

# THE TOPOLOGY OF THE SUN'S MAGNETIC FIELD AND THE 22-YEAR CYCLE

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Received October 20, 1960

## ABSTRACT

Shallow submerged lines of force of an initial axisymmetric dipolar field of  $8 \times 10^{21}$  maxwells are drawn out in longitude by the differential rotation (after the suggestion of Cowling) to produce a spiral wrapping of five turns in the north and south hemispheres after 3 years. The amplification factor approaches 45, with a marked dependence on latitude. Twisting of the irregular flux strands by the faster shallow layers in low latitudes forms "ropes" with local concentrations that are brought to the surface by magnetic buoyancy to produce *bipolar magnetic regions* (BMR's) with associated sunspots and related activity. The field intensity required for producing BMR's is reached at progressively lower latitudes according to the derived formula  $\sin \phi = \pm 1.5/(n + 3)$ , where  $n$  is the number of years since the beginning of the sunspot cycle. This accounts satisfactorily for Spörer's law and the Maunder "butterfly diagram." Sufficient flux rope for more than  $10^3$  BMR's is produced. "Preceding" parts of BMR's expand toward the equator as they age, to be neutralized by merging; "following" parts expand or migrate poleward so that their lines of force neutralize and then replace the initial dipolar field. This process, which involves severing and reconnection of lines of force in the corona, as well as expulsion of flux loops, need be only 1 per cent efficient. The result, after sunspot maximum, is a main dipolar field of reversed polarity. The process repeats itself, so that the initial conditions are reproduced after a complete 22-year magnetic cycle. This model accounts for Hale's laws of sunspot polarity and provides a qualitative explanation of the preponderance of "preceding" spots, of the forward tilt of the axes of older spots, of the recurrence of activity in preferred longitudes, and of Hale's chromospheric "whirls."

The aim of this paper is to develop a model to account for the varying configuration of the sun's magnetic field and to test the validity of this model in explaining a number of solar phenomena that are dependent on the field. This requires a synthesis of several current ideas and theories relating to isolated phases of the main problem. The most crucial of the observations to be accounted for are taken to be (1) the reversal of the main dipolar field, (2) Spörer's law of sunspot latitudes, (3) Hale's laws governing the magnetic polarity of sunspots (now known to apply to bipolar magnetic regions in general), and (4) the fact that bipolar magnetic regions (BMR's) disappear by expanding.

The data regarding the main dipolar field and the BMR's have been obtained through an extended series of observations with the solar magnetograph, beginning in 1952 (Babcock and Babcock 1955; H. D. Babcock 1959). Less than half of a complete 22-year magnetic cycle has elapsed since the beginning of these observations, so this attempt at a synthesis of the findings may be premature.<sup>1</sup> One reversal of the main dipolar field has been observed, however, and we proceed on the assumption that the field will reverse again near the time of the next sunspot maximum. In regard to precision, the available data leave much to be desired. Improved instrumental calibration and closer control of the quantitative aspects of the records are needed. Of particular value would be more and better measurements of the distribution and quantity of magnetic flux in BMR's as a function of age.

The concept of lines of force is indispensable. Since  $\text{div } \mathbf{H} = 0$ , each line of force is regarded as a continuous loop, without beginning or end. Because of the high conductivity, motion of the ionized gas, or plasma, across the lines of force is generally regarded as insignificant. Exceptions are made, however, for special cases where extended intervals of time are available and where the conductivity is appreciable. For example, severing

<sup>1</sup> To quote Carrington (1858): "... our knowledge of the sun's action is but fragmentary, and the publication of speculations on the nature of his spots would be a very precarious venture."

and reconnection of lines of force in the corona are considered essential, and some slow transverse slipping of lines of force through the photospheric plasma is permitted. The lines of force do not restrict the longitudinal motion of the plasma.

The variations in the main magnetic pattern of the sun and especially the reversal of the high-latitude dipolar field occur on a time scale that is vastly shorter than theory permits if the field is assumed to decay by Joule heating in the usual manner (Cowling 1945). Therefore, it is essential that the observed variations be accounted for by motions or circulation of the highly conducting plasma in which the lines of force are imbedded. This implies that the pertinent internal fluid motions, with their associated magnetic fields, occur in a relatively shallow layer near the sun's surface.

The plan followed is to list in Section I a series of phenomena or data related to the solar magnetic field. Then in Section II a model is developed in various stages to account for the phenomena in terms of the variation of the field and the motions of the plasma to which it is coupled.

#### I. THE DATA

1. The main dipolar solar field is essentially axisymmetric and has a mean intensity in the photosphere of the order of 1–2 gauss. It shows much irregular fine structure, and it is usually limited to high latitudes, although the boundaries are quite irregular and variable. The total magnetic flux was estimated in 1954 to be about  $8 \times 10^{21}$  maxwells, and the polarity was then opposite to that of the earth's field. The foregoing measurements depend upon the Zeeman effect; they are supported as to order of magnitude by estimates of the field intensity (about 1 gauss) required to maintain local condensations along the lines of force in the corona (van de Hulst 1950) and more recently by radio observations which set an upper limit of about 2.5 gauss on the high-latitude photospheric field (Högbom 1960). The polar streamers of the corona emanate from just those zones in which the surface fields are observed, and there is every reason to believe that they delineate the lines of force in the high solar atmosphere (Gold 1958).

2. The main dipolar field weakened and then underwent a reversal of magnetic polarity, first (1957) in the south and over a year later in the north (H. D. Babcock 1959). Since reversal, the field has appeared in somewhat lower latitudes ( $45^\circ$ – $50^\circ$ ) than it did at sunspot minimum. The reversal occurred very nearly at the date of maximum activity in the current sunspot cycle, and Waldmeier (1960) has pointed out that the spot maximum as well as the field reversal occurred about a year later in the northern than in the southern hemisphere.

3. The angular velocity of rotation of the sun is greatest at the equator and decreases toward the poles; thus the period is about 25 days at the equator and 29.3 days at latitude  $60^\circ$ . Results from studies of sunspots, filaments, and Doppler effect are in fair, though not precise, accord. Newton and Nunn (1951) derived from sunspots the formula

$$\omega = 14^\circ.38 - 2^\circ.77 \sin^2 \phi, \quad (1)$$

where  $\phi$  is the latitude and  $\omega$  is the daily rate. M. and L. d'Azambuja (1948) found from studies of filaments the result

$$\omega = 14^\circ.42 - 1^\circ.40 \sin^2 \phi - 1^\circ.33 \sin^4 \phi. \quad (2)$$

Adams (1908) found from measurements on  $\lambda 4227$  (Ca I) and  $H\alpha$  that the elevated layer in which  $H\alpha$  is produced appears to rotate about 3 per cent faster than the reversing layer.

4. The frequency of sunspots varies in a somewhat irregular cycle of roughly 11 years' duration. That there is a change in the mean latitude of spots during the cycle was discovered by Carrington (1858). Spörer's (1894) law, illustrated by the well-known "butterfly diagram" of Maunder (1922), describes the general decrease of latitude.

The spots occur in zones parallel to the equator, the width of a zone being  $15^{\circ}$ – $20^{\circ}$ . The mean latitude of the first spots is about  $30^{\circ}$  north and south; of the last spots of a cycle, roughly  $8^{\circ}$  north and south.

5. Hale (1908) showed that sunspots invariably have strong magnetic fields (100–3800 gauss; about 1–2 kilogauss on the average). The total magnetic flux varies widely, but for a typical spot of good size we may adopt  $10^{21}$  maxwells. Spots tend to occur in bipolar groups, with “preceding” (*p*) and “following” (*f*) members (in the sense of the east-west rotation) showing opposite magnetic polarity. Hale (1913) found that, for the great majority of spot groups, the characteristic polarities of the *p* and *f* spots were opposite in the northern and southern hemispheres and that they were reversed from one 11-year sunspot-frequency cycle to the next. Hale and Nicholson (1938) therefore concluded that the magnetic cycle of the sun has a duration of 22 years. In the present cycle (and other odd-numbered cycles), which had its maximum in 1958, *p* spots in the northern hemisphere and *f* spots in the southern hemisphere have positive polarity. (For a field of positive polarity the magnetic vector is directed out of the sun.)

6. Preceding spots of a group tend to be larger and to have a longer lifetime than *f* spots (Grotrian and Künzel 1950). The mean ratio of the effective flux *in the spots* is about 3:1.

7. An east-west asymmetry in the apparent number of sunspots has been found. This may be interpreted as a forward tilt of the spot axes in the direction of rotation, as was pointed out by Mrs. Maunder (1907). Minnaert (1946) found that the spots of greatest age were tilted most—about  $0^{\circ}.44$  for all spots (average age 1 day) as compared with  $7^{\circ}.6$  for spots of age 27 days or more.

8. The orientation of spot groups is such that the *p* spot is generally closer to the equator than the *f* spot. The orientation angle between the axis of the group and the east-west line depends on the latitude of the group, being about  $19^{\circ}$  for groups at latitude  $30^{\circ}$ – $40^{\circ}$  and smaller for groups closer to the equator (Brunner 1930).

9. Sunspots tend to recur in longitudes where there has been prior activity. As Wolfer (1899) has stated, the persistence of activity in certain regions is often well marked. Losh (1939) concluded that both the regions of sunspot activity and the free regions tend to concentrate into small intervals of longitude.

10. Observations of filamentary chromospheric structure, made in the light of the hydrogen atom, sometimes show characteristic clockwise or counterclockwise spiral “whirls” around well-defined large spots; usually large preceding or unipolar (unaccompanied) spots are involved. Hale (1927) and later Richardson (1941) showed that a majority of such whirls in the northern hemisphere have the same sense of spiraling independent of the magnetic polarity of the spot or of the parity of the 11-year sunspot cycle and that the opposite sense of the spiraling prevails in the southern hemisphere. Hale (1927) showed that the prevailing sense of rotation of the whirls is as if the spot rotates in the same sense as does the sun; i.e., the equatorward edge of the spot moves ahead in the direction of the sun’s rotation. Richardson concluded that the whirls resulted from a hydrodynamic rather than an electrodynamic cause.

11. Extended sequences of solar magnetograms show that the emergence of bipolar magnetic regions (BMR’s) underlies the development of spot groups and centers of activity on the sun. Weaker BMR’s, however, may show little or no associated activity. The BMR’s, more generally than spots, obey Hale’s polarity laws (5). Within the rather loose limits set by the precision of the observations, and in accord with the requirements of theory, equal amounts of magnetic flux, of opposite polarity, are observed in the *p* and *f* parts of BMR’s. While exceptions occur, most BMR’s are initially compact and show rapid growth in total flux until a maximum is reached, after which this quantity probably remains nearly constant while the area of the BMR increases and its field strength correspondingly decreases.

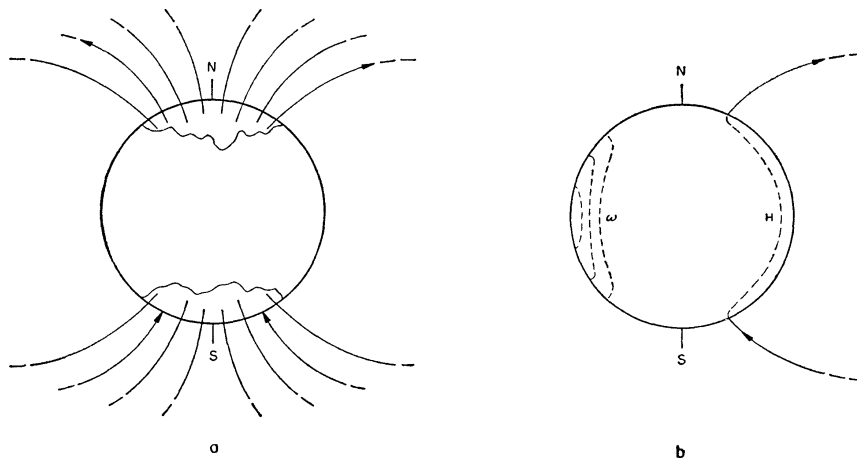
12. Calcium flocculi, hydrogen flocculi, faculae, and sunspots appear in that order

in a growing BMR as the magnetic-field intensity increases from about 10 to over 100 gauss. These features disappear in reverse order as the area expands and the field weakens. In later stages, the BMR can be identified only by its weak field ( $< 10$  gauss) after the dependent phenomena have vanished. Occasionally, large BMR's of low field intensity, which seem never to show optical activity, have been detected. Long stable filaments or prominences are characteristically supported at right angles to the magnetic lines of force above older BMR's; they divide the region into its  $p$  and  $f$  parts. Other stable filaments seem to delineate the poleward boundaries of some BMR's.

## II. FIELD VARIATIONS DURING THE SOLAR CYCLE

### *Stage 1. The Initial Dipolar Field*

The field of the model approximates an axisymmetric dipole with lines of force lying in meridional planes as in Figure 1. The polarity of the field is opposite of that of the



STAGE I

FIG. 1 —(a) The main dipolar field of stage 1 has a mean intensity in the polar caps of 1–2 gauss, and its total magnetic flux is  $8 \times 10^{21}$  maxwells. The lines of force emanate only from the polar regions. (b) The submerged part of each line of force is relatively shallow; together they lie in a layer which is thinnest between the  $30^\circ$  parallels of latitude. The lines labeled by  $\omega$  are cross-sections of the surfaces of constant angular velocity or "isotachs."

earth, so that the situation corresponds to that of the sun in 1952, about 3 years before the beginning of a new sunspot cycle. Any low-latitude active regions at this stage are regarded as residual effects of the preceding cycle. The coronal streamers near the poles partially delineate the lines of force of the dipole. The lines of force loop out from the north polar cap (with positive polarity), cross the equatorial plane at a distance of at least several solar radii, and re-enter the sun in the southern polar cap. At the surface, only latitudes greater than  $\pm 55^\circ$  are affected.

The total flux is  $8 \times 10^{21}$  maxwells, as estimated in 1954. All internal lines of force are taken to lie in a relatively thin submerged layer or sheath at a depth of the order of  $0.1 R$  that stretches between  $\phi = +55^\circ$  and  $\phi = -55^\circ$ . We assume for purposes of calculation that the sheath is uniform and that its thickness, between latitude limits of  $\pm 30^\circ$ , is  $0.05 R$ ; at higher latitudes the thickness increases as the sheath is "faired in" to the polar cap. The field intensity will be proportional to the ratio of the area of the polar cap to the cross-section of the sheath. We have

$$H_\phi = H_0 \sec \phi \quad (-30^\circ < \phi < +30^\circ), \quad (3)$$

where  $H_0$ , at the equator, is about 5 gauss.



*Stage 2. Amplification*

The submerged lines of force are drawn out by the differential rotation of the sun, an effect represented with the aid of surfaces of constant angular velocity—"isotachial surfaces"—which are taken to be surfaces of revolution about the sun's axis, symmetrical about the equatorial plane. According to Ferraro's (1937) "law of isorotation," each magnetic line of force must lie entirely on an isotachial surface if the field is to remain undistorted. But it is evident that the field is, in fact, increasingly distorted, and this comes about through breakdown of the conditions of isorotation.

Evidently the isotachial surfaces cut more deeply into the sun than do the magnetic lines of force, for only on this assumption will the lines of force be drawn out in longitude in the appropriate direction. The elongation of the submerged part of each line of force

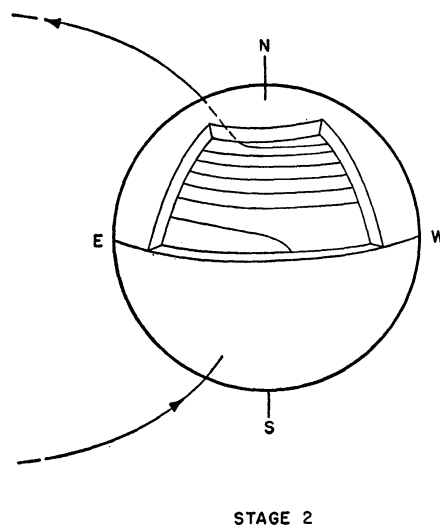


FIG. 2.—The submerged lines of force have been drawn out in longitude and wrapped around the sun by the differential rotation, with a consequent amplification of field strength that depends on latitude.

in response to the hydrodynamic flow results in the line's gradually assuming a nearly east-west direction. Thus on each side of the equator there is formed a fairly tight spiral—essentially a toroidal field—that is really another aspect of the general field. This part of the development has been adopted directly from Cowling (1953). Related ideas have been advanced by Bullard (1955) and Elsasser (1956).

Let us suppose that the foregoing amplification process, beginning with stage 1, proceeds for 3 years prior to the onset of an odd-numbered sunspot cycle. After 3 years, the equator will have gained in its rotation about 5.6 turns on the latitude circles at  $\phi = \pm 55^\circ$ , and each line of force will have made nearly as many turns about the sun in both northern and southern hemispheres (Fig. 2).

We now wish to find the degree of amplification of the field as a function of latitude. From equation (1) the differential advance in longitude in radians is

$$\theta = 17.6 (n + 3) \sin^2 \phi, \quad (4)$$

where  $n$  is the elapsed time in years measured from the beginning of the new sunspot cycle. The angle  $\psi$  that the spiraling line of force makes with the meridian is given by

$$\tan \psi = \frac{d\theta}{d\phi} = 35.2 (n + 3) \sin \phi \cos \phi. \quad (5)$$

We have, for the field intensity in the spiral,

$$H = H_0 \sec \phi \sec \psi . \quad (6)$$

When  $\psi$  is large, an adequate approximation is

$$H = H_0 \sec \phi \tan \psi = 35.2 (n + 3) H_0 \sin \phi . \quad (7)$$

Taking  $H_0 = 5$  gauss and  $n = 0$ , we find

$$H = 528 \sin \phi \quad (2^\circ < \phi < 30^\circ) . \quad (8)$$

The upper limit to which equation (8) applies is really an empirical limit set by observation. It is justified on our model by weakening of the field toward higher latitudes as the submerged lines of force spread upward to merge into the polar cap. One finds that after 3 years of amplification ( $n = 0$ ) the field in the sheath at  $\phi = \pm 30^\circ$  has reached a critical value of 264 gauss. This is believed to be adequate (as will be justified later) to induce sunspot activity at this latitude and, at the same time, by reason of this activity, to terminate amplification at the same latitude. It should be noted that activity can proceed for some time at a given latitude after amplification ceases. But at lower latitudes the critical field intensity required for the onset of magnetic activity will be attained later, so that amplification can proceed for a longer time. Placing  $H = 264$  and  $H_0 = 5$  in equation (7), we find that the relation between the number of years elapsed since the beginning of the cycle and the latitude at which the field is attaining the critical value is

$$\sin \phi = \pm \frac{1.5}{n + 3} . \quad (9)$$

Equation (9) is represented by the graph of Figure 4.

The distribution of lines of force cannot be expected to be smooth, as so far assumed. Local irregularities in the fluid motion due to convection will produce some distortion of the isotachial surfaces, which in turn will result in irregularities and occasional concentrations in the magnetic field. The fluid shear will be affected by the increased magnetic viscosity of local field concentrations, and these will be twisted into more or less discrete flux strands or "ropes." The greater forward velocity of the shallower layers of the sun near the equator will induce relatively rapid twisting of the flux ropes in the direction in which the sun rotates. In this connection the flux ropes may be visualized as roller bearings. Because this process occurs in a convective region in which a certain randomness of fluid motion prevails, it must be expected that the cross-section of each rope will be non-uniform along its length. If there is a local constriction due to excess twisting, the plasma within the rope will be squeezed along it, causing adjacent sections to swell. The field strength within the rope will be inversely proportional to the cross-section, while the magnetic pressure will be inversely proportional to the fourth power of the diameter of the rope.

Lundquist (1951), following a suggestion by Alfvén (1950), has shown that sufficient twisting of a flux rope results in instability, which occurs when the increase in magnetic energy due to twisting becomes of the same order of magnitude as the energy of the original field in the tube. The instability may manifest itself in the formation of a loop.

In recognizing that the magnetic flux of the amplified field is concentrated into ropes instead of a uniform sheath of some 260 gauss, we see that the field intensity in the ropes may very probably be several times greater than that in a sheath—say 1–2 kilogauss. Then, if the ropes are twisted, further amplification by a factor of 2 is admissible before instability sets in. The field intensity of such a rope of magnetic flux is then in the correct range to provide for the fields of BMR's and of the sunspots that occur

within them. The limit of the amplification of stage 2 is set by conditions of instability and buoyancy of the flux ropes which prevail when the field is sufficiently strong. Evidently the limit will be reached gradually, first in those latitudes ( $\sim 30^\circ$ ) where the field is stronger, and later in lower latitudes. Only after the flux ropes have been fragmented and distorted in stage 3 will the amplification come to a halt.

*Stage 3. Formation of Bipolar Magnetic Regions (BMR's) from the Flux Ropes*

Loops in the submerged flux ropes may be formed as instabilities due to twisting, as considered by Lundquist, but the magnetic buoyancy of locally intense concentrations of flux is probably of equal or greater importance here. Magnetic buoyancy will tend to lift concentrated flux ropes to the surface when the magnetic pressure,  $H^2/8\pi$ , becomes comparable to the gas pressure. For this phase of the development we adopt those elements of Parker's (1955) theory concerned with magnetic buoyancy.

If the gas pressure internal and external to the flux rope is  $p_i$  and  $p_e$ , respectively, we have

$$p_e = p_i + \frac{H^2}{8\pi}. \quad (10)$$

The magnetic pressure term is always positive, so that  $p_i < p_e$ . Now if the temperature is the same inside and outside the flux rope, it follows that the rope is lighter than its surroundings and tends to rise. Parker showed that flux strands of 1 kilogauss in intensity will experience significant buoyant forces when brought sufficiently close to the surface.

When loops or "stitches" of the toroidal flux ropes rise to the surface, they emerge to form BMR's (Sec. I, paragraph 11). Such regions are revealed by the magnetograph as contiguous areas of equal flux of opposite polarity. The lines of force cutting the surface in these regions arch upward into the higher atmosphere. Sunspots occur within BMR's (especially while they are young and compact) wherever the lines of force are sufficiently concentrated to have a substantial inhibiting effect on convection, according to the theory of Biermann (1941). The first such BMR's mark the onset of a new sunspot cycle corresponding to the beginning of stage 3. The stronger the initial field, the more rapidly will the BMR's appear, and the sooner will the maximum of surface activity be attained. The over-all pattern is established by the large-scale configuration of the submerged fields developed in stage 2, but considerable randomness in time and location of the BMR's follows from the disturbances due to twisting and convection of the flux ropes and to the gradual curtailment of the amplification process wherever BMR's are formed.

It is important to note that the varying topology of the solar magnetic field can be properly accounted for only through the emergence and development of individual BMR's. Averaging in such an extreme way as to neglect the BMR's is inadequate. A crucial point is the disposition of the lines of force as they expand into the high atmosphere; this will be considered later in stage 4.

In this stage we consider in some detail how BMR's, developed according to our model, can account for a number of the observed phenomena as listed in Section I.

*a) Sunspot polarity.*—A flux loop rising to the surface in the northern hemisphere (Fig. 3) will produce a BMR with equal amounts of positive and negative flux. The  $p$  and  $f$  parts will show positive and negative magnetic polarity, respectively, as will all normal BMR's in the present and other odd-numbered cycles. The opposite polarity will be found in BMR's of the southern hemisphere. In even-numbered cycles, as will be shown later, characteristic polarities will be reversed. Thus Hale's sunspot polarity laws (Sec. I, paragraph 5) are accounted for.

*b) Spörer's law (Sec. I, paragraph 4).*—It has been shown that, because the sun's rotation has a term proportional to  $\sin^2 \phi$ , a toroidal magnetic field of sufficient intensity

to produce BMR's will first be formed in intermediate latitudes ( $\phi \approx 30^\circ$ ) and that surface activity will proceed gradually to lower latitudes, reaching  $\phi = \pm 8^\circ$  some 8 years later. This is illustrated by Figure 4, which is based on equation (9). The amplification of the toroidal field in a given latitude will be terminated gradually when the stitching effect with the resultant formation of BMR's reaches a maximum, owing to fragmentation of the flux ropes. Because the dependence of BMR's on field strength is not precise, however, and because of the other random influences already mentioned, the limitation of BMR's to specific latitudes will be rather mild, and the mean  $20^\circ$  width of the sunspot zone is readily permitted. For the same reason, BMR's can appear before the field strength specified by equation (8) has been attained, and they will continue to be formed

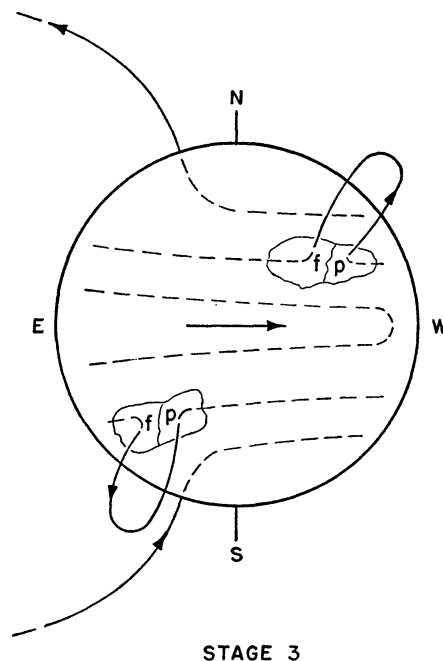


FIG. 3.—Bipolar magnetic regions (BMR's) are formed where buoyant flux loops of the submerged toroidal field are brought to the surface. The BMR's continue to expand, and the flux loops rise higher into the corona

after amplification at given latitude has been terminated, until the flux ropes have been used up. With these qualifications, equation (9) can be regarded as a derivation of Spörer's law.

*c) Number of groups.*—The total magnetic flux in the polar caps— $8 \times 10^{21}$  maxwells—is some eight times greater than that of a typical spot group or BMR. To estimate the number of BMR's that can occur during stage 3, we assume that the submerged flux consists of eight ropes and that each rope, of  $10^{21}$  maxwells, which initially has a length comparable to  $2 R$  in stage 1, is drawn out to make five turns around the sun in each hemisphere in the first 3 years of stage 2. At this point, then, the total length of flux rope is some  $450 R$ . But the amplification continues with diminishing effectiveness for another decade. Therefore, the total length of magnetic rope available to produce BMR's during the 11-year cycle may well approach  $10^3 R$ , and this would appear to be ample for more than  $10^3$  BMR's. It may be noted that the number of spot groups occurring in each cycle is about 3000 (Hale and Nicholson 1938), and this is just what would be expected if each unit length of flux rope,  $R$ , produces three or four BMR's.



*d) Preponderance of "p" spots.*—The fact that spots observed in the  $p$  parts of BMR's greatly outnumber those in the  $f$  parts (Sec. I, paragraph 6) can be accounted for by considering the distortion by differential rotation of the flux ropes that have risen to the surface (Cowling 1951). The requirements of compatibility here permit us to estimate that the submerged flux rope lies initially at a depth comparable to the radius of a "young" but well-formed BMR—say 50000–100000 km. Let the flux rope rise relatively quickly to the surface, thus stitching together the layers through which it passes. The overriding differential rotation of the shallower layers will distort the stitch of magnetic strands increasingly as time goes on, with the result that the rising section

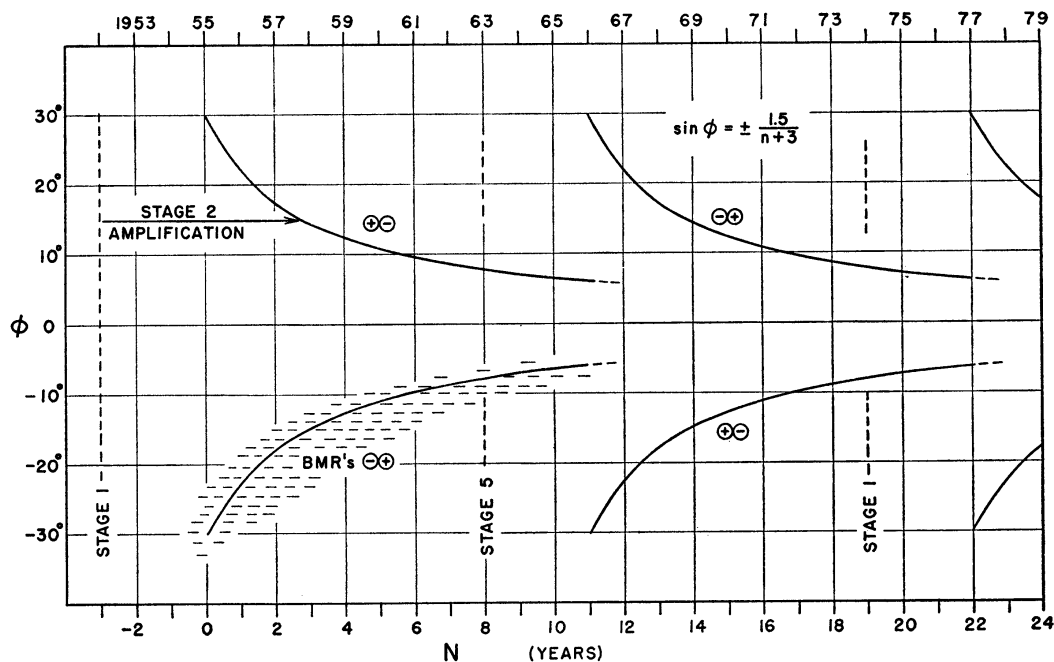


FIG. 4.—Equation (9), which represents Spörer's law of variation of sunspot latitudes, is plotted here. The amplified toroidal field of stage 2 reaches a critical value, required for the formation of sunspots, first near  $\phi = \pm 30^\circ$  and later in lower latitudes. In the formula,  $n$  is the number of years since the beginning of the 11-year sunspot cycle. Formation of numerous BMR's at a given latitude terminates amplification at that latitude by fragmenting the flux ropes.

of the flux rope in the  $p$  part of the stitch will assume a nearly vertical or even a forward-slanting attitude below the surface, whereas the  $f$  part will acquire an increasingly greater inclination (Fig. 5). The surface cross-section of the vertical rope in the  $p$  part will then tend to be minimal, and the field strength within it will be higher, so that it can produce a well-defined spot. But in the  $f$  part of the BMR, where the slanting flux rope intersects the surface at an increasingly large angle, the flux density will be lower, and there will be less likelihood that a spot can form, or, once formed, that it can persist as long as spots in the  $p$  region. Supporting evidence is found in the observed forward inclination of spot axes (Sec. I, paragraph 7), which increases with age.

If the foregoing explanation is correct, it permits us to estimate the depth of some of the isotachial surfaces. Suppose that the stitch rises from a depth of 70000 km (0.1 R) and that, after one solar rotation (27 days), the activity associated with the BMR is near maximum. Typically, the  $p$  spot will be prominent, while the  $f$  spot or spots will be smaller and on the point of breaking up if they have not already disappeared. We take the flux rope below the  $p$  spot to be vertical and estimate that, since the flux rope

first broke the surface, the BMR has been carried ahead about  $0.1 R$  ( $6^\circ$  in longitude) relative to the material at a depth of  $0.1 R$ . A simple calculation with the aid of equation (1) shows that, if the BMR has a latitude of  $15^\circ$ , the flux rope beneath it at a depth of  $0.1 R$  lies nearly on the isotach that intersects the surface at a latitude of  $\pm 31^\circ$ . This suggests that the isotachs are reasonably shallow and that most of the "equatorial acceleration" occurs rather close to the surface. We have already seen in stage 2 that the lines of force intersecting the isotachs lie in an even shallower layer, and this emphasizes again the remarks made earlier with respect to the limited depth of the phenomena under consideration.

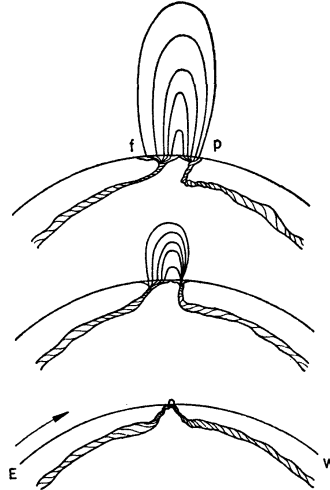


FIG. 5.—Formation of a BMR occurs when a constriction in a submerged flux rope is brought to the surface by magnetic buoyancy. *Lower*: the region is compact and symmetric; it may quickly produce spots and other forms of "activity" in both  $p$  and  $f$  parts. *Middle*: some days later, all the magnetic flux arches into the atmosphere, and on the surface the lines of force have begun to spread out. Differential rotation has advanced the BMR with respect to the submerged flux rope, so that the  $p$  part is more compact and has a higher field strength; spots are therefore more likely to be found within it. *Upper*: the  $p$  and  $f$  parts of the BMR continue to spread, and, as the field intensity diminishes, flocculi and other evidences of activity gradually disappear. Flow of plasma along the flux rope into the expanding BMR has resulted in new constrictions which may produce additional BMR's in the near vicinity.

*e) Chromospheric whirls (Sec. I, paragraph 10).*—It has been pointed out that the overriding effect of the faster-moving shallow layers near the equator will twist the submerged flux ropes in the direction of the sun's rotation. Such a twist would be propagated along the rope and would be expected to show generally in a counterclockwise twist for  $p$  spots in the northern hemisphere, and vice versa. This twist of the magnetic ropes would hardly be expected to produce observable effects in the higher-density regions, such as spot umbrae, but in the relatively tenuous chromospheric filaments in the near vicinity of the spot it is entirely plausible that the characteristic spiral whirls should appear, with their arms trailing the rotation of the central flux rope. This explanation of the whirls meets the requirement that the direction of winding is independent of the magnetic polarity of the spot, and it is in accord with Richardson's conclusion that the effect is a hydrodynamic one, as well as with the preferred direction of turning of the whirls as observed by Hale.

*f) Recurrence of sunspots in preferred zones (Sec. I, paragraph 9).*—When a submerged flux loop of high field intensity and small diameter is first formed, perhaps as a result of excessive twisting, the plasma within the tube or rope must be squeezed longitudinally away from the constriction. Thus a new BMR tends to be well defined and perhaps even

isolated. But after the flux loop has reached the surface, the BMR begins to spread, showing a gradual but pronounced increase in area. Presumably the affected depth is quite shallow, except in the core of the BMR. Because no significant amount of plasma can cross the lines of force in the time available, it follows that the plasma required for the expanding BMR can be supplied only by longitudinal movement of material along the flux rope into the BMR. Therefore, the adjacent sections of the flux rope will become constricted, with a consequent increase in field strength. The new constrictions, not greatly displaced in longitude from the larger and disappearing BMR, are likely to develop the field strength and buoyancy required to produce new BMR's in close proximity to the old one, but predominantly on the *f* or easterly side.

*g) Arching lines of force.*—The *p* and *f* parts of a BMR are the areas of opposite magnetic polarity on the sun's surface, generally contiguous, through which the magnetic lines of force representing one stitch of a flux rope emerge and return. Above a BMR the arching lines of force are related to condensations in the corona from which the disposition of the magnetic field can sometimes be inferred. It can be taken as well established that the BMR's expand and that the arching lines of force above them tend to loop outward with increasing height as time goes on. Active prominences also delineate lines of force, but stable, filamentary prominences frequently separate the *p* and *f* parts of BMR's, lying nearly at right angles to the lines of force (Babcock and Babcock 1955). From this it has been inferred that such filaments are supported in troughs or hammock-line depressions in the magnetic arches. The theory has been developed by Menzel (1951) and by Kippenhahn and Schlüter (1957).

It has been proposed that corpuscular streams are accelerated upward from the photospheric level along lines of force, having attained their energy essentially from hydromagnetic shock waves generated by turbulence in the convective layer. Confirmatory evidence for this is found in the increased density of material that collects in prominences, in the identification of localized radio noise sources above BMR's, and in the optical excitation of special coronal radiations above BMR's.

A more or less precarious imbalance prevails between the mechanical and magnetic forces, but it is taken to be a fact that the tension in the lines of force above BMR's is in the end overpowered by the gas pressure, so that the local magnetic flux loops become greatly extended radially from the sun. In so doing, they force the large equatorial loops of the initial dipolar field (stage 1) outward to increasing distances (and with increasing disorder), so that within a few radii of the sun the polar lines of force of the dipolar field likewise assume a nearly radial attitude. This situation is exemplified by photographs of the coronal streamers made near sunspot maximum, when the predominantly radial distribution of streamers is typical.

#### *Stage 4. Neutralization and Reversal of the Main Dipolar Field*

This stage is concerned first with the disposition of magnetic flux of disappearing BMR's—the lifetime of each being measured in weeks or months—and finally with the cumulative effect of the thousands of BMR's that occur in the course of a sunspot cycle. Thus the processes of stage 4 are distributed over a large part of the sunspot cycle, but many BMR's in passing through this stage are presumed to contribute their effect toward the end result: the cancellation of the initial main dipolar field and its replacement by another of reversed polarity. A secondary result is the liberation of magnetic flux loops in the corona.

The essential idea of stage 4 was anticipated in a paper of 1955 (Babcock and Babcock, p. 362) from which we quote:

We could speculate at this stage that the polar magnetic field is the result of a poleward migration of the *f* portions of disintegrating BMR's in the first few years of each sunspot cycle. If this were true, the main poloidal field should reverse its polarity every  $11\frac{1}{2}$  years, but out of phase with the frequency-curve for sunspots. On this theory the residual *p* portions of BMR's

should either be neutralized by merging of low-latitude fields of the northern and southern hemispheres, or there should be evidences of a general quadrupolar field. The intensity of the average low-latitude components of the quadrupolar field should be of the order of 0.3 gauss, which could hardly have escaped observation.

The evidence is to the effect that the process occurs in a piecemeal or tesseral fashion and that no persistent quadrupolar field is formed.

The magnetograms show that the respective  $p$  and  $f$  parts of BMR's generally expand and draw apart, while the flux loops rooted in them are being pushed outward into the corona. Each portion of a typical BMR tends to become elongated, with its low-latitude end preceding toward the west (Figs. 6 and 7). Evidently the fluid shear related to the differential rotation is influential in drawing out BMR's into the long, narrow, characteristically slanted configurations that are rather frequently found. It is often observed that the  $p$  part of a BMR expands or shifts toward the equator, while the  $f$  part expands or migrates in the direction of the pole. (The orientation effect observed in spot groups [Sec. I, paragraph 8] continues and is enhanced in the BMR's after the spots have vanished.) At this phase of the process, the fields are generally weak, and there is frequently confusion or interference with adjacent regions. Therefore, interpretation of the observations is difficult, and more quantitative data are desirable. But it is definitely established that the expansion of BMR's is not regressive.

It has been suggested that the orientation effect in spot groups is the result of Coriolis forces, which induce a vorticity in the whole of the fluid associated with a BMR when it rises to the surface. Possibly the magnetic tension ( $H^2/8\pi$  dynes/cm<sup>2</sup>) in the submerged flux ropes is a factor in the separation of the  $p$  and  $f$  parts of BMR's.

The cause of the migration in latitude of the respective parts of BMR's remains rather obscure, but this migration implies the existence of meridional flow toward the equator in low latitudes and toward the poles in moderate latitudes. Supporting evidence for this is seen in the finding by Richardson and Schwarzschild (1953) that individual sunspots show slow drifts in latitude away from the 20° latitude circles. The measured rate is only 0°01–0°02 per day, on the average, but it is perhaps not surprising that the spots, which occur early in the lifetime of a BMR and which represent concentrated portions of the deeply rooted flux ropes, should show slower meridional drifts than do the shallow, widespread parts of BMR's in their later phases.

In the high corona the full-blown flux loops above old BMR's continue to expand toward each other and toward the enveloping lines of force that connect the polar caps. Over considerable distances and for extended intervals of time, large segments of field lines that are rooted in different regions of the sun are nearly antiparallel and are gradually pushed closer together. The result is a neutralization of long segments of the standing legs of the flux loops, with severing and reconnection of the ends, as indicated in Figure 8. Large, low-intensity loops of the field lines are liberated to drift into the interplanetary region with the tenuous plasma of the corona.

During the progress of stage 4, as more and more BMR's form, grow, and vanish, the initial dipolar field of stage 1 is first neutralized by the poleward drift of the  $f$  parts of BMR's and then supplanted by a new dipolar field of opposite polarity (Sec. I, paragraph 2). Observation shows that the neutralization is effected at the time of sunspot maximum and that the new, reversed field is in evidence shortly thereafter. Indications are that, over an interval of some years, the intensity of the new field grows and the affected latitudes become somewhat higher.

The submerged segments of those flux ropes that have contributed to the new main dipole are now oriented with their poleward ends toward the west and their low-latitude ends lagging. In this situation, differential rotation brings them gradually parallel with the meridian; at the same time, as already suggested, the low-latitude end of each such section of rope merges with the corresponding end of a flux rope on the opposite side of the equator. This follows a severing and reconnection of lines of force above the sur-



face. The elevated portions of the reconnected lines, which stretch across the equator, are then required to sink and to become submerged below the photosphere. That there is a general submergence of plasma in the vicinity of the equator is suggested by the observed equatorward drift of individual sunspots having latitudes less than  $\pm 20^\circ$  (Richardson and Schwarzschild 1953).

The total amount of magnetic flux of  $f$  polarity appearing in a given hemisphere during one sunspot cycle is far greater than the flux existing initially in the dipolar field of stage 1. This follows from the amplification of stage 2 and from the fact that so many BMR's arise in various longitudes around the sun. The flux that supplants the

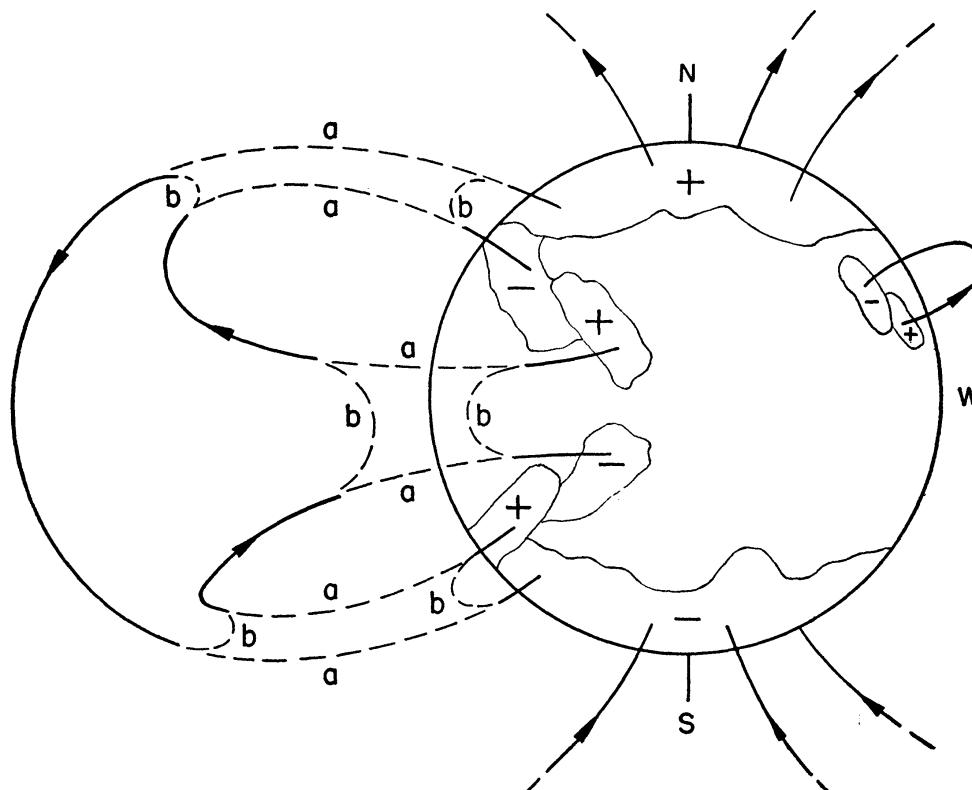


FIG. 8.—The expanding lines of force above older BMR's move out to approach the lines of force of the main dipole field. Severing and reconnection gradually occur, so that parts *a* are replaced by parts *b* and a portion of the main field is neutralized. Also a large flux loop of low intensity is liberated in the corona. Continuation of the process results in the formation of a new main dipolar field of reversed polarity.

initial dipolar field need be only  $16 \times 10^{21}$  maxwells in order to complete stage 4 by leaving the sun with an equivalent but reversed dipole as compared with stage 1. Yet the number of BMR's available is of the order of  $3 \times 10^3$ , and each has about  $10^{21}$  maxwells of flux. Taking account of the fact that the same lines of force thread BMR's on opposite sides of the equator, one finds that the efficiency of the essential process of stage 4 need be only about 1 per cent. Evidently a small fraction of the magnetic flux in the  $f$  parts of BMR's (mainly those occurring in the early years of the cycle before the preferred zone of emergence has receded to lower and less effective latitudes) is adequate for reversal of the dipolar field.

The disposition of the remaining 99 per cent of the lines of force of the BMR's, which is not required for the formation of a new main dipole, remains to be considered. A

survey of many magnetograms indicates that the general expansion of the BMR's as they approach their terminal stages results in a merging of most of the magnetic flux in the  $p$  part of one BMR with that of the  $f$  part of the next BMR to the west, and so on around the sun. Merging with other BMR's of lower latitude may also occur, as suggested by Cowling (1960). Such merging implies that there is a severing of the flux loops above BMR's, with reconnection according to the new pattern. A major part of the initially submerged toroidal field is thus gradually converted to individual coronal flux loops, such that the lowest segment of each loop temporarily links the photospheric layers of the sun. Because of the considerable time available and because of the reduced conductivity of the photosphere, these weak flux loops are eventually liberated to move outward with the plasma of the corona.

In the process just described, the relatively short segments of flux rope that join the  $p$  and  $f$  parts of different BMR's are for the most part at a shallower level than the submerged segments of the newly forming main dipole; thus there is no severe interference when the former segments are brought to the surface. If such interference does occasionally occur, it may well lead to complex magnetic regions giving rise to  $\gamma$  sunspot groups.

As solar magnetic activity frequently shows an imbalance between the northern and southern hemispheres, it would be expected that occasional temporary accumulations of  $p$  parts of BMR's might occur near the equator, where they would remain for some time before being neutralized by their counterparts from the opposite side. This provides an explanation for the unipolar magnetic regions (UMR's), three of which were observed in the declining years of the last solar cycle (1953). The most prominent UMR, with a magnetic flux estimated to be about  $10^{21}$  maxwells, was observed for six successive rotations of the sun and was evidently related to and responsible for a series of 27-day recurrent geomagnetic disturbances and for variations in cosmic-ray intensity (Simpson, Babcock, and Babcock 1955). Presumably a well-defined bundle of magnetic-field lines of great radial extent was rooted in this UMR.

#### *Stage 5. The Reversed Dipolar Field*

The reversed dipolar field is the residual of the foregoing processes. It is attained in full after about 11 years and is similar to stage 1 except for the reversed polarity. The differential rotation, of course, continues in the same sense, so that the analogues of stages 2, 3, and 4 then take place to complete the whole 22-year magnetic cycle, with return of the configuration of the field to that of stage 1.

### III. CONCLUDING REMARKS

The model described here is a freely running oscillator that lacks stabilization. There appear to be two main parameters affecting the frequency. The first is the rate of differential rotation, which determines the length of time required to amplify the dipolar field of stage 1 up to the intensity that will produce buoyant flux loops and BMR's. The second is the rate of transverse slipping of the lines of force through the plasma in stage 4.

The very low efficiency required of the conversion process of stage 4, whereby the reversed dipolar field is formed from the BMR's, suggests that it will be extremely sensitive to disturbing influences, random or otherwise, which may have a relatively large effect on the amplitude or phase of the magnetic cycle. Both frequency and amplitude may be affected by the randomness of the BMR activity and possibly by other, more obscure influences as well. Evidently a strong initial field will permit an early maximum of activity to be attained, and such a maximum is likely to be a high one. In general, one expects a cycle of high or of low activity to be followed by another of the same kind, but there is no apparent reason to expect stabilization of amplitude over a long term.

The differential rotation is obviously of fundamental importance to the model, for it provides the energy that goes into the magnetic field when the lines of force are drawn out longitudinally and twisted in stage 2. Because this process is repeated in every sunspot cycle, the question of the maintenance of the differential rotation is important. It is possible to make an order-of-magnitude comparison of the energy put into the magnetic field in each sunspot cycle and of the kinetic energy of the differential rotation. In this calculation it is assumed (perhaps with inadequate justification) that the conversion of kinetic to magnetic energy is fully efficient and that there is no feedback from the magnetic to the kinetic mode. If stage 2 results in the formation of a length 1000 R of magnetic rope having a flux of  $10^{21}$  maxwells and an assumed r.m.s. field strength of 400 gauss, the magnetic energy required in one sunspot cycle is about  $10^{36}$  ergs. The kinetic energy of differential rotation is difficult to estimate, but we have seen that on the present model the isotachial surfaces are fairly shallow, so that the faster-rotating equatorial belt need not contain a large fraction of the sun's mass. Assuming that the situation can be approximated by an equatorial belt  $40^\circ$  wide and 0.15 R in depth, which has a period of rotation of 25 days compared with 29 days for the remainder of the sun, one finds that the energy of differential rotation is some  $6 \times 10^{38}$  ergs. Because this is insufficient to drive the solar magnetic cycle for more than a few thousand years, the problem of the maintenance of the differential rotation of the sun assumes a new interest.

Schwarzschild (1947) derived a numerical solution for a slowly rotating star having a convective core and found the solution to give an equatorial acceleration on the surface. But it is now known that a model with a convective core is not applicable to the sun. Therefore, it remains to be shown how differential rotation can be maintained in a star like the sun, in which substantial dissipative processes are present.

In stage 3 of the model, the energy required for the expansion of the magnetic flux loops above growing BMR's is provided by the kinetic energy of the atmospheric gas or by the energy of turbulence near the surface and ultimately by the fundamental nuclear energy sources of the sun. It appears quite unlikely, however, that feedback from the flux loops can contribute significantly to the maintenance of the differential rotation.

Cowling (1960) has suggested the elements of a theory in terms of magnetic activity, assuming that some of the material ejected from the sun in moderate latitudes falls back on it near the equator. While it is well away from the sun, any magnetic field which may still connect it with the sun will tend to establish equality of angular velocity along the lines of force. When it returns to the sun, angular velocity tends to be conserved; thus the material may reach the sun near the equator with a much increased angular velocity.

In stage 4, transverse slipping of the lines of force through the plasma is required high in the corona, where in intervals measured in years there is a neutralization of extended segments of lines of force of the main dipolar field by antiparallel lines originating in BMR's. Transverse slipping is also required of those low-level parts of the coronal flux loops that link the photosphere in merging BMR's. If it were not for turbulence, the rate of slipping would be far too slow. The characteristic time intervals ( $t_0$ ) are measured in weeks or months, while the linear dimensions ( $l$ ) are of the order of 0.03 R. If the material is at rest, the order of magnitude of the diffusion time for a magnetic field is given by  $t_0 = 4\pi\sigma l^2$  (Cowling 1953). Taking  $\sigma = 10^{-9}$  e.m.u. for the photosphere, it is found that the diffusion time would be some thousands of years. But in the presence of turbulence the diffusivity may be enormously increased, as has been shown by Elsasser (1956). Therefore, in the convective zone of the sun, a very large increase in the rate of slipping of the lines of force may be expected. Although the theory is not as yet rigorous, the rate of slipping of the lines of force demanded by the model appears not implausible.

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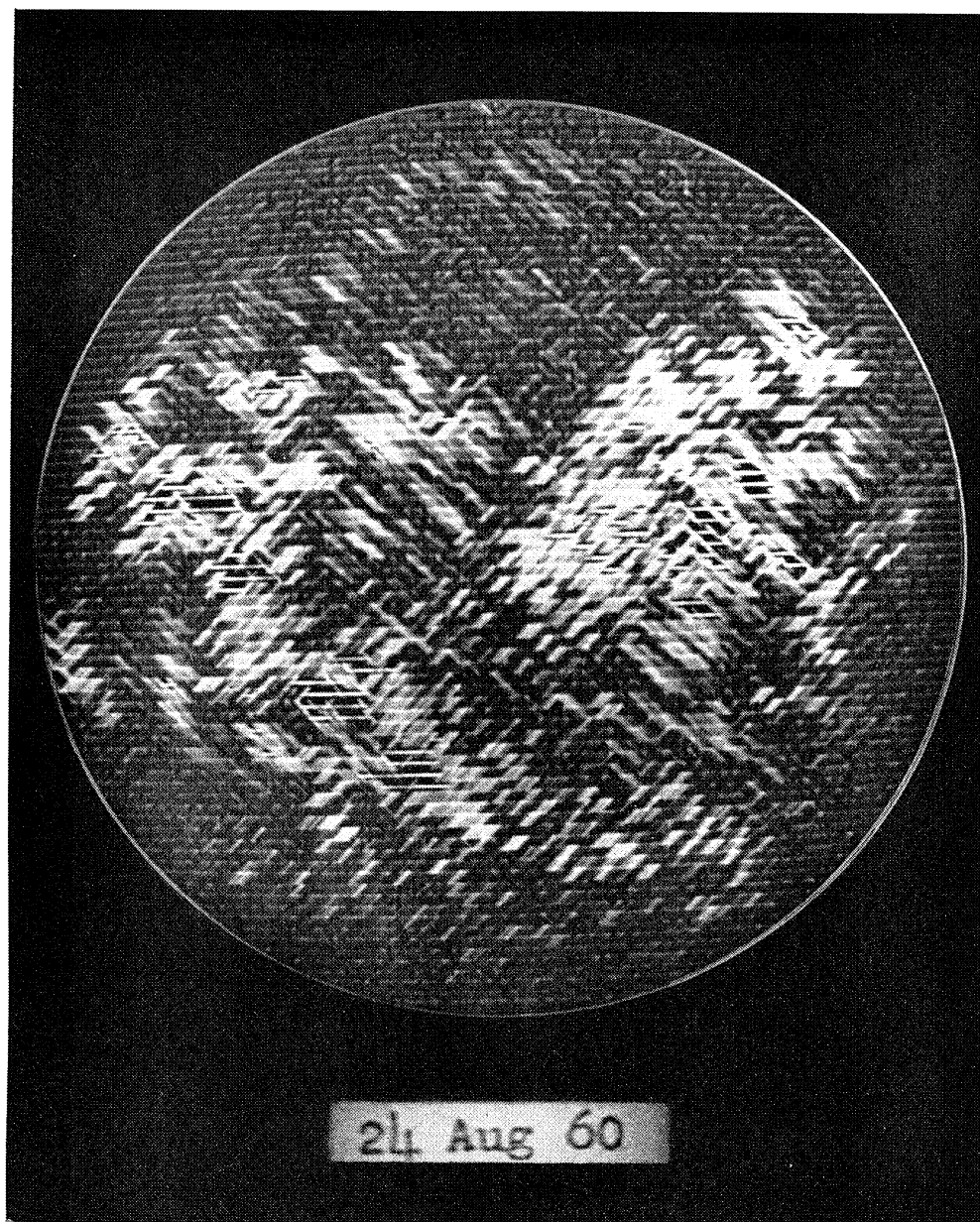


FIG. 6.—In this magnetogram the brightness of the trace is roughly proportional to magnetic-field intensity. Lowest level of brightening is about 5 gauss here, so that only the stronger parts of the dipolar field in the polar caps are shown. Where the recording spot produces a narrow horizontal line, the field is stronger than about 40 gauss. The short recording line slants to the right for positive polarity, and vice versa. North is at the top, east to the right. (Magnetogram by J. O. Hickox)



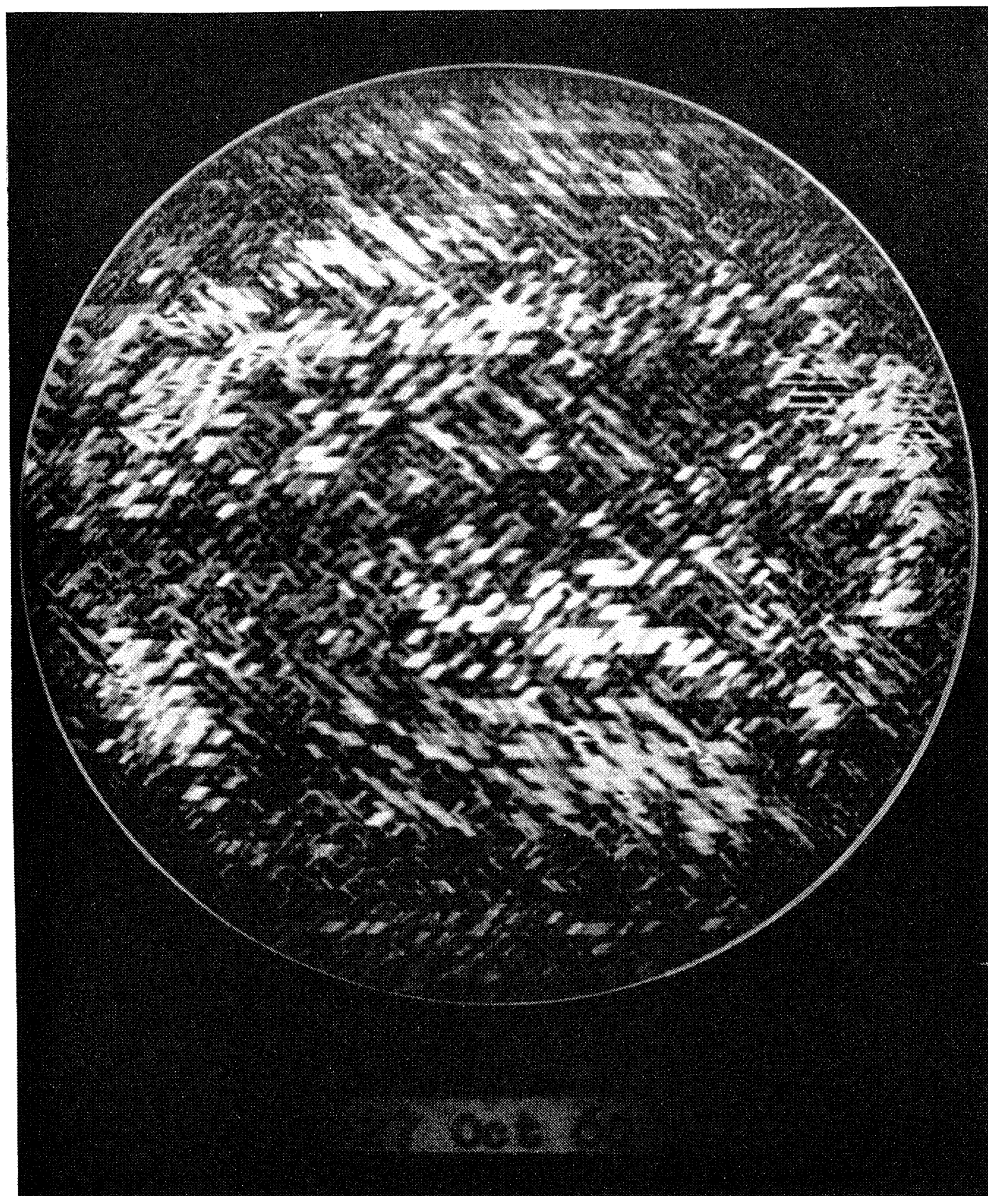


FIG. 7.—A recent magnetogram showing evidence for the main dipolar field in high latitudes. Opposite magnetic polarity of the two polar caps is indicated by the slant of the short recording line. The more intense local magnetic regions in moderate latitudes often show an elongated, diagonal disposition. The instrument measures only the line-of-sight component of the field (October 27, 1960).