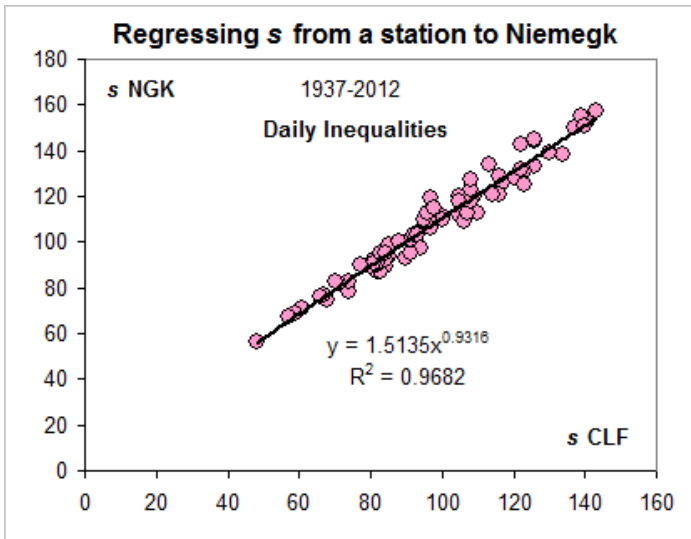


Figure 11: Annual means of the Summed (Absolute) Ranges, s , based on the variation through each day for the French station at Chambon-la-Forêt (CLF) inversely regressed to that of Niemegek (NGK). A power law is used to cater for the slight curvature of the data cloud.

Selecting 19 stations with either very long series of data or data in the 19th century (see Table 1) we can similarly calculate a scaled $s(\text{day})$ for each, Figure 12:

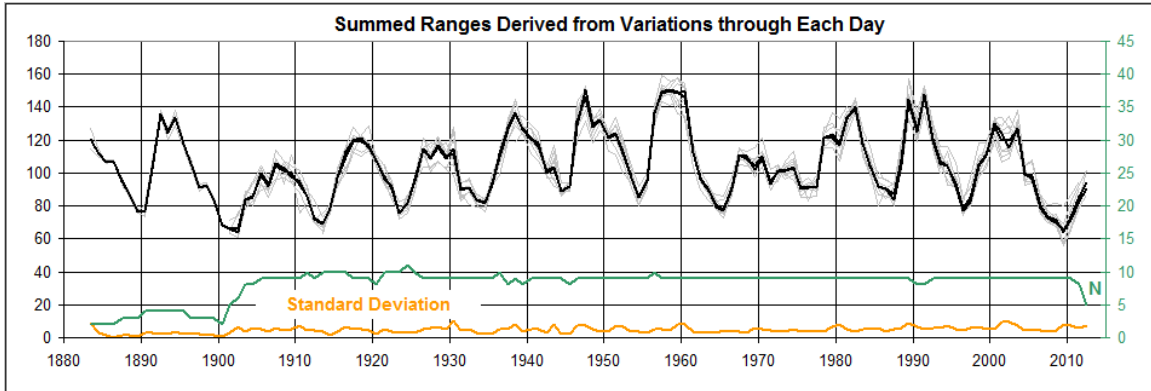


Figure 12: Annual means of Summed Ranges, s , derived from daily departures for the stations in Table 1 (faint grey lines). The heavy black curve is the all-station mean. The number of contributing stations is shown by the green curve [and scale at right]. At the bottom the orange curve shows the standard deviation of s .

It is evident that Moos' intuition that the summed ranges for daily departures would be preferable over the monthly departures bears out as the curves in Figure 12 have a strong resemblance to the variation of IDV. Figure 13 shows how close the fit is [$R^2 \sim 0.93$] over the entire interval 1883-2012. It thus appears that $s(\text{day})$ is a suitable proxy for IDV.

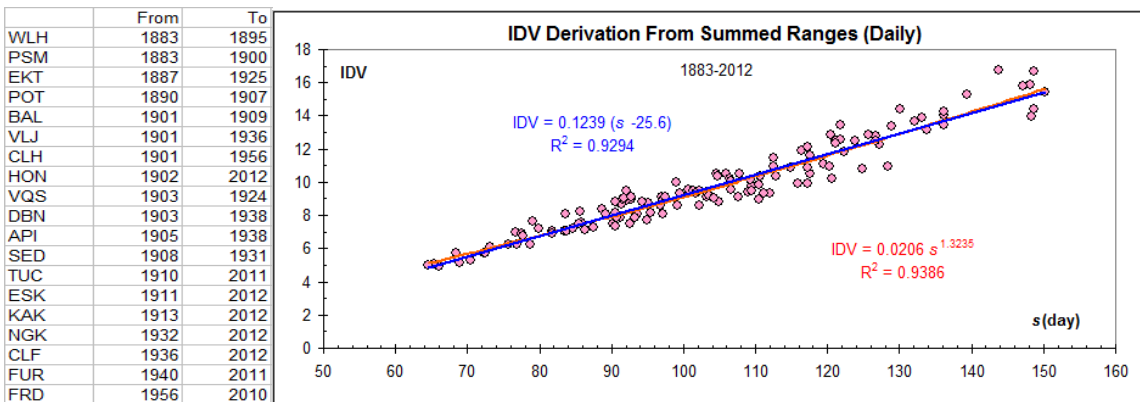


Table 1: Stations used.

Figure 13: Correlation between IDV09 and Summed Ranges s based of daily departures as suggested by Moos.

It is of interest to see if this good correspondence also extends to stations at somewhat higher latitude; in particular since we have digital hourly means for Helsinki [HLS: 60.173°N, 24.948°E] from 1844-1896 and for Saint Petersburg [SPE: 59.93°N, 30.33°E] from 1850-1862. The Finnish station Nurmijärvi [NUR: 60.508°N, 24.655°E], in operation 1953-present, is the modern replacement station for Helsinki. These stations have nearly the same corrected geomagnetic latitude as well [Table 2]:

HLS	1844	56.22°
	1896	55.43°
SPE	1850	55.16°
	1862	54.95°
NUR	1953	56.69°
	2012	57.06°

Table 2: Corrected Geomagnetic Latitude for Finnish-Russian stations, calculated using the GUFM1 main field coefficients [extending back to 1590] for the years indicated. Helsinki was on average about 1° further south than Nurmijärvi.

The relationship between IDV and s is remarkably strong for NUR [$R^2 = 0.9447$; $R^2 = 0.9345$ for linear fit] so it appears that we can also reconstruct IDV from $s(\text{NUR})$ with high confidence, extending the latitudinal range for IDV, Figure 14:

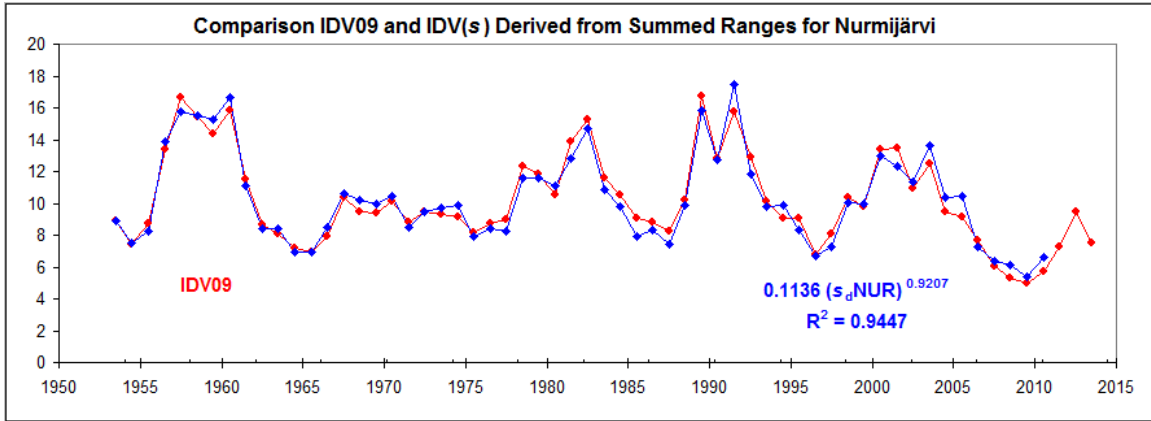


Figure 14: IDV09 (red curve with circles) compared with IDV(s) (blue curve with diamonds) constructed from summed ranges s (based on daily departures) at Nurmijärvi using the power-law fit $\text{IDV}(s) = 0.1136 s^{0.9207}$. A linear fit is just as good ($R^2 = 0.9345$).

So, Helsinki and Saint Petersburg also provide good proxies for IDV via their summed ranges. But before we exploit that avenue we must consider the problem of the changing scale-value of the H-variometer at Helsinki. It is well-known that the average *range*, i.e. the difference between the maximum and minimum values of the average diurnal variation is extremely well correlated with appropriate solar activity indices [e.g. F10.7 microwave flux, sunspot number, or the group number (number of active regions on the solar disk)], as was discovered by Wolf [1856] and subsequently extensively verified by many workers [e.g. Bartels 1946], in fact, having the highest correlation of all indices. Figure 15 shows the yearly average ranges for Declination D and Horizontal Force H at Helsinki:

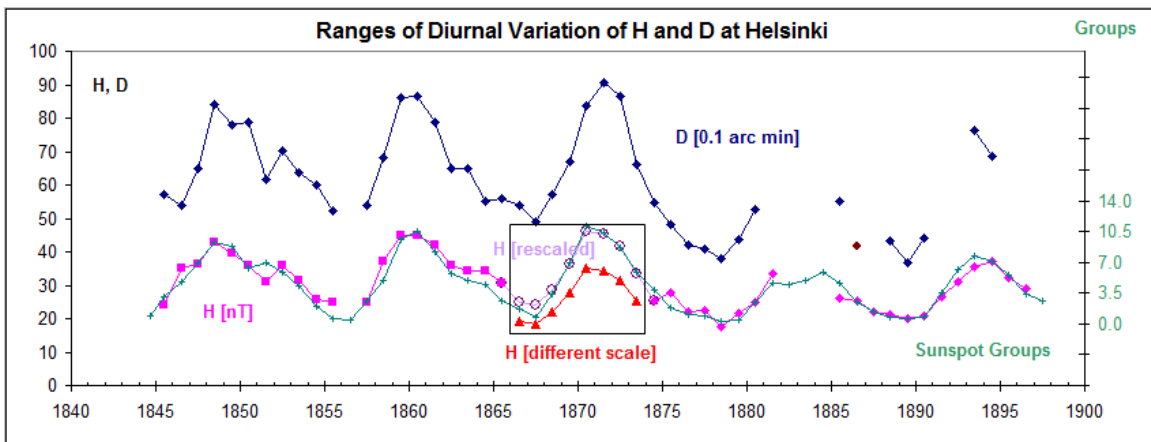


Figure 15: Yearly average Ranges for Declination D [in 0.1 arc minute units], blue curve, and for Horizontal Force [in nT units], pink curves. Because of the strong seasonal variation only years with no more than a third of the data missing are plotted. The green curve [with '+' symbols] shows the number of active regions

[sunspot groups] on the disk scaled to match the pink curves (H). As expected the match is excellent, except for the interval 1866-1873, where the H-range would have to be multiplied by 1.31 for a match, purple open circles.

The Group numbers used in Figure 1 are derived from the recent re-evaluation of solar activity [http://ssnworkshop.wikia.com/wiki/3rd_SSN_Workshop]. It seems that the scale value for H during the interval 1866-1873 must be different from that used for the rest of the H-data, specifically only $1/1.31 = 0.762$ of the value used by Nevanlinna et al. [2004] in constructing the Helsinki series. The range of the Declination during that interval matches that of H when H is re-scaled by the factor 1.31. The ranges of D and H generally vary together [with solar activity] being due to the same current system, indicating a problem with the scale-value of H. In a separate document we examine and corroborate this conclusion in detail.

Correcting the H scale-value for Helsinki and fitting HLS *s* to IDV09 for 1874 to 1896 (where we know that IDV09 is well-determined) we can reconstruct IDV(*s*) from HLS back to 1844, as shown in Figure 16:

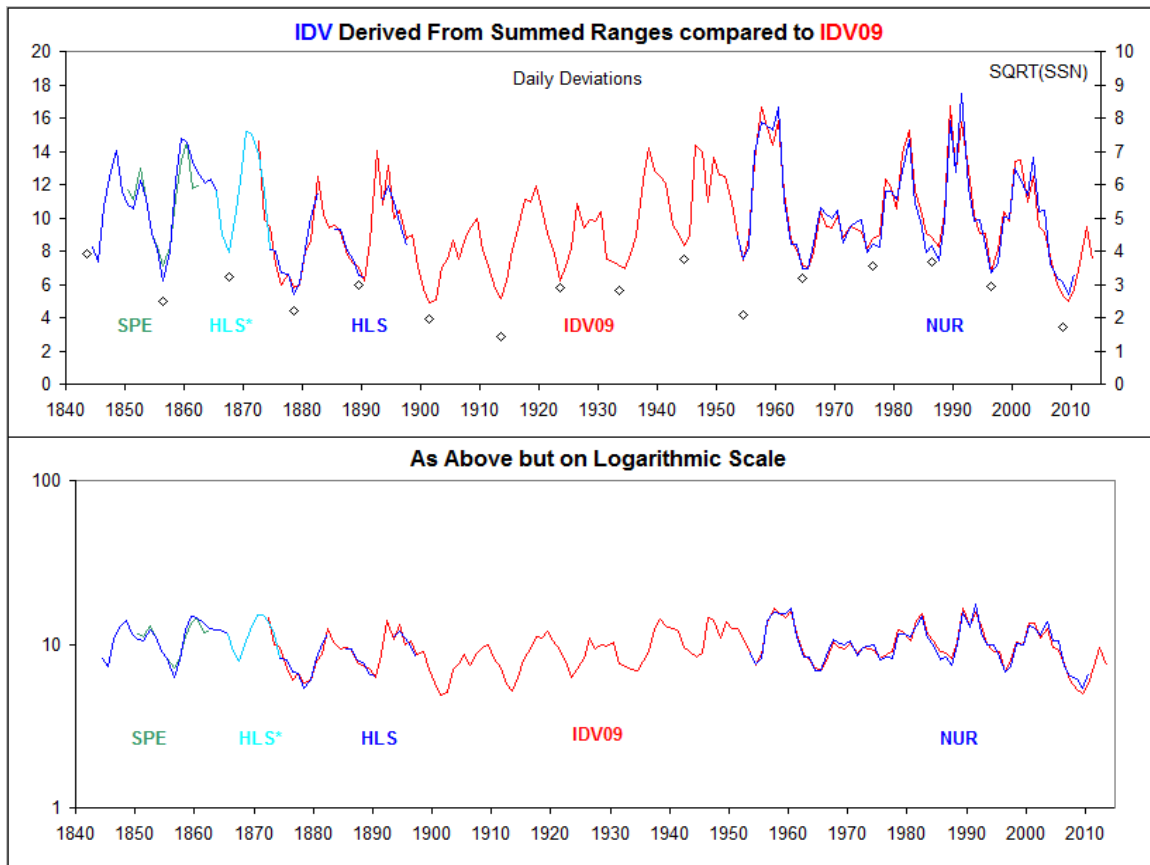


Figure 16: (Top) IDV derived from summed ranges [of daily departures] from Helsinki (HLS, blue – at left), corrected Helsinki (HLS*, cyan), Saint Petersburg (SPE, green, fitted to HLS), and Nurmijärvi (NUR, also blue – at right). IDV09 is shown for reference. Small black open diamonds show the square root of the (corrected) sunspot number [right-hand scale]. (Bottom) As above, but on a logarithmic scale showing that the relative scatter is scale-free.

The s -values from SPE closely match the data from HLS, so has been added to Figure 16. Data from other Russian stations [BRN, NER, EKA] from the 1850s do not match well and closer examination of the accuracy of the data is warranted.

We noted in discussion of Figure 10 that using summed ranges calculated [as Bartels and Moos did] by summing absolute departures of the monthly average diurnal variations was inferior to summing absolute departures of the daily diurnal variation directly. We now investigate one clear deficiency visible in Figure 17:

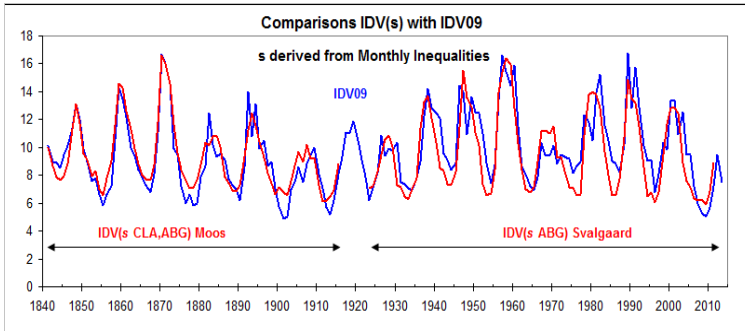


Figure 17: IDV calculated from s derived from *monthly* data for Colaba and Alibag (red curve) compared with IDV09 (blue curve). Before 1916 the data was given by Moos, after that calculated by us.

For solar cycles where there are significant recurrent stream activity on the declining branch of the cycle [e.g. 16 through 18, 1924-1954] the effect of the magnetic field in the streams, as evidenced by IDV09, is not picked up by s (month). Luckily, the effect is fully present in s (day) and shows itself as ‘shoulders’ on the curve, Figure 16. But why not simply calculate ‘real’ IDV directly from the hourly values of H? An early attempt gave discordant results because the scale-value problem was not known at that time. But now it makes sense to make a second attempt. Figure 18 shows ‘raw’ IDV derived from midnight values with a limit of 75 nT compared with IDV derived from s based on daily departures for HLS and SPE:

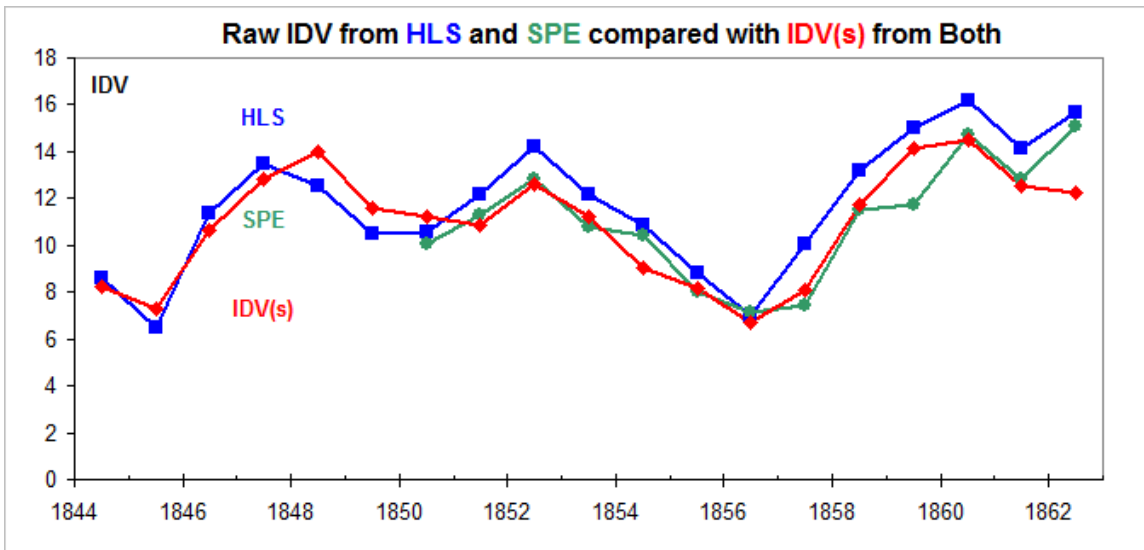


Figure 18: ‘Raw’ IDV from HLS (blue squares) and SPE (green dots) calculated the standard way using local midnight values of H compared with average IDV(s) (red diamonds) derived from summed ranges (Figure 16).

The limit of 75 nT has been found to mostly remove the effect of auroral zone contamination for stations near 55° corrected geomagnetic latitude. We scale the raw IDV to IDV(s) and include [with equal weight] the values in the ‘master’ IDV series: IDV13.

The sunspot cycle minima values of IDV do not return to the same level at each minimum, but rather to a value depending of the residual sunspot number, R_{\min} , [which usually does not go to zero] at minimum: $IDV(\min) = 2.9 + 1.45 R_{\min}^{1/2}$ with Coefficient of Determination $R^2 = 0.77$. The non-zero offset probably reflects the fact that the heliospheric magnetic field strength B does not go to zero (but reaches a ‘floor’ of 3.8 nT) even when the CME-rate and, presumably with it, the sunspot number go to zero [Crooker & Owens, 2011], Figure 19:

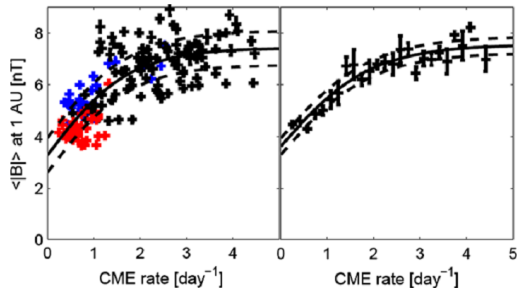


Figure 19: Scatter plots of Carrington-Rotation averaged magnetic field strength in the heliosphere at 1 AU against the CME rate. In the left panel, points from the recent solar minimum are red and points from the previous solar minimum are blue. In the right panel, points are binned by CME rate.

With IDV firmly in hand we can regress the field strength against IDV. The relationship is slightly non-linear and a power law $B = (1.45 \pm 0.10) IDV^{0.65 \pm 0.03}$ is marginally better ($R^2 = 0.91$) than the simple linear fit ($B = 2.32 + 0.412 IDV$; $R^2 = 0.87$). Figure 20 shows a preliminary result of inferring B [blue] from geomagnetism as well as the data [red] obtained from spacecraft near the Earth:

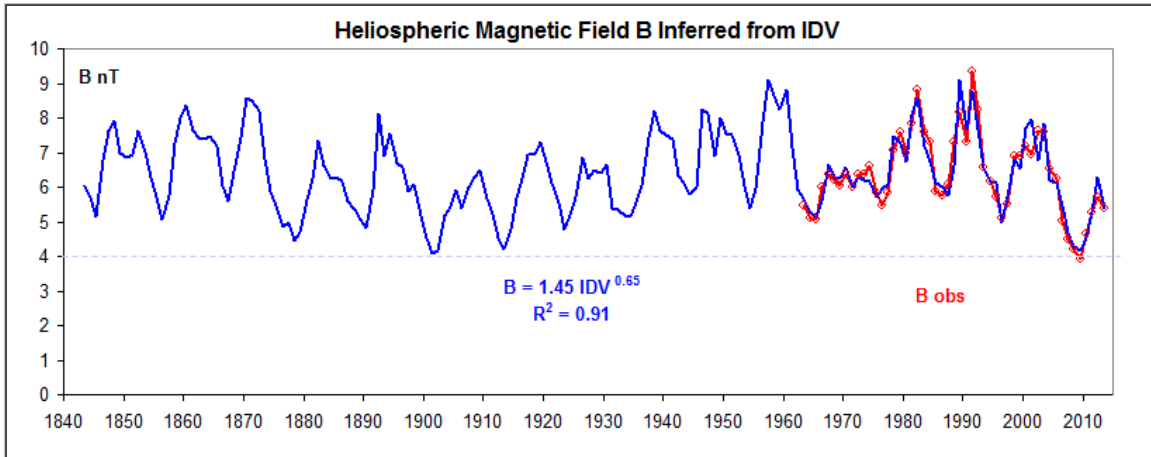


Figure 20: Current ‘best’ estimate of the Heliospheric Magnetic Field Strength B [blue] inferred from IDV13. The red points and curve show B measured *in situ* by spacecraft at 1 AU.

Now that we have begun to exploit hourly values before 1872 and no longer need to rely on the crude summed ranges derived from monthly inequalities, we may ask: “what has changed? Does it make any difference?” In Figure 21 we compare the newly derived B [call it B_{13}] to the B [call it B_{09}] as published in *Svalgaard & Cliver* [2010]:

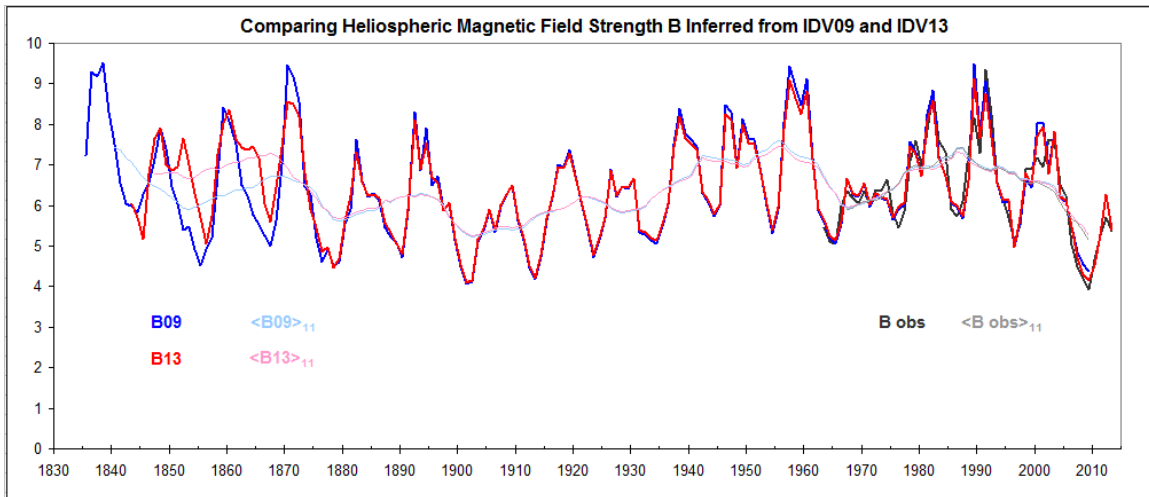


Figure 21: HMF B [red curve, B_{13}] inferred from IDV13 compared with B [blue curve, B_{09}] inferred from IDV09. The thin curves show 11-year running averages.

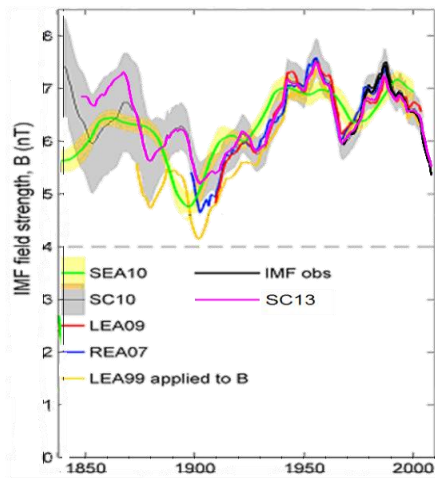


Figure 22: Our $\langle B_{13} \rangle_{11}$ (pink curve, SC13) in context of other solar cycle average B ; adapted from *Lockwood & Owens* [2010].

Although the maximum values for solar cycles 9 and 10 in 1845-1866 are well matched, the ‘shoulders’ on the declining branches raise the cycle averages by about 0.75 nT compared to B_{09} , bringing the mid-19th century level up to par with mid-20th century levels.

Putting the new estimate of B_{13} in the context of earlier estimates, Figure 22 compares 11-year cycle averages with other values from the literature. There is general agreement after 1910, but not before that year. In particular, ‘LEA99 applied to B ’ fails seriously, as expected [*Svalgaard & Cliver*, 2006]. Since B (and solar and geomagnetic activity generally) at present is as low as at a century ago, but no lower, the issue of the disparate values of B at that time looms large. Figure 22 also shows the field strength (SEA10) inferred by *Steinhilber et al.* [2010] with the same anomaly ~1880-1900.

The recent re-assessment of the cosmogenic evidence [^{10}Be] by *McCracken* [2013] offers some hope for reconciliation, Figure 23:

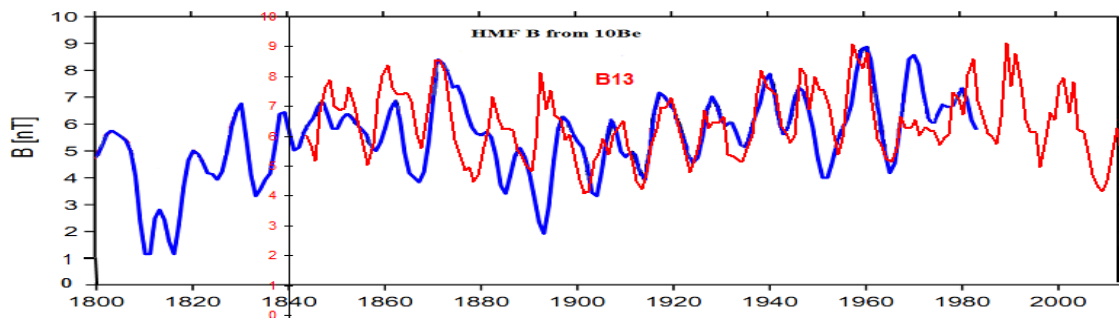


Figure 23: HMF B as derived from cosmogenic ^{10}Be flux from several ice cores (blue curve, preliminary composite *McCracken* [2013]) and B_{13} (red curve).

Although the scales are slightly different [this likely is a calibration issue for $B(^{10}\text{Be})$]: $B_{10\text{Be}} = (10/9) (B_{13} - 1) = 1.11 B_{13} - 1.11$, the agreement is quite reasonable, except (and this is the crucial point) for the two decades following ~ 1810 and ~ 1885 . It is perhaps not mere coincidence that large volcanic eruptions took place during those decades (1809 *Dai* [1991], 1814 *Mayon*, 1815 *Tambora*, 1883 *Krakatoa*). Alternatively, the calculation of the cosmic ray solar modulation parameter may not be quite correct for low solar activity – for which the assumption of a spherically symmetric heliosphere is not valid. The issue remains unresolved, although the recent low solar activity combined with an actual measurement of the intensity in the Local Interstellar Medium may eventually provide the empirical evidence needed to settle the matter.

On Usoskin's website (<http://cosmicrays oulu.fi/phi/phi.html>) the cosmic ray modulation parameter is available 1937-2012. The data since ~ 1952 [derived from neutron monitors] does seem to be reasonably well correlated with HMF B , Figure 24:

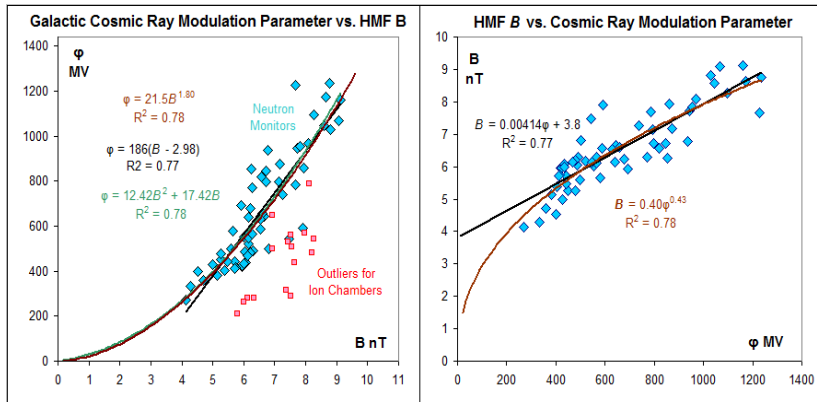


Figure 24: (Left Panel) Galactic cosmic Ray modulation parameter as a function of HMF B . (Right) the inverse relation. Several tries at functional relations are shown, including a quadratic and a power-law through the origin.

The modulation parameters calculated from Ion Chamber data do not match the magnetic field strength (nor the reconstruction by *McCracken* (Figure 23)) and are here considered to be outliers that should not be used, Figure 25:

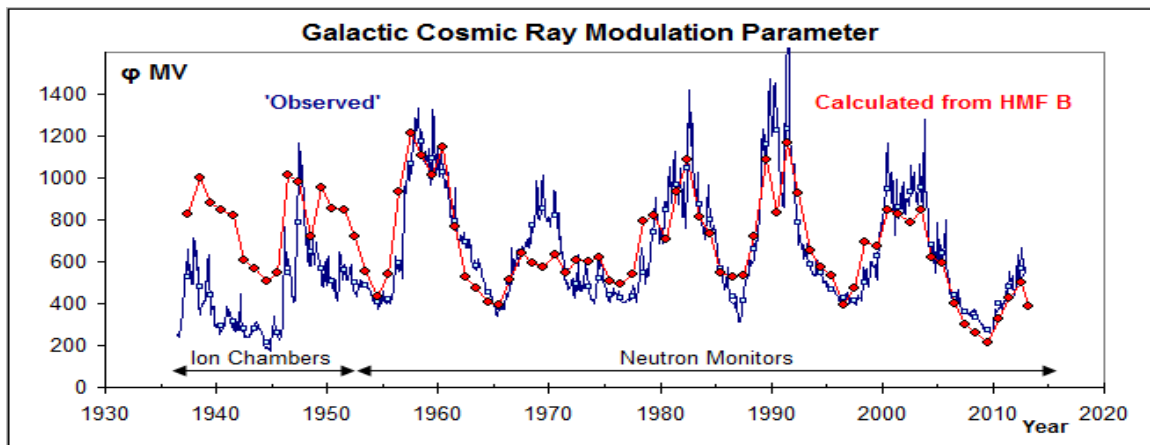


Figure 25: Modulation parameter (blue curve, monthly averages) and HMF B (red curve) estimated as the average values calculated from the various functional relationships shown in Figure 24.

For $B = 1.8$ nT during the Maunder Minimum [Steinhilber *et al.* 2010; Lockwood & Owens 2010] the linear relation yields the unphysical negative value of the modulation parameter $\phi = -220$ MV while the power law or polynomial fits yield 62 MV, both values being grossly at variance with the observed strong modulation during Grand Minima. Similar comments apply to the values of B between 1 and 2 nT suggested by McCracken (Figure 23). A [floor?] value of B of 3.93 nT corresponds to a modulation parameter of 253 MV (using the power-law fit) to be compared with the 271 MV ‘observed’ in the deep minimum of 2009. We would not expect close agreement since it is clear from Figure 25 that there can be significant departures from a close fit, e.g. during solar cycle 20 from 1965-1976.

The earlier finding that the HMF strength B would scale with the square root of the sunspot number or with the nearly equivalent number of active regions (sunspot groups) still holds, at least approximately, Figure 26:

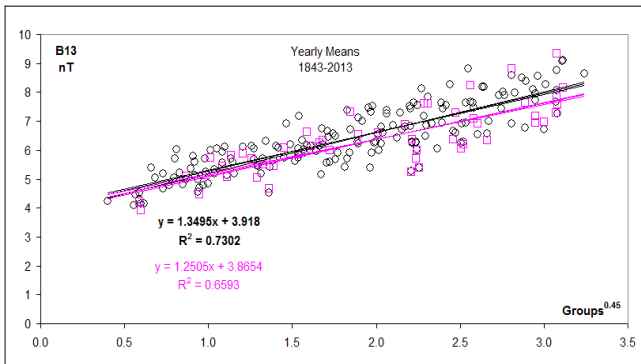


Figure 26: HMF B_{13} (black circles) vs. the number of sunspot groups to power 0.45 (best LS fit). Using only years with B measured *in situ* (pink squares) yields a slightly worse fit. As before, there is a non-zero (floor) offset for zero Groups. The number of groups comes from the SSN workshop revisions.

Using the regression equation of Figure 26, we get the red curve in Figure 27:

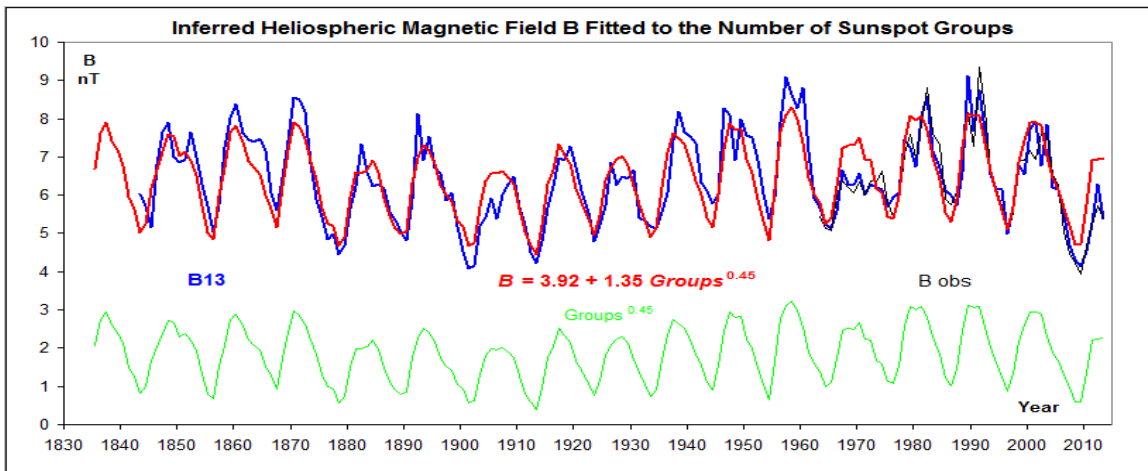


Figure 27: Comparing the HMF B inferred from geomagnetism (blue) with that (red) inferred from the relationship with the number of sunspot groups (green).

While generally good at representing the overall level of HMF B , the Group-derived values suffer from the same defect as IDV(s) derived from monthly data and shown in Figure 17, namely not picking up the ‘shoulders’ on the declining branch of the cycles. For this reason one should not construct a ‘composite’ data set of B from geomagnetic and sunspot data.

Continuing comparisons with other indices we now take a look at the newly proposed IDV(1d) index, based on calculating IDV for each hour of the day and averaging the result. The claim is that this procedure is meaningful at all latitudes, including the latitudes we normally consider to be too high for IDV. We calculate IDV [based on local midnight values] for Nurmijärvi [red curve] and for all hours of the day [IDV1d, orange]:

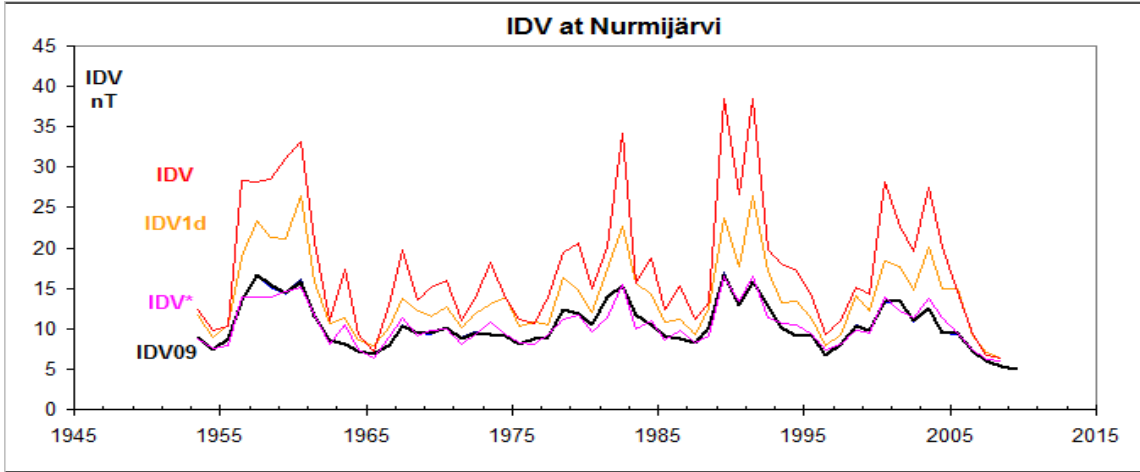


Figure 28: IDV_{NUR} for the midnight hour (red) for NUR at corrected geomagnetic latitude $57^\circ N$ and $IDV1d$ for all hours (orange) compared with IDV09 (black). The pink curve, IDV^* , shows IDV1d normalized to IDV (cf. Figure 29).

The relationship between global IDV09 [for about 50 stations] and IDV_{NUR} is not linear as seen in Figure 29:

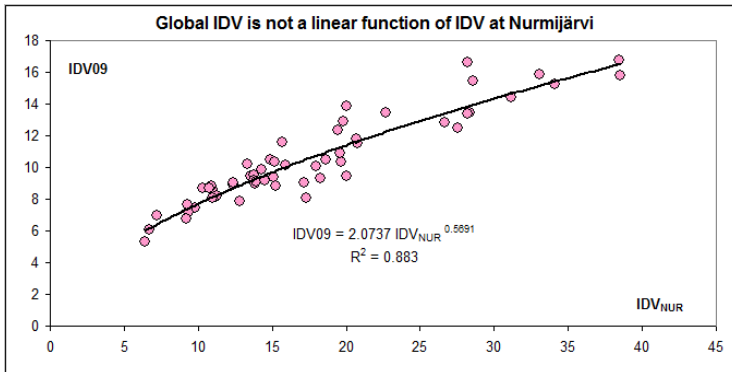


Figure 29: IDV09 as a non-linear function of IDV_{NUR} : $IDV09 = 2.074 IDV_{NUR}^{0.59}$ ($R^2 = 0.88$).

With this relationship we can scale IDV_{NUR} to match IDV09: IDV^* as shown in Figure 28.

The reason for the sharply increased values of IDV_{NUR} near sunspot maxima (where global IDV is high) is that Nurmijärvi is too close to the auroral oval. The size of the oval depends on activity itself, as *Milan* [2009] shows:

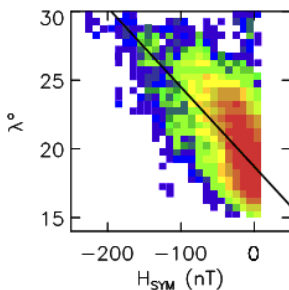


Figure 29: Occurrence distributions of radius of the auroral oval, λ , as a function of (a) ring current intensity H_{SYM} . From Stephen E. Milan, Both solar wind-magnetosphere coupling and ring current intensity control of the size of the auroral oval, *Geophysical Research Letters*, Volume 36, L18101, doi:10.1029/2009GL039997, 2009

According to Milan, the distance to the auroral zone is given by:

$L^\circ = 90^\circ - \text{CorrGeoMagLat} - (18.7^\circ - 0.058 H_{\text{SYM}}) \approx 14.4^\circ - 0.058|D_{\text{st}}| = 14.4^\circ - 0.13 \text{ IDV}$
 using the relationship between D_{st} and IDV from SC10. So, each increase of global IDV by 10 nT moves the auroral zone 1.3° closer to the station [equivalent to a corresponding increase of latitude], which in turn increases IDV_{NUR} by a further 30%, calculated from the following empirical formula for the IDV dependence on corrected geomagnetic latitude, L :

$$\text{IDV} = 10.87 + 197.1 / (1 + 10^{(12.4 - 0.1942 L^\circ)})$$

Activity increases very sharply when the auroral zone expands to become nearer to the station. Since the magnetic field B varies a lot [up to a factor of 20] more than the solar wind speed squared [factor of 4], B , rather than V , will drive IDV for a high latitude station and that non-linearly. We can see that sharp increase clearly if we plot (raw) $\text{IDV}(h)$ for each hour, h , of the 24 hours with a different color, Figure 30:

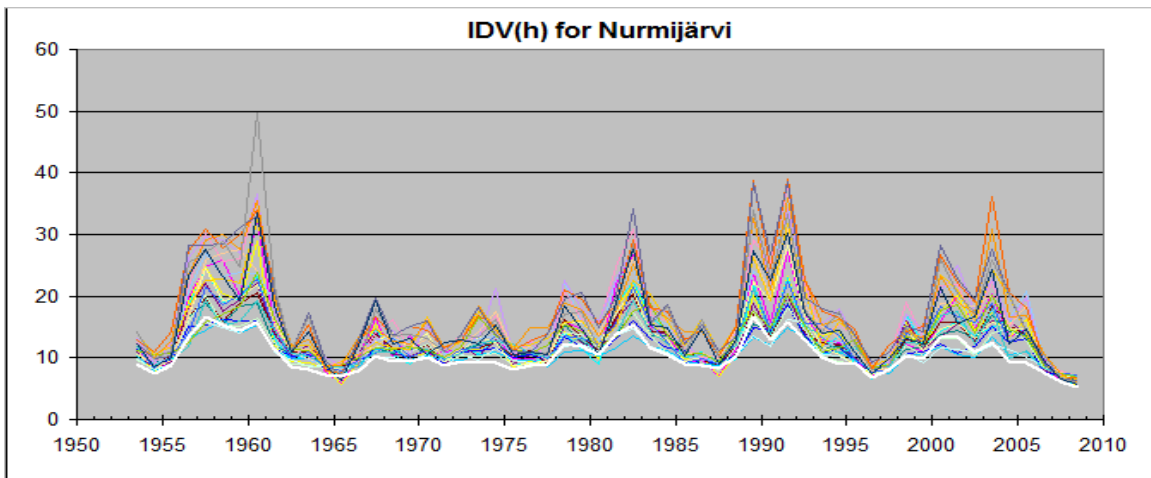


Figure 30: $\text{IDV}(h)$ for each hour, h , of the (UT) day for Nurmijärvi for each year 1953-2008. The white thick curve is IDV09.

It should be obvious that the large variation during the day [by a factor of two] is not due to a corresponding variation of the ring current, but simply reflects the varying distance from the auroral oval as it during the day sweeps through the sky north of the station. The heavy white curve is simply global IDV09 showing the variation of the ring current. The $\text{IDV}(h)$ values for Nurmijärvi approach IDV09 asymptotically reaching the white curve for local hours $h = 10$ and $h = 11$, just before noon when the station is the farthest away from the oval.

Since the dayside oval is at latitude 10° higher than the nightside oval, the contamination by auroral currents which we in SC10 determined to begin at latitude $\sim 50^\circ$ is expected to be absent on the dayside up to a latitude some 10° higher, above 60° . This means that $\text{IDV}(h)$ computed for the *dayside* for Nurmijärvi and Helsinki would not be seriously affected by auroral currents, while, on the other hand, $\text{IDV}(h)$ computed for the *nightside* for these stations certainly would be, as is evident from Figures 28 and 30.

At least statistically, one can compensate for this uneven sensitivity to auroral effects and S_{R} currents by calculating for each hour and each station a normalization factor for $\text{IDV}(h)$ to IDV09 [the latter based on ~ 50 stations at all local times]. This is the number

to divide $IDV(h)$ by to match $IDV09$. Normalizing and plotting as before in different colors, one gets a much more pleasant graph, Figure 31.

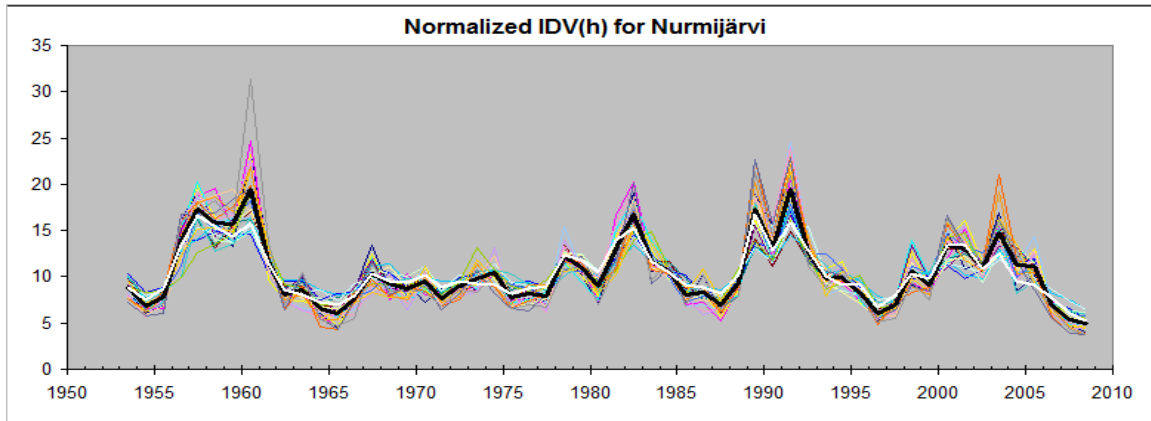


Figure 31: Normalized $IDV(h)$ for Nurmijärvi, compare with Figure 30.

The heavy black curve is the daily mean and the heavy white curve shows $IDV09$ as before. There are still a few ‘spikes’ as one would expect from using only a single station. There are standard ways to suppress spikes, so they are not a problem, or we can simply live with them. The spikes could also simply be because a single normalization factor for all levels of IDV is too simple.

We can do the same exercise for Lerwick [Corrected Geomagnetic latitude 58.2°] and again we see the $IDV(h)$ values for Lerwick approach $IDV09$ asymptotically reaching the white curve for local hours $h = 10$ and $h = 11$, just before noon when the station is the farthest away from the auroral oval, Figure 32:

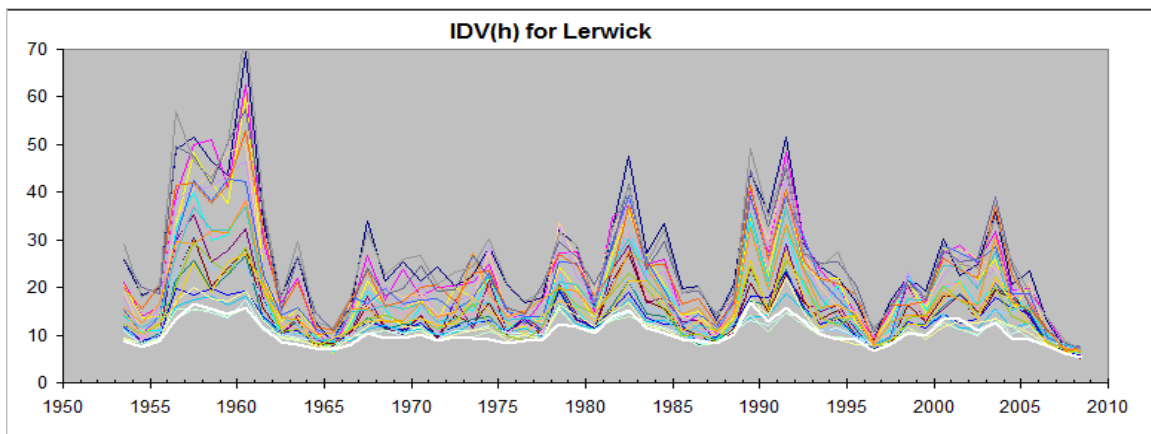


Figure 32: $IDV(h)$ for each hour, h , of the (UT) day for Lerwick for each year 1953-2008. The white thick curve is $IDV09$.

And again we can compensate for the uneven sensitivity to auroral effects and S_R currents by calculating for each hour a normalization factor to $IDV09$. In this and several similar Figures to follow we use the same time interval 1953-2008 for easier comparisons. Normalizing and plotting as before in different colors, one again gets a more pleasant graph. The heavy black curve is the daily mean and the heavy white curve shows $IDV09$, Figure 33:

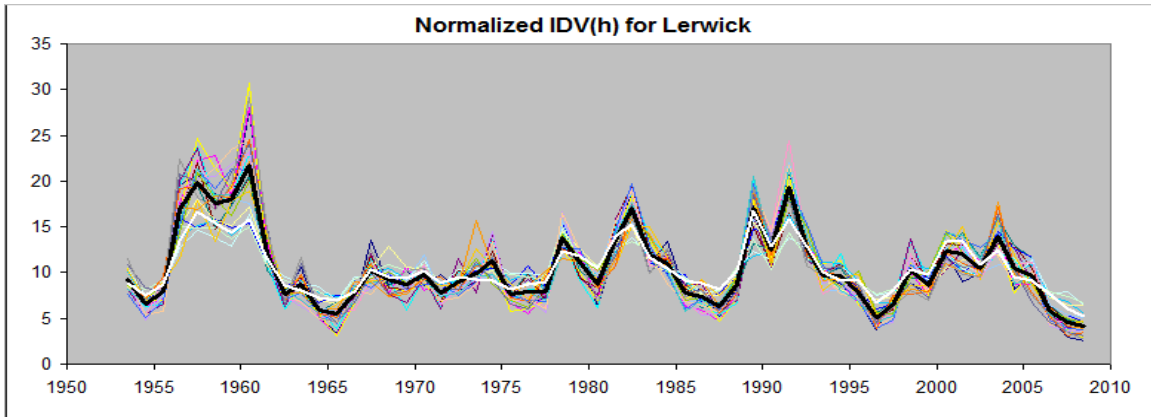


Figure 33: Normalized IDV(h) for Lerwick, compare with Figure 32.

As before, the average normalization [being the same for all solar cycles, *i.e.* for all levels of activity] does not quite remove the auroral effect for strong solar activity cycles [like for cycle 19 peaking in 1958], especially not since Lerwick is at even higher corrected geomagnetic latitude than is Nurmijärvi.

As we move to higher and higher latitudes the auroral contamination gets worse and worse, as the following examples show:

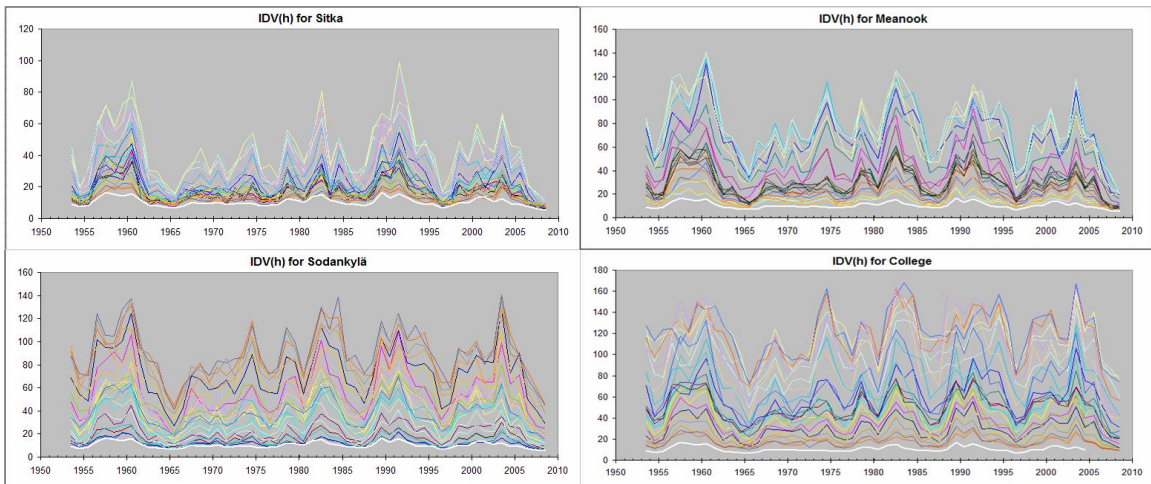


Figure 34: IDV(h) for each hour h through the day for Sitka SIT (corrected geomagnetic latitude 59.9°), Meanook MEA (62.5°), Sodankylä SOD (63.5°), and College Alaska CMO (64.9°).

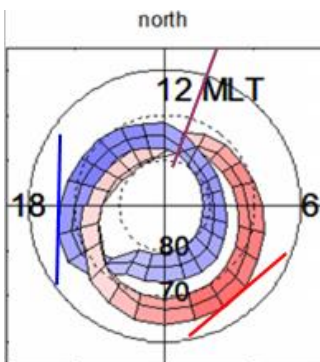


Figure 35: Just a reminder of where the auroral ovals and their lower latitudinal limits are (for the Northern Hemisphere).

We expect maximum contamination to occur when the station is nearest to the electrojets in the early morning [red line] and in the late afternoon [blue line], with a minimum around 11^{h} MLT [purple spur], and that is what is indeed observed, Figure 36:

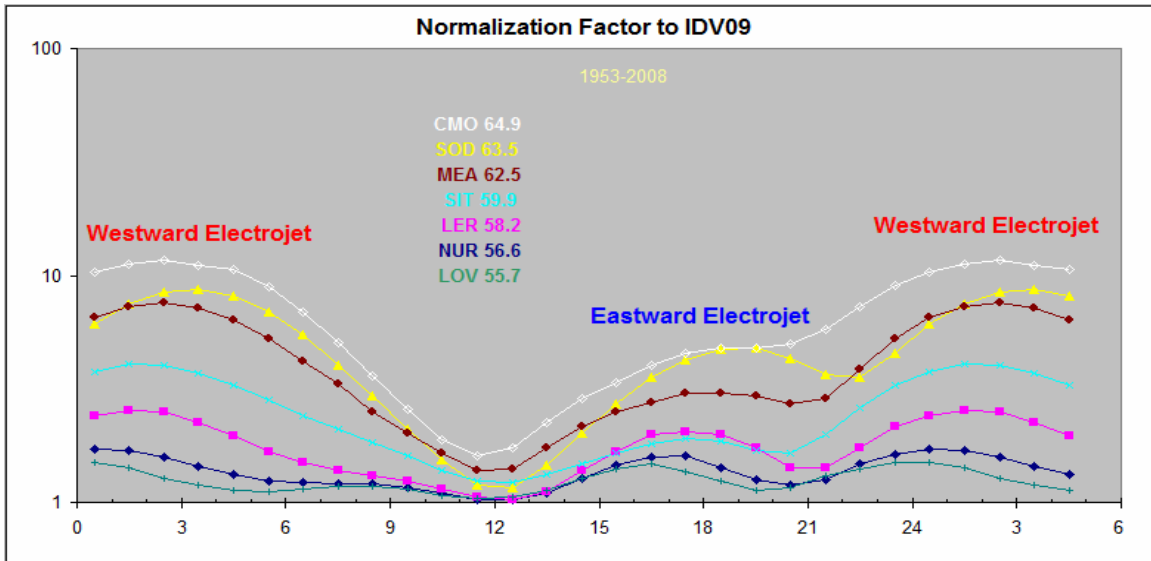


Figure 36: The ratio between $IDV(h)$ and $IDV09$ for several stations with CGML ranging from 64.9° down to 55.7° as a function of hour, h , expressed in magnetic local time, MLT (as the oval is somewhat irregular we have allowed the timing to be adjusted an hour either way to make the minima line up). Just before noon MLT, the stations are farthest away from the electrojets and the contamination is at a minimum. At other times, the contamination becomes severe (note that the scale is logarithmic) at higher latitudes.

It now becomes of extreme importance to see how Helsinki fares in this game, Figure 37:

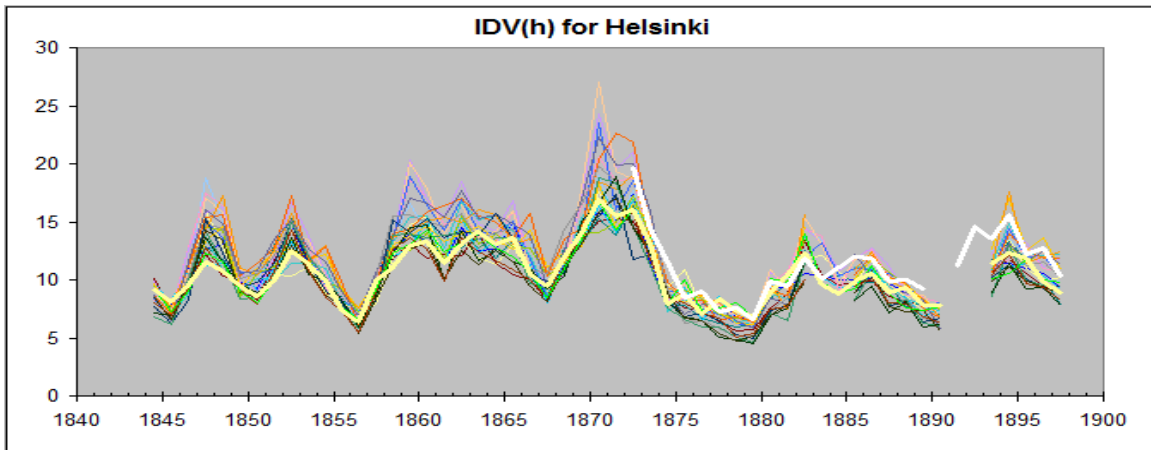


Figure 37: $IDV(h)$ for each hour, h , of the (UT) day for Helsinki for 1844-1897 (corrected for scale-value problem). The yellow thick curve is $IDV(h)$ at local noon. The white thick curve is $IDV09$.

Apart from the noon values being a bit lower than $IDV09$ the agreement is reasonable. There is another systematic difference: the noon values before ~ 1875 form the lower envelope of the data, while later the noon values are in general higher than most of the data. There could be several reasons for this: 1) the scale-values may be slightly different before 1866 and after 1873, 2) the CGM latitude for Helsinki has changed enough over time to influence IDV , or 3) the variation of the normalization factor [or simply $IDV(h)$

itself as IDV09 does not vary with h] through the day is different for stations at lower latitudes. To this latter possibility we turn next.

We start at the equator and work our way polewards. First for M'Bour (at 14.38°N, 343.03°E; CGM Lat $\approx 5^\circ$ N), Figure 38:

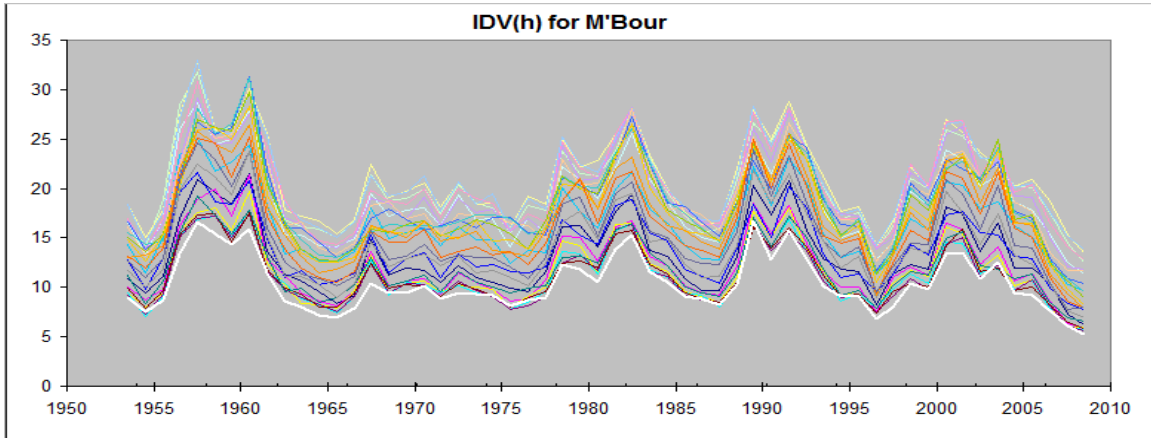


Figure 38: IDV(h) for each hour, h , of the day for M'Bour for each year 1953-2008. The white thick curve is IDV09.

The values of IDV(h) approach IDV09 asymptotically, reaching the white curve for several hours just after local midnight, in stark contrast to the behavior at CGM latitudes above 55° where the best match with IDV09 is just before noon.

For these comparisons we try to stay within the European longitude sector. For the purpose of these comparisons we have not adjusted IDV(h) for variations in latitude as they are small compared to the time variations of IDV.

Further north we have L'Aquila, Italy (at 42.383°N, 13.317°E; CGM Lat 36.27°N), Chambon-la-Forêt, France (at 48.025°N, 2.260°E; CGM Lat 43.80°N), Hartland, England (at 50.990°N, 355.516°E; CGM Lat 48.14°N), and Eskdalemuir, Scotland (at 55.314°N, 356.794°E; CGM Lat 53.06°N, all CGMs for 1980), Figure 39:

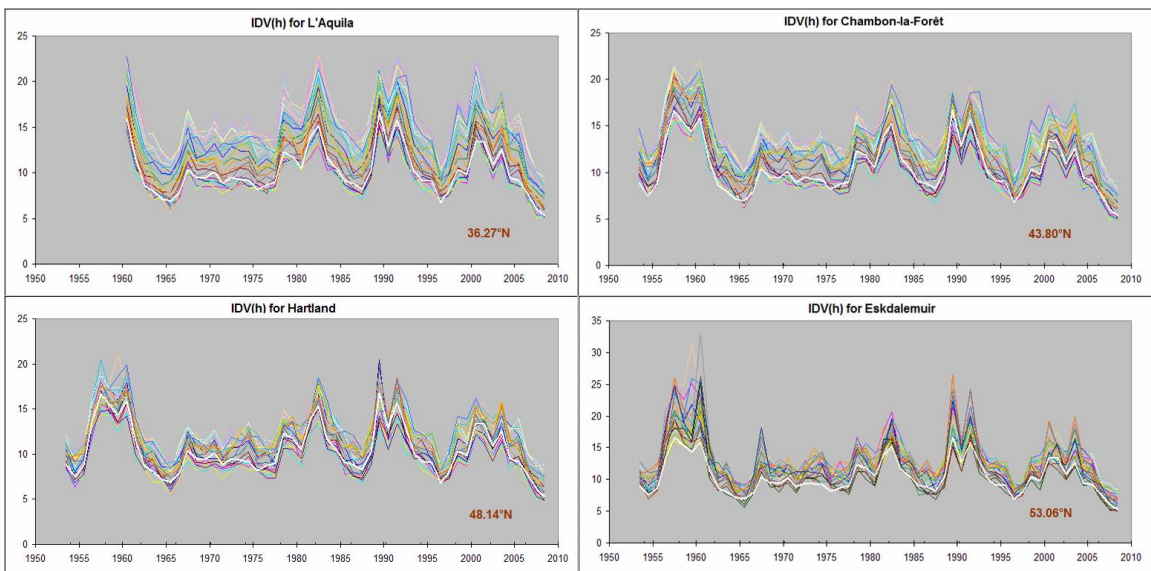


Figure 39: IDV(h) for each hour, h , of the day for AQU, CLF, HAD, and ESK for each year 1953-2008. The white thick curve is IDV09.

Just as in Figure 36 we can calculate the ratio between IDV(h) to IDV09 for these stations, adding RSV/BFE, Denmark (at 55.7°N, 12°E; CGM Lat 52.13°N) and SPE, Russia (at 59.93°N, 30.30°E; CGM Lat 55.06°N), Figure 40:

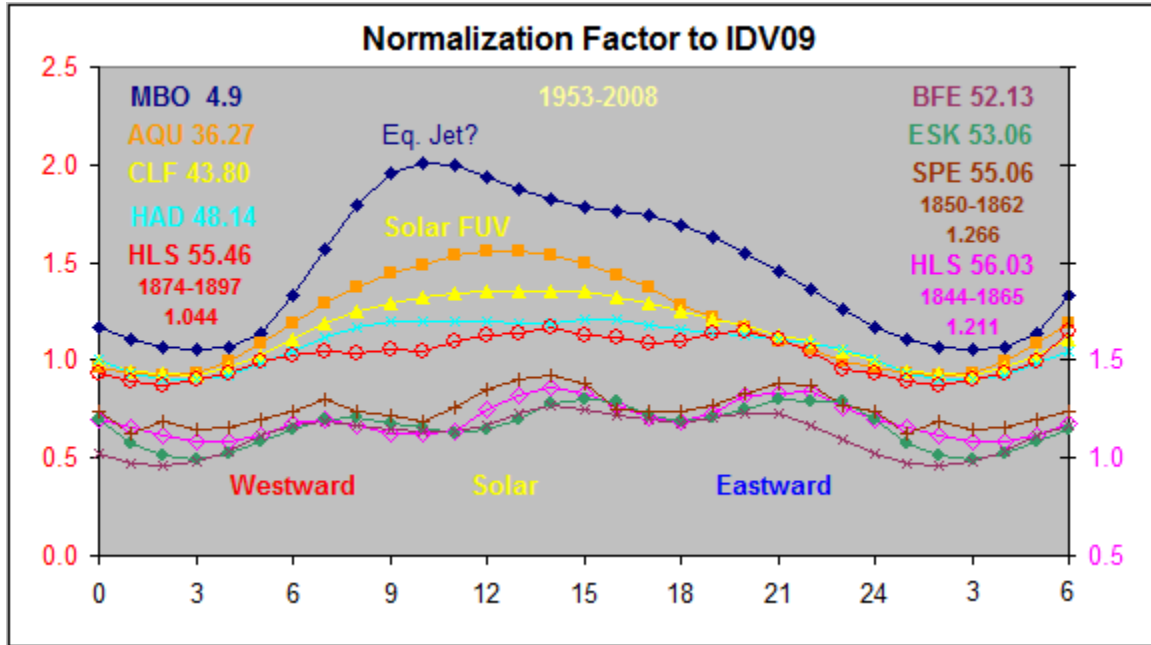


Figure 40: The ratio between IDV(h) and IDV09 for several stations with CGML ranging from 5° up to 56° as a function of hour, h , expressed in solar local time. The right-hand scale applies to the stations mentioned next to that axis while the left-hand scale applies to stations next to that axis. Note that the RH scale is displayed 0.5 units down relative to the LH scale to avoid confusing overlaps of curves. At the lower latitudes solar FUV dominates and controls the conductivity of the E-layer giving rise to enhanced IDV while at the higher latitudes the FUV influence diminishes and the auroral electrojets begin to make their effect felt. See the text for further details.

The effect of Solar Far Ultraviolet is, not surprisingly, largest near the equator and diminishes to insignificance with **increasing** latitude until about CGM latitude ~55° where it effectively disappears. In Figure 36 we saw that the effect of the auroral electrojets shows the opposite trend: largest in the auroral zone and diminishing to insignificance with **decreasing** latitude until the same CGM latitude about ~55°

For Helsinki we plot the result for the interval before 1866 (pink) separately from the interval after 1873 (red). The former [for which the average CGM latitude of HLS was 56.03°] exhibits the ‘auroral’ signature, with an average value of 1.193, while the latter [for which the average CGM latitude of HLS was 0.6° lower, 55.46°] shows the ‘FUV’ signature with an average value of 1.044, albeit with a hint of beginning ‘auroral bumps’. The average ratio IDV(h)/IDV09 was thus 16% higher at the 0.6° higher latitude (1.211/1.044), twice of what would be expected using the Milan formula, but it is not clear how far this quantitative guesswork can go or how well the conditions 1844-1897

can be compared with 1953-2008 any better than just a qualitative suggestion. Thus a possible explanation for the difference of IDV-levels seen in Figure 37 might simply be the change of the CGM latitude Λ of HLS. Figure 41 shows the evolution of Λ at Helsinki and Nurmijärvi:

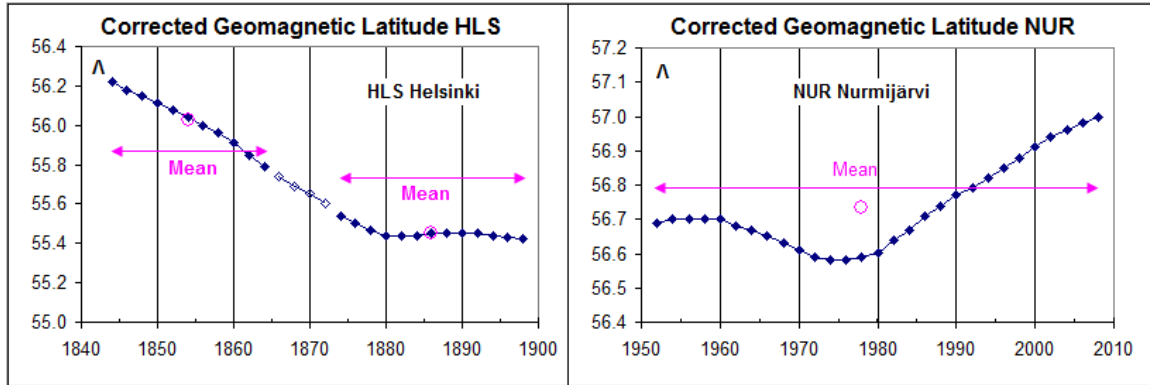


Figure 41: Corrected Geomagnetic Latitude for Helsinki HLS and Nurmijärvi NUR.

In any event, it seems clear that there is a CGM latitude, Λ_0 [$\approx 55.7^\circ$, approximately where Helsinki is] where the contaminations from the influence of the auroral zone and from the solar-diurnal variation are both minimal. Above that latitude, auroral zone contamination can be avoided by measuring IDV near noon, while below that latitude, the solar-diurnal contamination can be avoided by measuring IDV near midnight or at only a fixed hour, assuming the diurnal variation is roughly constant from one day to the next. The average IDV(h) for the day, IDV(1d), at latitudes even just a little away from Λ_0 will thus always be contaminated by ‘non-IDV’ effects and is not appropriate for serious work.

Mayaud [1980, page 2] pointed out that a geomagnetic index must monitor a single class of geomagnetic variations deriving from a distinct physical phenomenon and not be a mixture of effects from different physical causes. A heavily contaminated index like IDV(1d) is thus apt to confuse the unwary rather than to illuminate matters. Because of the non-physical nature of IDV(1d) and the problems with the scale-value for Helsinki, the IDV(1d) as it now stands [based on LEA13 HLS+ESK] should not be used in our composite for IDV derived from geomagnetism.

Mayaud [1980, page 13] noted: “the u index ... certainly suffers from intrinsic defects ... (2) One might suspect a contamination by the regular variation S_R , since its day-to-day variability should contribute to the interdiurnal variability defined by U. However, we tried to evaluate the importance of this contamination and were astonished at its relative smallness. Even if Mayaud was ‘astonished’ he still found the S_R contribution to be of the order of at least a third. Using the technique outlined above it seems possible to be able to use a station several degrees polewards of the limit recommended by SC10, by in the same step normalizing away the S_R dependence and the auroral zone proximity. This is important if there is data for only one or very few such stations, but one would, obviously, not use this technique if several low- and mid-latitude stations were available.