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Axial rotation, orbital revolution and solar spin–orbit coupling

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ABSTRACT

The orbital motion of the Sun has been linked with solar variability, but the underlying physics remains unknown. A coupling of the solar axial rotation and the barycentric orbital revolution might account for the relationships found. Some recent published studies addressing the physics of this problem have made use of equations from rotational physics in order to model particle motions. However, our standard equations for rotational velocity do not accurately describe particle motions due to orbital revolution. The Sun's orbital motion is a state of free fall; in consequence, aside from very small tidal motions, the associated particle velocities do not vary as a function of position on or within the body of the Sun. In this note, I describe and illustrate the fundamental difference between particle motions in rotation and revolution, in order to dispel some part of the confusion that has arisen in the past and that which may yet arise in the future. This discussion highlights the principal physical difficulty that must be addressed and overcome by future dynamical spin–orbit coupling hypotheses.

Key words: celestial mechanics – Sun: interior – Sun: magnetic fields – Sun: rotation.

1 INTRODUCTION

The first published description of the Sun's orbital revolution about the barycentre of the Solar system appeared in 1687, in Newton's *Principia* (Cajori 1934): '. . . since that centre of gravity is continually at rest, the Sun, according to the various positions of the planets, must continually move every way, but will never recede far from that centre.' In 1965, P. D. Jose published curves showing substantial agreement of Hale-cycle sunspot numbers and the rate of change of the solar orbital angular momentum $\mathrm{d}\boldsymbol{L}/\mathrm{d}t$ (Jose 1965). In subsequent years, this and other parametrizations of the solar motion have been related to the occurrence of solar prolonged minima (Fairbridge & Shirley 1987), to torsional oscillations in long-term sunspot group clustering (i.e. in active longitudes) (Juckett 2003), to short-term variations in solar luminosity (Shirley, Sperber & Faribridge 1990), to violations of the Gnevyshev–Ohl rule and to variability of the solar differential rotation (Javaraiah 2005) and to various other indices of solar variability (Wood & Wood 1965; Landscheidt 1999; Charvátová 2000).

The solar axial rotation plays a fundamental role in dynamo theories constructed to represent and replicate the solar magnetic and sunspot cycles. Some form of coupling between the solar rotation and the solar orbital revolution has long been suspected, in order to account for the observed relationships linking the orbital revolution with indices of solar variability. However, past attempts to identify a coupling mechanism have not met with success.

Zaqarashvili (1997) and Juckett (2000) present specific solar spin–orbit coupling hypotheses. However, the mechanism of spin–orbit coupling presented by Zaqarashvili (1997) reflects a subtle but still fundamental misapprehension of the nature of solar particle motions associated with the orbital revolution. The mechanism presented by Juckett (2000) involves an exchange of angular momentum, and thus differs from that of Zaqarashvili, but it too must be disqualified for similar reasons. Our objective in this note is to help to prevent the recurrence of future errors of this type.

2 THE SUN IN FREE FALL

Fig. 1 provides a simplified polar view of the system under consideration. The circles represent the body of the Sun at two times (designated T_1 and T_2). The curved solid arrow represents the trajectory of the centre of mass of the Sun about the barycentre (β) of the Solar system. The label $\boldsymbol{R}$ identifies the orbital radius vector linking the centre of the Sun and the Solar system barycentre. $\boldsymbol{r}$ is the position vector for the location A; this vector is referred to and originates at the centre of the Sun.

In order to isolate the motions of revolution, we will initially suppose that our subject body is not rotating. Thus, the locations labelled A and A′ on the figure represent the same location on the surface, and the dashed line gives the trajectory of that point during the interval T_1–T_2.

Also, shown in Fig. 1 is a bold arrow representing the position of the location A′ at time T_2 with respect to the system barycentre β. We see immediately that the radial distance (given by $\boldsymbol{R}+\boldsymbol{r}$) separating the points A and A′ from the barycentre varies significantly over the interval T_1–T_2.

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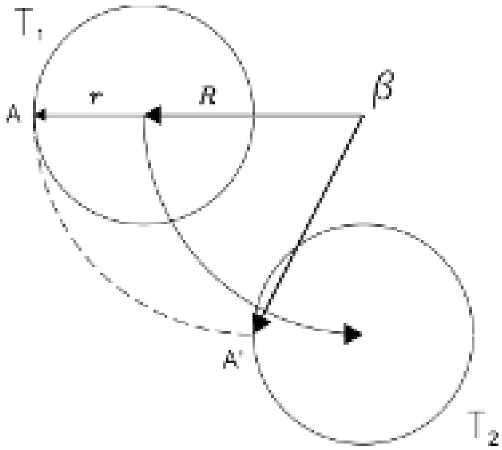

Figure 1. 'Revolution without rotation'. The circles represent the body of the Sun as viewed from a position to the north of the orbital plane. The curved arrow represents the orbital trajectory of the centre of the Sun about the centre of mass (β) of the Solar system. The dashed line represents the parallel trajectory of an arbitrary point A located on the Sun's surface. During the interval from T_1 to T_2, the Sun traverses an arc of 90°.

3 'COUPLING' OF ORBITAL AND ROTATIONAL VELOCITIES

The rotational velocity of a particle is proportional to the perpendicular distance of the particle from the axis of rotation; this may be obtained from

$$\boldsymbol{V} = \boldsymbol{\omega} \times \boldsymbol{r}, \tag{1}$$

where $\boldsymbol{\omega}$ represents the angular velocity of rotation. It seems quite reasonable to apply this equation to the case of orbital revolution, by employing an appropriate value for $\boldsymbol{\omega}$ and substituting $\boldsymbol{R} + \boldsymbol{r}$ for $\boldsymbol{r}$ in the above equation. If we do this (as in equation 1 of Zaqarashvili 1997), we obtain different inertial system orbital velocities for particles found at different locations within the body of the Sun. These differences are hypothesized to give rise to material flows within the Sun, thereby altering the rotational velocities, and thus coupling the orbital and rotational motions.

However, the use of the above equation for representing particle motions associated with the solar motion is incorrect. To see why this is so, we must recognize a fundamental difference between rotation and revolution. In rotation, the constituent particles of a subject body move in concentric trajectories with velocities that depend upon their position in relation to the axis of rotation (equation 1). In revolution, the particles of the body move in parallel trajectories with identical velocities (aside from small differences produced by the gradients that give rise to the tides). In gravitational physics, this motion is identified as a state of free fall (Misner, Thorne & Wheeler 1973).

Fig. 1 serves to illustrate the essential point. The velocity with respect to the barycentre β of the location A (or A′) is at all times identical to that of the solar centre of mass (CM_S). The orbital velocities of A and CM_S are identical, but their curvilinear trajectories are not concentric. In effect, each particle of the subject body revolves about its own unique centre of revolution. Thus, there can be no relative acceleration of any two constituent particles of the body of the Sun that is solely due to the revolution of the Sun about the Solar system barycentre; and the spin–orbit coupling hypothesis of Zaqarashvili (1997) must be discarded.

4 'COUPLING' OF ORBITAL AND ROTATIONAL ANGULAR MOMENTA

Juckett (2000) presents a solar spin–orbit coupling mechanism that involves a transfer of angular momentum between the orbital and rotational reservoirs. The spin angular momentum for particles may be written as

$$l = m\,\omega r^2. \tag{2}$$

Substituting $R + r$ for r in this equation (see Section 3 of Juckett 2000), and referring once more to Fig. 1, it is evident that the magnitude of the orbital angular momentum l for particles situated at the locations A and A′ must differ significantly. Juckett (2000) relates differences such as these to the observed variability of the solar differential rotation, suggesting that the differences in orbital angular momentum are in effect compensated by changes in the spin angular momentum. As noted in the introduction, there is circumstantial evidence to suggest that something of this sort may indeed be occurring.

However, in order for some external agency to alter the rotation state of an extended body or any of its parts, we require a torque, which may be represented most simply as a force with a non-vanishing moment arm when referenced to the rotation axis of the body. As previously described, the freely falling orbital motion of the Sun is unable to supply the required moment arm at any location; there are no differentials of force or acceleration within the Sun arising solely due to the orbital revolution. This has led many previous investigators to conclude that the motions of rotation and revolution are dynamically independent and uncoupled. Although the instantaneous orbital angular momentum for widely separated solar particles may differ significantly, this difference is considered to be without consequence for solar dynamics.

5 DISCUSSION

The inappropriate use of rotational equations for modelling particle motions due to orbital revolution is an ongoing problem (yet another example is found in Section 2 of De Jager & Versteegh 2005). The present discussion is intended to help to prevent the recurrence of future errors of this type.

The principal stumbling block for dynamical spin–orbit coupling hypotheses evidently lies in our identification of the solar motion as a state of free fall. To be successful, future solar spin–orbit coupling hypotheses must address and overcome this obstacle.

The disqualification of the particular hypotheses of Zaqarashvili (1997) and Juckett (2000) does not diminish the scientific interest of this problem. Evidence for the existence of some form of solar spin–orbit coupling has accumulated in recent years, and it is possible that some more successful hypothesis will in future resolve this puzzling conundrum.

The papers by Zaqarashvili (1997) and Juckett (2000) include valuable contributions that are unaffected by the issues discussed here; Zaqarashvili (1997) explores magnetohydrodynamics equations for periodic shear flows, and Juckett (2000) presents new findings on north–south asymmetries of solar variability. Finally, I note in passing that some of the details of the hypotheses of Zaqarashvili (1997) and Juckett (2000) have been neglected here in the interests of brevity. Zaqarashvili (1997) links the acceleration differences with the eccentricity of the solar orbit, while Juckett (2000) highlights north–south asymmetries in $\boldsymbol{R} + \boldsymbol{r}$ due to the obliquity of the solar rotation axis with respect to the invariable plane. The reader is directed to the original sources for further details.

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