Astronomiche Mittheilungen #38

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(page 378 ff)
Ca. 1875/76 looks to be a preliminary estimate of the epoch of the next sunspot minimum. The 12 spotless days in 1874 combined with the at least 36 spotless days in the first half of 1875 alone give the expectation that the minimum hardly could be later than 1876, and it looks then that this unusually short cycle is just where according to my epoch-table it should be if the 80-90 year period that {start emphasis} I already have suggested and done preparatory work on in Contribution #24 {end emphasis}, really will come to pass. That this would be of the greatest interest, I don't need to elaborate on for everyone who has been following my work; I just wish to remark that when this anomaly [the short cycle, Trans.] that is coming also shows up in the magnetic variations, then the last [doubting, Trans.] Thomas must accept the in parallel running sunspot number-curve and variation-curve.

The average relative-number that we derived above for 1874:

R = 44.6 corresponds to delta v = 0.045 r = 2.01

and the magnitude of the magnetic Declination variation in Central Europe should therefore, according to the work reported in Contribution #35 be 2.01 arc-minutes larger than their minimum values, which I have already determined as follows for

	Prague	Christiania	Munich
	5.89m	4.62m	6.56m
i.e.:	7.90m	6.63m	8.57m

In Prague, the observed value was 7.98m (according to lit.#333 from Hornstein), in Christiania 7.09m (according to lit.#332 from Fearnley), so that at the first location we have an extraordinary good agreement, at the second, at least a satisfactory agreement. The results of the Munich observations have not yet reached me.

Schiaparelli has added to his many worthy contributions the publication of the result of the series begun in 1836 of observations of variations of the magnetic declination at Milan in the Appendix to the third volume of "Memorie degli spettroscopisti Italiani" under the title "The eleven-year period in the diurnal variation of terrestrial magnetism considered in relation to the frequency of sunspots. Results of 38 years of observations done at Milan (1836-1873)", giving not only the annual means but also for each month, the daily difference between observed morning and afternoon values, taken close to those times of maximum deflection in the Declination.

Table I contains the data supplied by him with the sole difference that the six missing months of the year 1869 have been reconstructed from the average of corresponding months from the year before and the year after, supplemented by data from 1868-1870 for Prague; as a result of these interpolated data, the yearly mean for 1869, namely 8.42 is slightly different from the value, 8.78, given by Schiaparelli, but this small difference does not significantly alter the conclusions. Table II contains for the same years my recomputed, and until now - at least as far as monthly values are concerned - not wholly published Sunspot Relative-Numbers. Table III and IV, finally, contain the smoothed values of both the Milan-Variations and the Relative Sunspot Numbers, using the method already set forth in Contribution #33 and onwards. Comparison of those two tables, and, even more strikingly, curves constructed from them, show, anew, the parallel variation of the amplitude of the daily movement of the magnetic needle and of the frequency of sunspots; epochs of maxima and minima extracted from both tables corroborate the close agreement between the periods, allowing their duration to be inferred, even if not with the same certainty as from my already more than a quarter millennium-long table of epochs.

For construction of Tables III and IV, as well as for the execution of the, in no small measure, tedious numerical computations, the reason for which will become clear in what follows, I have had commendable help from my senior students (Mssrs. Leuch, Herzog, Wolfers [sic] and Keller). I here publicly express my gratitude for their help, and shall now continue with a report over the first calculations and results based on the above considerations [the variation of the magnetic needle, Trans.]: Table V contains first the from Table I and II derived yearly means v of the variations and r of the relative-numbers, and that for three groups of years 1836-1848, 1849-1861, and 1862-1873. For the first group r was primarily taken from Schwabe's observations, while for the two latter groups, my own observations dominate. For each year of each of these groups, the quantities a and b were now determined from the equation

$$v = a + b\,r \qquad (1)$$

and from those, the average values of a and b for each group. In this manner, we get for Milan the variation formulae

$v' = 6.990 + 0.0331\,r$	for	1836-1848
$v' = 5.039 + 0.0443\,r$		1849-1861
$v' = 4.383 + 0.0577\,r$		1862-1873

where v' is what is computed from these formulae, and placed in Table V' for comparison with v. The comparison yields the following standard errors { (Sum((v - v')^2)/n }

0.703	for	1836-1848
0.724		1849-1861
0.444		1862-1873

It is thus also possible to calculate the Milan variation from the sunspot number; however, the values for a and b are different from group to group as are the average errors. If these inconsistencies are related to the physical phenomenon or derive from instrumental problems I did not investigate at this time, although by comparison of the Milan series with other observatories one could resolve this problem. I would rather look at further consequences of the data in Table V: The values in column A are taken from Table A in Contribution #35, and the values in the column marked v-A show therefore location-dependent constant, alpha, in the usual variation formula for Central Europe

$$v'' = \alpha + 0.045\,r \qquad (2)$$

alpha had in Milan the values

6.76 as average for the years 1836-1848

5.05	1849-1861
5.04	1862-1873

so that the difference between the two last groups of years has now disappeared, while being even larger for the first group. If one uses these three values of alpha in eq.(2) to compute v'' and compare the result with v as done in Table V, one obtains for the standard errors

0.674	for	1836-1848
0.711		1849-1861
0.604		1862-1873

so that eq.(2) seems just as good as eq.(1) in expressing the Milan variations.

Schiaparelli's beautiful series of monthly means of the diurnal variation gave me the impetus to go ahead with a long-conceived plan, namely to derive variation formulae not only on a yearly basis but for each month. I limited, however, myself to only use the 25 years 1849-1873 during which I had personally observed the sunspots: I divided these 25 years in five groups according to the yearly average relative-number as follows

1870, 71, 72, 49, 60; 1859, 61, 69, 50, 73;
1851, 62, 58, 52, 64; 1863, 53, 68, 65, 57;
1854, 66, 67, 55, 56

and computed for each month in each group the average relative-number and the average Milan-variation. The result is given in Table VI and can be used in conjunction with eq.(1) to determine a' and b' for each month for each of the five groups. These values are entered into Tables VII and VIII. Next to each set I also entered the corresponding values a'' and b'' for the same groups of years for the Prague observations - and finally by dividing the a and b values by their averages forming the quotients q' and q'' and from the two corresponding quotients from Milan and Prague finally getting their average q. The two sets of a-values and the q-values derived from them show such a distinctive seasonal variation that it seems an obvious next step to express this variation by a periodic function, in fact by

qn = alpha + beta sin (gamma + n . 30degrees) (3)

where n is the number of the month. For qn values of the q-series one gets

alpha = 1.000 beta = 0.635 gamma = 279d 53m

and for the q'-series

alpha = 1.000 beta = 0.761 gamma = 277d 39m

from which eq.(3) yields the values in columns Q and Q' of Table VII [Wolf has by now surely lost the reader. Trans.]. The Q differ from the q on average +/-0.076, and the Q' differ from the q' on average by +/-0.092, so that the q seem to be only slightly better fitted than the q', meaning that the advantage of using the averages rather than just the Milan-series is only slight. And, as seen in Table VIII, substituting q' for b has a distinct negative influence on the result. So I have decided, at least for now, to use only the Milan-series and formed the A'-series from Q' by multiplication by 4.785. Comparing this series with the a'-values I get for d = a' - A' the average value +/-0.438. So, indeed, eq.(3)

in the form

A'n = 4.785m + 3.641m sin (277d 39m + n . 30d) (3)'

expresses very closely the observed average monthly variation of the Milan-series for a sun without spots, -- and this is quite interesting, {start emphasis} that, the angles for each month are within a few degrees of the value of the right ascension of the Sun for the middle of that month {end emphasis}, or that the formula for the part of the variation that does not vary with the state of the spottedness of the Sun gives its mean value shortly before the equinoxes (namely for n = 2.74 and n = 8.74), a maximum shortly before the summer solstice (namely for n = 5.74), but a minimum shortly before the winter solstice (namely for n = 11.74). This seems to corroborate my hypothesis, already expressed in Contribution #17 in 1865, that some variable associated with solar declination controls the magnitude of the daily variation of the Magnetic Needle, -- yes, I even had the hope, assuming that I was not misled by that preliminary data, to be able to present a more precise report in a not too distant future. It seemed to me that the residuals d still were somewhat systematic, with a semiannual variation, so I dared to try to express the residuals by the formula

dn = alpha' + beta' sin(gamma' + n . 60d) (4)

[Wolf had forgotten the mark on gamma. Trans.] I obtained in this way

alpha' = -0.006 beta' = 0.424 gamma' = 274d 42m

and when I then calculate D from the resulting equation

Dn = -0.006 + 0.424 sin (274d 42m + n . 60d) (4)'

and compare them with d, I found for the differences a mean value of +/-0.319, while the spread in d had increased to +/-0.438. There is thus an improvement, although not great, to be had. Anyway it is of interest that the phase angle gamma' came out so close to gamma that one might as well set them to be equal, and also that alpha' is equal to the average value of d, i.e. close to zero. -- The Milan-series of b' in Table VIII shows a variation similar to that of d, even more distinctly, with maxima at the equinoxes and minima at the solstices reminiscent of the annual variation of the frequency of aurorae. However, The Prague-series b'' has only weak vestiges of such a variation, almost lost in the averages q. I thought it therefore best to stay with only the Milan-series, and to calculate it from eq.(4) in the form

Bn' = 0.0495 + 0.01166 sin (239d 45m + n . 60d) (4)'

with the result in the B' column in Table VIII and in the differences b'-B'. As from the latter, the spread in the differences between b' and B' is so small, just +/-0.0067, it is clear that the Milan-series of b is quite well expressed by eq.(4)". But because of the disagreement with the results from the Prague data, that I had previously preferred, it is difficult to come to definite conclusion on this matter. -- However, the results of the above investigation, inclusive of the mysterious discrepancy between Milan and Prague, are interesting enough to warrant a more extensive study using several more stations. This is something that has been well underway already for quite some time, and only other un-delayable commitments have prevented the study to come to an end. I hope to be able to present, in a later Contribution, many interesting results of this work and close this one with just a further addition to the literature on sunspots: (....)