The Origin of Magnetic Fields

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Magnetic Fields *ab initio*

- The origin of magnetic fields in the universe is a very important unsolved problems in cosmology
- It is widely accepted that astrophysical magnetic fields reached their present state in a two stage process:
- First, the generation of a *seed* field, and
- Then dynamo action which amplified the field

- What are the possible sources for the origin of such seed fields?

Could the Magnetic Seed Fields simply be Primordial?

Several models of the early Universe predict the generation of primordial magnetic fields (PMF), either during inflation or during later phase transitions. PMF have an impact on cosmological perturbations and in particular on CMB anisotropy angular power spectra that can be used to constrain the PMF amplitude. Recent results from the Planck mission http://arxiv.org/pdf/1303.5076.pdf [Planck XVI] show that the cosmological parameters are in agreement with those estimated assuming no PMF, so we have to look elsewhere for a mechanism for generating magnetic field from ‘nothing’.

Biermann to the Rescue

Ludwig Biermann
1907-1986

Ludwig Biermann, Zeitschrift für Naturforschung, vol. **5a**, p 65 (1950)

proposed a mechanism “The Biermann Battery Process” by which a weak seed magnetic field can be generated from zero initial conditions by the relative motion between electrons and ions.

[I have managed to get a copy of his original paper].

Derivation of the Biermann Equation

Force Balance on a single electron in a fluid

$$m_e \, d\boldsymbol{v}_e/dt = -e\boldsymbol{E} + e\,\boldsymbol{j}/\sigma - \nabla p_e/n_e - e\,\boldsymbol{v}_e \times \boldsymbol{B}/c + \boldsymbol{f} = 0$$

e = elementary charge

$\boldsymbol{E}$ = electric field

$\boldsymbol{B}$ = magnetic flux density

$\boldsymbol{j}$ = electric current density

σ = conductivity

Fluid properties:

∇p_e = pressure gradient

n_e = number density

$\boldsymbol{v}_e$ = velocity

All pertaining to electrons

$\boldsymbol{f}$ = whatever other forces, e.g. Compton scattering and momentum transfer by photo-ionization

Assume a Proton – Electron Plasma

The fluid velocity is basically the proton velocity $\boldsymbol{v}_\mathrm{p}$:

$$\boldsymbol{v} \equiv (n_\mathrm{p} m_\mathrm{p} \boldsymbol{v}_\mathrm{p} + n_\mathrm{e} \cancel{m_\mathrm{e}} \boldsymbol{v}_\mathrm{e}) \,/\, (n_\mathrm{p} m_\mathrm{p} + n_\mathrm{e} \cancel{m_\mathrm{e}}) \approx \boldsymbol{v}_\mathrm{p}$$

because the electron mass m_e is so much smaller than the proton mass m_p.

Since the current density is $\boldsymbol{j} = e n_\mathrm{e} (\boldsymbol{v}_\mathrm{e} - \boldsymbol{v}_\mathrm{p})$, the electron velocity can be approximated by

$$\boldsymbol{v}_\mathrm{e} \approx \boldsymbol{v} - \boldsymbol{j} / (e n_\mathrm{e})$$

Ohm's Law in a [two-fluid] Plasma

Inserting the electron velocity $\boldsymbol{v}_\mathrm{e} \approx \boldsymbol{v} - \boldsymbol{j}/(e\,n_\mathrm{e})$ into

$$0 = -e\boldsymbol{E} + e\boldsymbol{j}/\sigma - \nabla p_\mathrm{e}/n_\mathrm{e} - e/c\ \boldsymbol{v}_\mathrm{e} \times \boldsymbol{B} + \boldsymbol{f} \qquad \text{we get}$$

$$0 = -e\boldsymbol{E} + e\boldsymbol{j}/\sigma - \nabla p_\mathrm{e}/n_\mathrm{e} - e/c\ \boldsymbol{v} \times \boldsymbol{B} + \boldsymbol{j} \times \boldsymbol{B}/c\,n_\mathrm{e} + \boldsymbol{f}$$

Which, BTW, gives us a more general expression for Ohm's law:

$$\boldsymbol{j}/\sigma = \boldsymbol{E} + \nabla p_\mathrm{e}/en_\mathrm{e} + \boldsymbol{v} \times \boldsymbol{B}/c - \boldsymbol{j} \times \boldsymbol{B}/c\,en_\mathrm{e} - \boldsymbol{f}/e$$

$$\boldsymbol{j}/\sigma = \boldsymbol{E} + n_\mathrm{e}\nabla p_\mathrm{e}/en_\mathrm{e}^2 + \boldsymbol{v} \times \boldsymbol{B}/c - \boldsymbol{j} \times \boldsymbol{B}/c\,en_\mathrm{e} - \boldsymbol{f}/e$$

$$0 = -e\boldsymbol{E} + e\boldsymbol{j}/\sigma - n_e\nabla p_e/n_e^2 - e/c\, \boldsymbol{v} \times \boldsymbol{B} + \boldsymbol{j} \times \boldsymbol{B}/c n_e + \boldsymbol{f}$$

Apply $-(1/e)\nabla\times$ to both sides:

$$0 = \nabla\times \boldsymbol{E} - (\nabla\times \boldsymbol{j})/\sigma + \nabla n_e \times \nabla p_e / e n_e^2$$

$$+ 1/c\nabla\times(\boldsymbol{v} \times \boldsymbol{B}) - 1/c\nabla\times(\boldsymbol{j} \times \boldsymbol{B}/n_e) - 1/e\nabla\times \boldsymbol{f}$$

Maxwell tells us that $\nabla\times \boldsymbol{E} = -1/c\, \delta\boldsymbol{B}/\delta t$ so we get:

$$1/c\, \delta\boldsymbol{B}/\delta t = -(\nabla\times \boldsymbol{j})/\sigma + \nabla n_e \times \nabla p_e / e n_e^2$$

$$+ 1/c\nabla\times(\boldsymbol{v} \times \boldsymbol{B}) - 1/c\nabla\times(\boldsymbol{j} \times \boldsymbol{B}/n_e) - 1/e\nabla\times \boldsymbol{f}$$

Multiplying by c:

$$\delta\boldsymbol{B}/\delta t = -c(\nabla\times \boldsymbol{j})/\sigma + c\nabla n_e \times \nabla p_e / e n_e^2$$

$$+\nabla\times(\boldsymbol{v} \times \boldsymbol{B}) - \nabla\times(\boldsymbol{j} \times \boldsymbol{B}/n_e) - c/e\nabla\times \boldsymbol{f}$$

$$\delta \boldsymbol{B}/\delta t = - c(\nabla\times \boldsymbol{j})/\sigma + c\nabla n_e \times \nabla p_e / e n_e^2$$

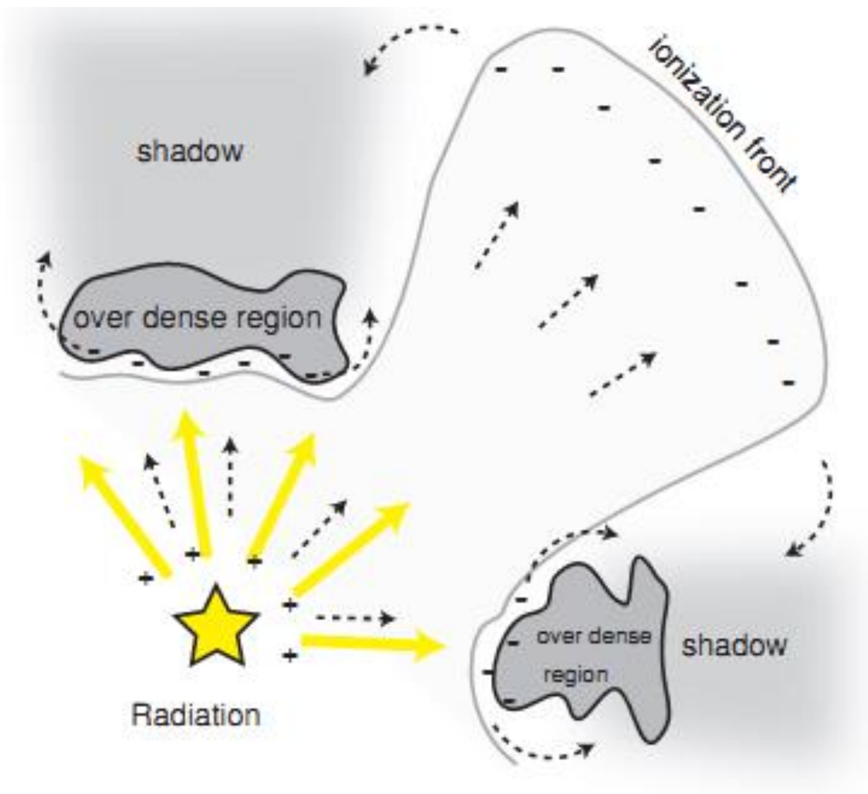
$$+\nabla\times(\boldsymbol{v}\times\boldsymbol{B}) - \nabla\times(\boldsymbol{j}\times\boldsymbol{B}/n_e) - c/e\,\nabla\times\boldsymbol{f}$$

The argument is now that in a plasma $\boldsymbol{j} \approx 0$, so that

$$\delta \boldsymbol{B}/\delta t = \nabla\times(\boldsymbol{v}\times\boldsymbol{B}) + c/e n_e^2 \nabla n_e \times \nabla p_e - c/e\,\nabla\times\boldsymbol{f}$$

The first term, $\nabla\times(\boldsymbol{v}\times\boldsymbol{B})$, is advection of magnetic flux, but even if there initially isn't any magnetic flux, i.e. $\boldsymbol{B} = 0$, the second term, $c/e n_e^2 \nabla n_e \times \nabla p_e$, could be non-zero if the gradients ∇n_e and ∇p_e are not co-linear. This is the Biermann Battery Effect that can generate a magnetic field from 'nothing' but gas dynamics. The third term, $c/e\,\nabla\times\boldsymbol{f}$, could be important if there is strong radiation pressure.

Radiation Field



If the radiation from a star ionizes an interstellar cloud the force ***f*** can change very rapidly across an ionization front and ∇× ***f*** can become large, but in general the Biermann Battery Effect will dominate.

Estimate of the Seed Field

$$B \sim \frac{c}{n_e^2 e}\left(\frac{n_e}{\Delta r}\right)\left(\frac{p_e}{\Delta r}\right)\sin\theta\, t_{\rm age}$$

$$\sim 5.0 \times 10^{-17}{\rm G}\left(\frac{t_{\rm age}}{2{\rm Myr}}\right)\left(\frac{\sin\theta}{0.1}\right)\left(\frac{\Delta r}{1{\rm pc}}\right)^{-2}\left(\frac{T}{10^4{\rm K}}\right)$$

where Δr denotes the typical length of n_e and p_e change significantly, θ is the angle between ∇n_e and ∇T.