

# Grand Minima of Solar Activity and the Mean-Field Dynamo

I.G. Usoskin · D. Sokoloff · D. Moss

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**Abstract** We demonstrate that a simple solar dynamo model, in the form of a Parker migratory dynamo with random fluctuations of the dynamo governing parameters and algebraic saturation of dynamo action, can at least qualitatively reproduce all the basic features of solar Grand Minima as they are known from direct and indirect data. In particular, the model successfully reproduces such features as an abrupt transition into a Grand Minimum and the subsequent gradual recovery of solar activity, as well as mixed-parity butterfly diagrams during the epoch of the Grand Minimum. The model predicts that the cycle survives in some form during a Grand Minimum, as well as the relative stability of the cycle inside and outside of a Grand Minimum. The long-term statistics of simulated Grand Minima appears compatible with the phenomenology of the Grand Minima inferred from the cosmogenic isotope data. We demonstrate that such ability to reproduce the Grand Minima phenomenology is not a general feature of the dynamo models but requires some specific assumption, such as random fluctuations in dynamo governing parameters. In general, we conclude that a relatively simple and straightforward model is able to reproduce the Grand Minima phenomenology remarkably well, in principle providing us with a possibility of studying the physical nature of Grand Minima.

**Keywords** Magnetic fields · Stars: late-type · Stars: magnetic fields · Sun: activity · Sun: magnetic fields

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I.G. Usoskin (✉)  
Sodankylä Geophysical Observatory (Oulu Unit), 90014 University of Oulu, Finland  
e-mail: [Ilya.Usoskin@oulu.fi](mailto:Ilya.Usoskin@oulu.fi)

D. Sokoloff  
Department of Physics, Moscow State University, Moscow, 119992, Russia  
e-mail: [sokoloff@dds.srcc.msu.su](mailto:sokoloff@dds.srcc.msu.su)

D. Moss  
School of Mathematics, University of Manchester, Manchester M13 9PL, UK  
e-mail: [moss@ma.man.ac.uk](mailto:moss@ma.man.ac.uk)

## 1. Introduction

Cyclic solar activity in form of the “11-year” activity cycle is thought to be a manifestation of solar dynamo action, driven by differential rotation and the mirror asymmetry of solar convection. Solar dynamo theory can explain at least the most basic features of the solar cycle. However, the temporal behavior of the large-scale solar magnetic field is more complex than just a regular cycle. From time to time in solar history cyclic solar activity became very weak or was even almost interrupted by events known as Grand Minima. The most recent of such minima is the well-known Maunder Minimum, which occurred during the second half of the 17th and the beginning of the 18th centuries. It is far from being obvious *a priori* that such long-term dynamics can be related to solar dynamo action.

A mechanism that can in principle produce the long-term dynamics of the solar cycle in the form of Grand Minima has been suggested by Hoyng (1993), namely stochastic fluctuations of the dynamo driving parameters. The idea is as follows. Mirror asymmetry contributes to dynamo action through the “ $\alpha$ -effect,” which gives rise to an electromotive force component  $\mathcal{E} = \alpha \mathbf{B}$ . This is parallel to the mean magnetic field  $\mathbf{B}$  (rather than perpendicular, as is usual). The  $\alpha$ -effect arises in the course of averaging of the magnetic field over an ensemble of turbulent pulsations. The point is however that the number of turbulent cells in the Sun is small in comparison with Avogadro’s number in statistical physics, and so the fluctuations of  $\alpha$  remain important. Direct numerical simulations of MHD turbulence as the driver of a dynamo (Brandenburg and Sokoloff, 2002; Otmianowska-Mazur, Koval, and Hanasz, 2006) support the idea that an  $\alpha$ -effect that is generated by a mirror-asymmetric turbulent flow is associated with substantial fluctuations on time scales comparable with the magnetic cycle length.

Moss *et al.* (2008) applied this idea to simple models of the solar dynamo, using the Parker migratory dynamo as well as similar, somewhat more elaborate, models to demonstrate that sequences of Grand Minima can be produced that appear random and have statistical properties that are broadly compatible with the analysis of the isotopic reconstruction of solar activity over past 10 000 years, as suggested by Usoskin, Solanki, and Kovaltsov (2007). An obvious next step is to explore how robust and specific this explanation is; this we investigate here.

## 2. The Dynamo Model

We use a simple dynamo model, as studied in Moss *et al.* (2008), which is a straightforward generalization of the initial Parker (1955) migratory dynamo. The governing equations can be written in standard dimensionless form

$$\frac{\partial B}{\partial t} = D \sin \theta \frac{\partial A}{\partial \theta} + \eta \frac{\partial^2 B}{\partial \theta^2} - \mu^2 \eta B, \quad (1)$$

$$\frac{\partial A}{\partial t} = \alpha B \frac{\partial A}{\partial \theta} + \eta \frac{\partial^2 A}{\partial \theta^2} - \mu^2 \eta A. \quad (2)$$

Here  $B$  represents the toroidal component of magnetic field,  $A$  is the azimuthal component of the vector potential of the poloidal component,  $\theta$  is co-latitude (*i.e.*, polar angle),  $D$  is the dimensionless dynamo number (a measure of the intensity of dynamo action),  $\mu$  is a measure of the thickness of the convective zone (we study the case  $\mu = 3$ , as representing the thickness of the solar convective envelope),  $\alpha$  is the alpha-effect coefficient, and  $\eta$  is

the turbulent diffusivity. Further background and justification of this model are given in Moss *et al.* (2008). In that paper we considered random fluctuations and a naive algebraic quenching of the  $\alpha$ -effect in the form

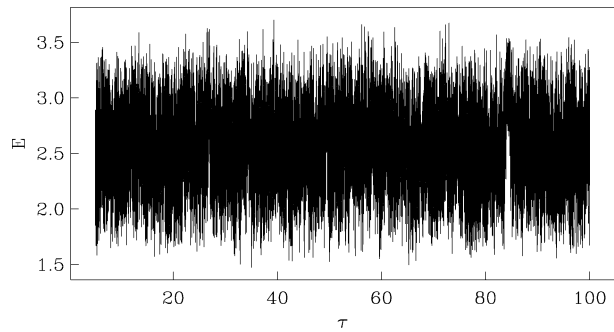
$$\alpha = \frac{\alpha_0 \cos \theta}{1 + B^2/B_0^2} [1 + r(t)], \quad (3)$$

where  $\alpha_0$  is the nominal value of the alpha-effect, which is incorporated into the definition of dynamo number  $D = R^3 \alpha_0 \Omega_0 / \eta_0^2$  (where  $\Omega_0$  is an angular velocity), and  $r(t)$  are random fluctuations that we take to be independent in the Northern and Southern hemispheres. [In Equation (3) we have reverted temporarily to dimensional variables, where  $B_0$  is the equilibrium magnetic field strength at which the nonlinear effects become important – below  $B_0$  is used as the unit for magnetic field.] To check the robustness of the result we try here also, quite arbitrarily, an alternative suppression of the dynamo action, by enhancement of the turbulent diffusivity in the form  $\eta = \eta_0(1 + B^2/B_0^2)$  or  $\eta = \eta_0(1 + B^2/B_0^2)^{1/2}$ . (The nominal value  $\eta_0$  appears in the definition of  $D$ .) The fluctuations are quantified by their memory time  $T$  and standard deviation  $\sigma$ . We note in advance that these alternative treatments do not, in general, significantly alter the results. Moss *et al.* (2008) demonstrated that the particular form of the memory is not very important for the results obtained; for the sake of definiteness we take here model III from Moss *et al.* (2008).

### 3. Grand Minima and Dynamical Chaos

Solar dynamo models are known to yield chaotic regimes of magnetic field evolution for certain values of the dynamo governing parameters even in the absence of any fluctuations (*e.g.*, Tworkowski *et al.*, 1998). We check here, using an example of the simple Parker migratory dynamo with nonlinear suppression of  $\alpha$ , whether such a model can give sequences of Grand Minima. The model, which is governed by two parameters – dynamo number  $D$  and thickness of convective zone  $\mu$  – is that investigated by Moss *et al.* (2004). Regimes where the magnetic field evolution appeared irregular and at least apparently chaotic were obtained when  $D$  was chosen to be highly supercritical and the  $\alpha$ -quenching was implemented in the form  $\alpha = \alpha_0 \cos \theta (1 - B^2/B_0^2)$  (where  $B$  and  $B_0$  are as defined earlier). Irregular oscillations associated with a traveling wave were obtained provided that  $\mu$  was large enough (*i.e.*, the shell is very thin). The long-term evolution of the model is shown in Figure 1 for  $D = -3 \times 10^5$  and  $\mu = 10.0$ . Irregular variations of the cycle amplitude are visible in the figure. Sometimes the magnetic energy deviates substantially from its typical values, but no pronounced Grand Minima are visible. Application of a formal procedure, employed by Usoskin, Solanki, and Kovaltsov (2007) to identify Grand Minima from the isotopic data, confirms this conclusion. We conclude that, at least in the framework of simple Parker-type models of the solar dynamo, and in the absence of fluctuations of the dynamo governing parameters, we are unable to find regimes with Grand-Minima-like behavior. [Deterministic models exist that *can* exhibit intervals superficially resembling Grand Maxima and/or Minima, but they are of a rather different type from those considered here, and it is not apparent that they are relevant to the solar behavior (Tobias, 1996; Moss and Brooke, 2000; Brooke, Moss, and Phillips, 2002).]

**Figure 1** Temporal evolution of the energy  $E$  of the toroidal magnetic field for a model with a nonfluctuating dynamo;  $D = 3 \times 10^5$  and  $\mu = 10.0$ . Behavior is irregular, but there are no Grand Minima.

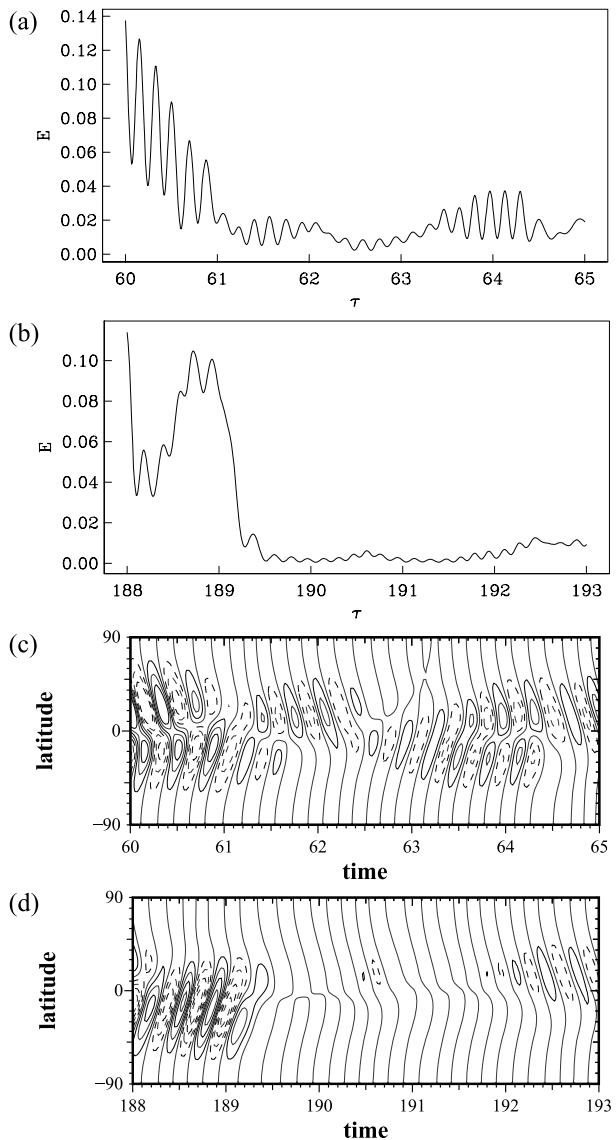


#### 4. Cycles inside a Grand Minimum

The question of whether solar cycles persist during a Grand Minimum and, if so, whether their phase and frequency remain locked is important for understanding the physical nature of Grand Minima. There is some evidence that the solar cycle continued, at least in some form, inside the Maunder Minimum. Solar, geomagnetic, and heliospheric activities were dominated by the 22-year cycle (Usoskin, Mursula, and Kovaltsov, 2001) with indications of the underlying 11-year cycle (Frick *et al.*, 1997; Beer, Tobias, and Weiss, 1998; Fligge, Solanki, and Beer, 1999; Usoskin, Mursula, and Kovaltsov, 2001; Miyahara *et al.*, 2004; Sokoloff, 2004). While the very existence of the solar cyclic variability (even though at a barely detectable level) during the Maunder Minimum is beyond doubt, the question of the cycle length is not yet resolved. Although some analyses of sunspot and  $^{10}\text{Be}$ -based data suggest that the cycle length was roughly the same as usual, that is, about 11 years (Beer, Tobias, and Weiss, 1998; Hoyt and Schatten, 1998; Fligge, Solanki, and Beer, 1999), data on the  $^{14}\text{C}$  cosmogenic isotope suggest that cycles may have been somewhat longer than usual during the Maunder and Spörer Grand Minima (Miyahara *et al.*, 2004, 2006) and some evidence for the presence of such behavior in sunspot data was reported by Frick *et al.* (1997). Therefore, the direct (telescopic sunspot numbers) and indirect (cosmogenic isotopes) data suggest that the solar cycle persists during a Grand Minimum with the cycle length being either comparable to or slightly longer than the nominal length.

Here we investigate what happens to the cyclic behavior inside a Grand Minimum in our models. We analyze the model presented in Figure 4 of Moss *et al.* (2008) (where  $T = 0.12$  and  $\sigma = 0.15$ ; we note a misprint in the figure caption in that paper where values of  $T$  and  $\sigma$  were interchanged). We show in Figure 2 the detailed temporal evolution of the energy of the toroidal magnetic field and the butterfly diagram for the toroidal field during two Grand Minima events in simulation number 36 of Moss *et al.* (2008). Here the fluctuations of  $\alpha$  in the two hemispheres are independent. It is clearly seen in Figures 2a and 2b that the oscillation persists within the Grand Minimum. The cycle amplitude there remains very low in comparison with that outside the Grand Minima, so the result is compatible with the possible absence of clear observed cycle manifestations in the record of sunspot activity. Note that in each case the figure includes an epoch just before the Grand Minimum as well as the Grand Minimum itself. We emphasize that the examples considered here are typical of our simulations. Correspondingly, the model was not tuned to reproduce all details of the solar Grand Minima. In particular, the Grand Minima produced by our simple Parker-type model are perhaps too long in comparison with solar Grand Minima. We acknowledge that the magnetic energy presented here may not be straightforwardly translated into sunspot number, which is probably a threshold phenomenon, and our determination of the activity

**Figure 2** Magnetic field evolution inside Grand Minima. Panels (a) and (b) show the evolution of the energy of the toroidal magnetic field in two Grand Minima (run 36). Panels (c) and (d) show the butterfly diagram for the toroidal magnetic field corresponding to panels (a) and (b), respectively. In panel (d) the field strength in the Grand Minimum is so much smaller than that immediately before that for some cycles no contours are visible, although they can be seen if the lowest contour level is reduced, and the butterfly diagram does persist at a much lower intensity. The continuous contours indicate positive or zero values, and broken contours indicate negative values. The contours extending from top to bottom of (c) and (d) are the zero contours.



level is thus only qualitative (Moss *et al.*, 2008). On the other hand, the episodes we identify as Grand Minima in the simulations by reason of their reduced energy are so clear that no ambiguity remains concerning their difference from the normal activity.

We conclude from Figure 2 that, in the framework of the model considered, the cycle length remains more or less stable during Grand Minima, whereas the cycle amplitude varies very substantially. We recognize strong deviations from the North–South asymmetry and in particular butterfly diagrams located in one hemisphere only during low activity episodes. Such butterfly diagrams are known from the end of the Maunder Minimum (Sokoloff and Nesme-Ribes, 1994). The Grand Minima episode presented in Figures 2b and 2d reproduces the abrupt transition into the minimum reported for the Maunder Minimum by Frick *et al.*

(1997) as well as substantial deviations from North–South symmetry in the butterfly diagrams in an episode a few cycles before the Grand Minima. Such behavior is deduced from the available, admittedly rather sparse, sunspot data from just before the Maunder Minimum (Nesme-Ribes *et al.*, 1994; Sokoloff, 2004) and may represent a general feature of Grand Minima (Usoskin and Mursula, 2003).

It is noteworthy that sometimes an additional cycle appears in the magnetic field evolution in one hemisphere compared with the other. Such episodes can perhaps, to some extent, be related to the so-called lost solar cycle at the beginning of 19th century (Usoskin, Mursula, and Kovaltsov, 2003; Zolotova and Ponyavin, 2008). We did not find indications of such behavior during medium- or high-activity times in selected intervals of one of our simulations, suggesting that if such events occur away from Grand Minima they are rare.

A visual inspection of Figure 2 suggests that the cycle length remains roughly constant, 0.15–0.2 dimensionless time units, in all regimes in our model and a more detailed examination of the data suggests that the maximum variation from the mean period is about 20%.

Therefore, we can conclude that the cyclic behavior of solar magnetic variability during Grand Minima, as produced by our model, is generally consistent with the observational results.

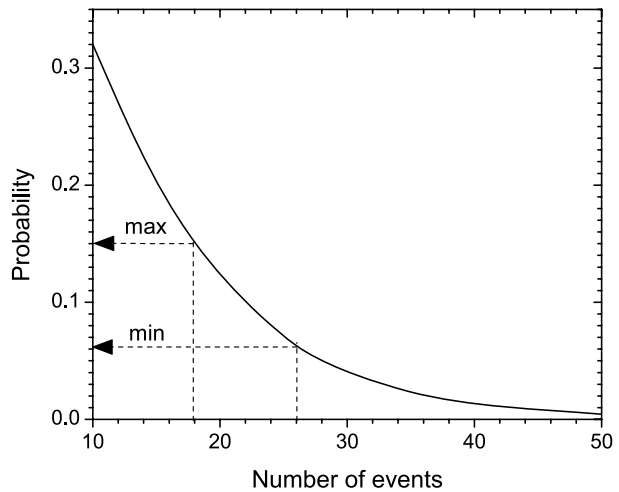
## 5. Statistics of Grand Minima

Of particular interest is an analysis of the distribution of waiting times (WTD – waiting time distribution) between consecutive Grand Minima, which can shed light on the nature of Grand Minima occurrence, whether as being a result of a periodic, chaotic/stochastic memoryless process or a non-Poissonic process (Usoskin, Solanki, and Kovaltsov, 2007; Moss *et al.*, 2008). Generally, an exponential WTD implies that the process resulting in Grand Minima is a Poissonic memoryless process with time-constant probability, whereas significant deviations from an exponential form suggest the existence of “memory” or self-organization in the process. It was reported earlier (Usoskin, Solanki, and Kovaltsov, 2007) that the WTD for the solar Grand Minima, as reconstructed from cosmogenic isotope  $^{14}\text{C}$  data for the Holocene (Solanki *et al.*, 2004; Usoskin, Solanki, and Korte, 2006), deviates significantly from exponential form, thus implying a kind of “memory” in the long-term solar activity evolution. A total of 27 clear Grand Minima have been identified by Usoskin, Solanki, and Kovaltsov (2007) in the reconstructed series, which covers 11 millennia or roughly 1000 solar cycles. Here we investigate the possibility that an essential deviation from the exponential WTD can appear in the data because of the limited length of the data series analyzed.

First, we studied the probability of obtaining a nonexponential (power-law) WTD for a limited subset taken from data that have, by definition, an exponential WTD (*i.e.*, a Poissonic process). This was done as follows:

1. We generated an artificial series of length  $n$  of pseudo-random numbers with an exponential distribution (see Figure 4),  $R_{\text{exp}} = -\lambda \ln R_u$ , where  $R_u$  is a pseudo-random number uniformly distributed between 0 and 1.
2. For each such series we computed the cumulative distribution  $C(x)$ , defined as  $\nu(x)/n$ , where  $\nu(x)$  is the number of elements of the analyzed  $R_{\text{exp}}$  series with value exceeding  $x$  (see Figure 4 as an example). The first and last two points, corresponding to the shortest and longest waiting times, respectively, were excluded.

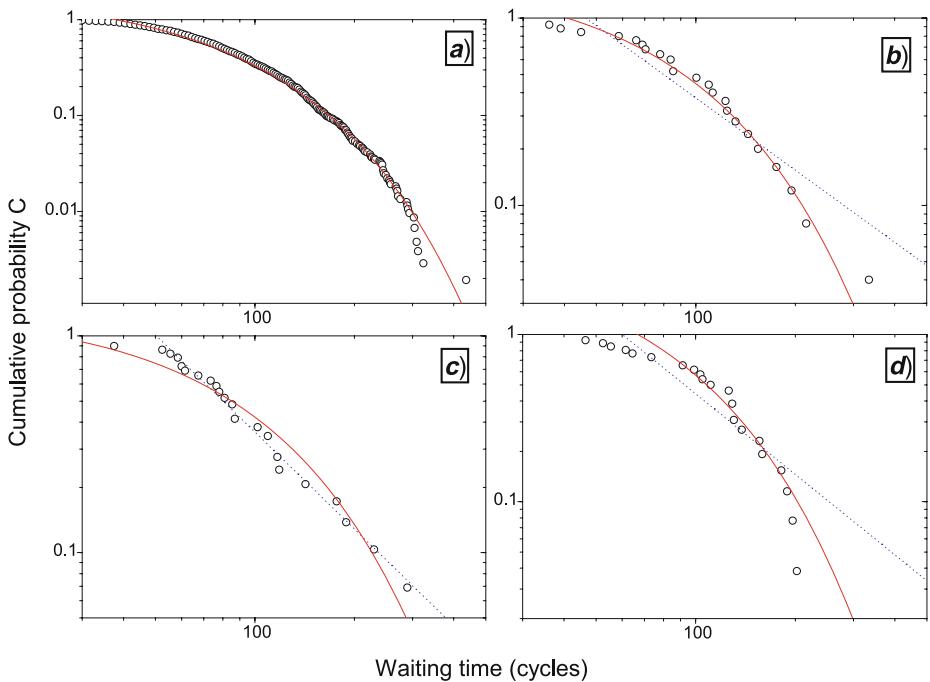
**Figure 3** The probability of finding an apparent power-law distribution in a subset of the truly exponentially distributed data, as a function of the sample length  $n$  (see text). Arrows “min” and “max” denote the corresponding probabilities for the data on Grand Minima and Maxima, respectively, as identified in the reconstructed solar activity (Usoskin, Solanki, and Kovaltsov, 2007).



3. The cumulative distribution  $C(x)$  was formally fitted (by using the logarithmic least square method) by power-law and exponential functions. Examples of the apparent exponential and power-law WTD in different subsets of the same data set are shown in Figures 4b and 4c, respectively.
4. If the (logarithmic) rms for the power-law fitting appears smaller than the (logarithmic) rms for the exponential fit, the corresponding  $R_{\text{exp}}$  series was identified as having a power-law distribution.

This procedure (steps 1–4) was repeated  $N = 10^5$  times, and the number  $N_{\text{PL}}$  of realizations with a power-law distribution (step 4) was counted. Finally, the probability  $P(n)$  of misinterpreting the truly exponential distribution as being a power law is defined as the ratio of  $N_{\text{PL}}/N$ . In this case the seemingly power-law distribution, with enhanced tails of the distributions, appears solely as a result of statistical fluctuations of the analyzed data subset, and the probability of misinterpretation is greater for shorter samples and vanishes with increasing subset length. As an indirect support for this interpretation, we recognize that sometimes WTDs with “reduced tails” (see Figure 4d), which would imply a “memory effect” different from that of a power-law WTD, appear as random fluctuations. The result of this simulation is shown in Figure 3. One can see that the probability of misinterpreting the real data is small but not negligible. The WTD for Grand Minima (27 Grand Minima correspond to 26 waiting intervals), which was identified as a power law (Usoskin, Solanki, and Kovaltsov, 2007), can in fact appear as a result of unlucky sampling of a Poissonic process, with a probability of about 6%. The probability of misinterpretation is significantly higher (about 15%) for Grand Maxima [with 19 Grand Maxima identified by Usoskin, Solanki, and Kovaltsov (2007)].

With this in mind, we have studied the correspondence between the statistics of Grand Minimum occurrence as simulated in different models and the real data. Although all the simulated series do possess Poissonic behavior (exponential WTD) when continued for long enough (similar to Figure 4a), seemingly power-law WTDs may appear in subsets with lengths comparable to that of the real data. To study this in detail we randomly took subsets of 1000 nominal cycle lengths from the modeled series and, similarly to steps 2–4, computed the probability of finding an apparent power-law distribution. The results for different models are shown in Table 1. The probability varies between 8% and 16% depending on the series. These variations arise because, although the length of the samples is



**Figure 4** Distribution of the cumulative probability of the weighting time exceeding the given value. Time is given in nominal solar cycles. The solid and dotted curves depict the best-fit exponential and power-law approximations, respectively. Panels show (a) the entire simulation 1114 (33 000 cycles), (b) an example of an exponential WTD (run 1115, sample 28), (c) an example of a power-law WTD (run 1114, sample 17), and (d) an example of a “reduced tails” WTD (run 1115, sample 29).

**Table 1** Statistics of occurrence of Grand Minima in the simulations, showing the parameter  $T$  and the total length of the simulated series (in nominal cycles). The last row represent the probability  $p$  of finding an apparent power law WTD for Grand Minima within a randomly taken subset of length 1000 nominal cycles (see text for definition). The values of  $D = -10^3$ ,  $\mu = 3$ , and  $\sigma = 0.15$  are the same for all runs. Runs 1112, 1114, and 1115 use quenching of dynamo generation in the form  $\eta = \eta_0(1 + B^2/B_0^2)$ , whereas run 1113 uses  $\eta = \eta_0(1 + B^2/B_0^2)^{1/2}$ . Runs 36, 37, and 38 are  $\alpha$ -quenched.

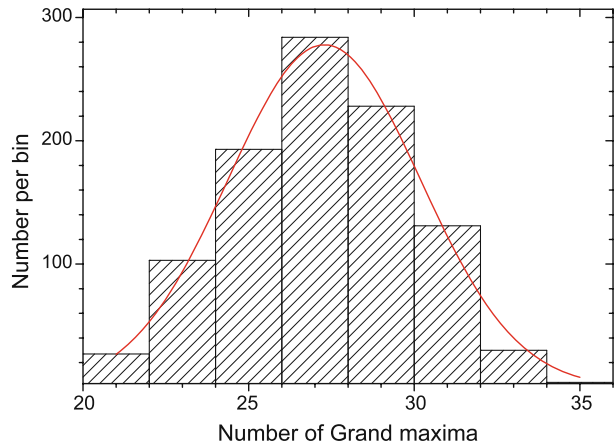
Run	1112	1113	1114	1115	36	37	38
$T$	0.12	0.12	0.06	0.2	0.12	0.06	0.2
Length	17 000	33 000	33 000	33 000	33 000	33 000	33 000
$p$	0.11	0.14	0.16	0.10	0.11	0.17	0.08

fixed at 1000 nominal cycles, the number of Grand Minima in each sample may vary. A histogram of the number of Grand Minima per 1000-cycle-long sample for the model run 1114 is shown in Figure 5. While the mean value is 27 (as set by adjusting the level of the Grand Minima defining threshold), there are samples containing 35 or 20 Grand Minima. Since the probability of misinterpretation of the WTD grows nonlinearly with decreasing number of Grand Minima (see Figure 3), this spread leads to an overall enhanced probability.

Thus, we can conclude that the true exponential WTD for the Grand Minima occurrence, produced by the model considered, is not entirely inconsistent with the seemingly power-



**Figure 5** Histogram of the number of Grand Minima identified in randomly selected 1000-cycle-long samples of the model 1114.



law WTD, obtained from cosmogenic isotope data. The limited statistics of the real data leave a small but not negligible chance that the true WTD nevertheless corresponds to a Poissonic process. Of course, we admit in principle that were a longer isotopic record for stellar activity to be available, a contradiction between the phenomenology of Grand Minima and exponential statistics might be proven and then a more sophisticated interpretation, such as discussed by Moss *et al.* (2008), would be inevitable.

## 6. Discussion and Conclusions

The present paper develops a scenario in which solar Grand Minima are generated by the solar dynamo as the result of random fluctuations of the dynamo governing parameters, as suggested by Moss *et al.* (2008).

Our results can be summarized as follows:

1. A simple model based on random fluctuations in dynamo governing parameters for a simple Parker-type dynamo is proposed that can reproduce the main features of the solar Grand Minima as they are known or estimated from direct and indirect solar data.
2. The fine structure of Grand Minima obtained from the model is in good agreement with the existing telescopic and isotopic data for the Maunder Minimum and isotopic data for the Spörer Minimum. In particular, the model can reproduce strongly asymmetric butterfly diagrams at some stages of a Grand Minimum, abrupt transition to the Grand Minimum, and gradual recovery after the Grand Minimum.
3. The model predicts that magnetic field cyclic oscillations persist during Grand Minima, in agreement with the results obtained from direct and indirect data for Maunder and Spörer Minima of solar activity. The greatly reduced amplitude of the solar cycle during a Grand Minimum is consistent with the very sparse occurrence of sunspots during the Maunder Minimum.
4. The model successfully reproduces the observed fact that the length of the activity cycle is much more stable than the cycle amplitude. We find that the cycle length variations produced by the model are of order of 10%–20%.
5. We have confronted the statistics of Grand Minima from our model with that from isotopic data. While the modeled WTD is truly exponential, implying a memoryless Poissonic process of the Grand Minima occurrence, it is not totally inconsistent with the

- power-law WTD found in the reconstructed solar activity evolution during the Holocene (Usoskin, Solanki, and Kovaltsov, 2007). We found that the restricted length of the isotopic record available (about 1000 cycles) gives about a 10% probability of finding an apparent power-law WTD in the series even if its WTD is truly exponential.
6. We have demonstrated that the model is robust in the sense that it not sensitive to the type of dynamo saturation used.
  7. We recognize that the fluctuations in dynamo parameters can also enlarge (rather than diminish) the amplitude of the activity cycle. In this context, it is tempting to look for Grand Maxima in our synthetic data and to make a comparison with the Grand Maxima claimed to exist in the solar record. We appreciate however that the definition of solar Grand Maxima from the available observations is not as straightforward as for Grand Minima (Usoskin, Solanki, and Korte, 2006). Analysis of our simulation data would present similar difficulties. Thus we have left examination of this problem for a future, more detailed work.

Summarizing these results, we conclude that a simple model, based on random fluctuations of the dynamo governing parameters in the framework of the standard Parker dynamo, can reproduce to some extent all the main features of the occurrence of Grand Minima of solar activity as known or estimated from direct and indirect solar data. Of course, the Parker dynamo (and indeed all mean-field dynamo models) omits much of the physics of stellar convection zones, but a pragmatic view suggests that some, at least, of the underlying behavior of the magnetic field is captured. State-of-the-art direct numerical simulations are highly noisy and lend some support to the idea that stochastic effects may play an important role in the long-term behavior of the solar magnetic field.

Note that the interpretation presented here is restricted to some extent by the limited nature of our present knowledge of solar cycle phenomenology. In particular, it looks plausible that, apart from the Grand Minima phenomena, solar activity sometimes demonstrates less pronounced episodes such as the Dalton Minimum, which is claimed to have occurred at the very beginning of the 19th century. Another episode of this type, which could be referred to as a “semi-Grand Minimum,” was possibly observed by P. Gassendi a few decades before the Maunder Minimum. However, because of bad luck the quality of the corresponding observations is substantially lower than for the Maunder Minimum (for a review, see Sokoloff, 2004). We could in principle isolate some episodes similar to these (for example, near  $\tau = 188$  in Figure 2b); however, our feeling is that it is better to reserve our opinion on such issues until there is substantial improvement of the observational basis.

Obviously, we do not reject *a priori* a possibility that the phenomenon of Grand Minima has some more specific nature rather just random fluctuations of dynamo governing parameters and stress that we consider the attempts to develop such an alternative explanation as very important.

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