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PERIOD AND PHASE OF THE 88-YEAR SOLAR CYCLE AND THE MAUNDER MINIMUM: EVIDENCE FOR A CHAOTIC SUN

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Abstract. The problem of whether the solar dynamo is quasi-periodic or chaotic is addressed by examining 1500 years of sunspot, geomagnetic and auroral activity cycles. We find sub-harmonics of the fundamental solar cycle period during the years preceding the Maunder minimum and loss of phase of the subharmonic on emergence from it. These phenomena are indicative of chaos. They indicate that the solar dynamo is chaotic and is operating in a region close to the transition between period doubling and chaos. Since Maunder type minima reoccur irregularly for millennia, it appears that the Sun remains close to this transition to and from chaos. We postulate this as a universal characteristic of solar type stars caused by feedback in the dynamo number.

1. Introduction

Developments in theories of nonlinear equations and chaos have brought forward the question of whether the solar cycle is chaotic or quasi-periodic (Ruzmaikin, 1981; Zel'dovich, Ruzmaikin, and Sokoloff, 1983; Weiss, Cattaneo, and Jones, 1984; Spiegel, 1985). Highly simplified numerical models of the solar dynamo (Weiss, Cattaneo, and Jones, 1984) have exhibited chaos in which intervals showing apparently quasi-periodic-like behavior are interrupted by irregularly spaced intervals of suppressed activity. It has been suggested that the low solar activity from 1645 to 1715, the Maunder minimum (Eddy, 1976), is such an interval for the Sun (Ruzmaikin, 1981; Zel'dovich, Ruzmaikin, and Sokoloff, 1983; Weiss, Cattaneo, and Jones, 1984; Speigel, 1985) indicating that the solar dynamo is chaotic. Here the problem of choosing between the chaotic and quasi-periodic hypothesis for the solar dynamo is addressed by examining the behavior of solar variations during the last 1500 years. Two phenomena that are seen in models in association with chaos are examined; the existence of sub-harmonics of the fundamental period when the control parameter is near the bifurcation to chaos (Weiss, Cattaneo, and Jones, 1984; Thompson and Stewart, 1986; Swinney, 1986) and the loss of phase coherence in cyclic variations as the system emerges from suppressed activity (Thompson and Stewart, 1986; Weiss, 1988).

Numerical studies of nonlinear differential equations have delineated characteristic behaviors that many systems exhibit as variation in the control parameter brings them to a bifurcation to chaos (cf. Thompson and Stewart, 1986). One of the most well-studied of these behaviors is period doubling in which sub-harmonics with twice, four, eight, etc., times the period of the fundamental are seen as the control parameter of the system approaches values at which bifurcation to chaos occurs. Typically if the control parameter is further increased the system enters a chaotic state. When the chaotic state

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appears but the control parameter is still close to the bifurcation value the spectral lines of the periodic regime still appear but they are broadened (Thompson and Stewart, 1986; Weiss, 1988). Doubled periods for control parameters near the chaotic bifurcation have also been observed in the laboratory in a wide variety of systems (Swinney, 1986).

In this note we compare observations of solar periodicities to expectations based on the numerical models. As a particular example of such models we use that studied by Weiss, Cattaneo, and Jones (1984). They treated a highly simplified idealized nonlinear model of an oscillatory solar dynamo. The model was constructed to be the simplest that incorporated the essential physics of the dynamo processes together with the crucial nonlinearities associated with the Lorenz force. The effects of the field in influencing the fluid motion was included as well as the effects of the fluid motion in producing the fields. The system of equations were studied by varying the dimensionless dynamo number D (Parker, 1979),

$$D = \alpha \frac{v'}{2} \eta^2 K^3 \, ,$$

where α and η are constants describing the regeneration of the field through helicity and the turbulent dissipation respectively, v' is an average value of the velocity shear and K is the wave number of a dynamo wave satisfying the model equations. Weiss, Cattaneo, and Jones (1984) then made various further simplifying assumptions to allow the study of approximations to the equations in order to explore the transition from periodic to aperiodic oscillations. They studied the behavior of their simplified dynamo as a function of the control parameter in some detail and found a rich bifurcation structure. For example, for a control parameter of about 3.47 there was an oscillatory bifurcation after which trajectories lay on a 3-torus. With increasing dynamo number further bifurcations led to frequency locking, followed by a period-doubling cascade that led to chaos for dynamo number greater that 3.84. The period-doubled-chaotic bifurcation will be of particular interest in this study. In the Weiss, Cattaneo, and Jones model the chaotic region apparently persisted for larger dynamo numbers. In the chaotic regime they found intervals of quasi-periodic-appearing behavior interrupted by intervals of suppressed activity. The time between successive intervals of suppressed activity varied. In the nearby periodic regime the intervals of suppressed activity were evenly spaced. Here we compare observations of solar variations with the behavior of the model in the vicinity of this bifurcation to chaos.

During the 17th century Maunder minimum solar activity became very low as measured by reports of sunspots and mid-latitude aurora (Eddy, 1976) and 14C anomalies (Stuiver and Braziunas, 1988; Damon, 1988). In the centuries before the Maunder minimum, solar activity as measured by auroral activity showed an 11- and 22-yr variation (Attollini *et al.*, 1988) and a well-established strong 88-yr cycle (Gleissberg, 1955; Siscoe, 1980; Feynman and Fougere, 1984). Recently a period of about 87 years has been confirmed for 10000 years of 14C anomaly data (Stuiver and Braziunas, 1988). The 88-yr variation was not seen in the Eltanin varve records but that has recently been shown not to be a solar activity proxy (Williams, 1989). Thus the 88-yr cycle has now been seen in all data sets which are solar activity proxies (Feynman, 1983).

Examination of a 10000 yr record of 14C has resulted in the identification of 9 minimums of the Maunder type (Stuiver and Braziunas, 1988) at irregular intervals. In the rest of this note the term 'Maunder minimum' is reserved to describe the 17th century minimum although it can be expected that all of the minima will have the same physical nature.

If the Sun shows apparent periodic type behavior but is chaotic it can be expected to display behavior characteristic of systems near bifurcation to chaos. This includes the broadening of the periodic lines existing in the system for control parameters in the periodic region but near the bifurcation to chaos. Here the hypothesis that the 88-yr Gleissberg cycle was part of such a sequence of period doublings is tested.

In order to test this hypothesis it must be established that the Gleissberg period was 8 times the period of the 11-yr cycle to within the accuracies of the determinations. The 11- and 22-yr variations and their periods are first discussed and it is concluded that the Gleissberg cycle qualifies as a third subharmonic of the fundamental within the accuracies of both determinations. If the fundamental period of the Sun is taken to be 22 yr then the 88-yr variation is the second doubling. These results indicate that the solar dynamo is near the period doubled regime but do not test whether the dynamo is chaotic. If the solar dynamo is periodic the phase of the variations will be maintained across intervals of suppressed activity. As a test of this point the phase of the 88-yr cycle before and after the Maunder minima is discussed and it is shown that the observed phase change is beyond the uncertainty in the phase determinations, an indication of the chaotic nature of the suppressed solar activity interval. Furthermore, since the 17th century minimum was an example of recurrences of suppressed activity typical of chaotic systems, and since the intervals of quiet are not regularly spaced, then all other Maunder-type minimums indicate a chaotic dynamo. The presence of the series of period doubled variations shows that the dynamo has remained close to the bifurcation value. The final section of the paper presents a discussion of the results including a suggestion as to why the Sun appears to hang in the region of bifurcation to and from chaos.

2. The 88-Year Cycle as a Subharmonic

In order to show that the 88-yr variation can qualify as the third period double of the 11-yr cycle it must be shown that their periods are in the correct ratio to the accuracy with which they have each been determined.

2.1. THE 11- AND 22-YEAR CYCLES

The 11-yr cycle of solar activity expresses itself in many ways including a periodic variation of the number of sunspots which newly appear on the surface (Kopecký, 1979), the life time of the sunspots (Kopecký, 1979), the sunspot number and the intensity of auroral and geomagnetic activity. All of these solar activity cycle variables express somewhat different aspects of solar activity and any of them can act as a measure of the activity cycle (Feynman, 1983). Several authors have examined the frequencies of the 11-yr cycle since the year 1715, when solar activity returned after the Maunder minimum. The parameters most commonly used to measure solar activity during this period are the sunspot number and auroral observations. For the purposes of an analysis of chaos the important period is the steady-state oscillation period of the solar dynamo that can be found from a limit cycle analysis of the Sun's periodic behavior. Each of the actually observed cycles differ somewhat in period from the steady-state cycle (Gudzenko and Chertroprud, 1964). Using the sunspot data since 1715 Gudzenko and Chertroprud (1964) found the steady-state dynamic oscillations had a period of 11.2 with a standard deviation of plus or minus 0.25 years. This value is adopted here as being the most accurate available for studies of the chaotic solar dynamo.

The 22-yr cycle is generally accepted to have exactly twice the period of the 11-yr cycle, and corresponds to the switch in polarity of the magnetic poles of the Sun. It has been proposed that the 22-yr cycle be considered the fundamental solar dynamo period instead of the 11-yr period. That assumption would not change the arguments or results of this analysis, although if the 11-yr cycle is the fundamental, the 22-yr cycle is the first period doubling as suggested by Gough (1988).

2.2. THE 88-YEAR CYCLE

Information on the behavior of the solar activity during the centuries before the Maunder minimum can be obtained from the records of the occurrence of mid-latitude aurora collected from European and Oriental sources for the time from 450 AD to 1450 AD (Siscoe, 1980). It is generally accepted that auroral data may be used as a proxy for solar activity and this is the most complete data available on solar activity from the pre-Maunder minimum period. The Oriental records are completely independent of the European records but their agreement is remarkable (Siscoe, 1980). Typical power spectrums of the data on the number of auroral observations reported per decade is shown in Figure 1 (Feynman and Fougere, 1984). A clear strong line at about 88 yr is seen. Inaccuracies in the data itself due to wars, politics or plagues could not have produced the 11 cycles of 88-yr variation detected by the power spectral analysis.

In order to interpret the spectral line in Figure 1 as being due to a period doubling, the period of the Gleissberg cycle must be accurately determined. Figure 2 shows 14 determinations of the period obtained from a series of power spectral analyses (Feynman and Fougere, 1984). Two different methods of power spectral analysis were used, the Burg-Maximum Entropy Method and a nonlinear generalization of the Burg method. To test the stability of the spectral lines both MEM and nonlinear spectrums were calculated using a variety of weights. The effect of uncertainty in the data base was tested by adding noise of amplitude one to the data. The mean amplitude of the 88-yr variation was 2.2 recorded auroral sightings per decade. The most probable period was found to lie between 88.4 plus or minus 0.7 yr. Evidence has been presented that this

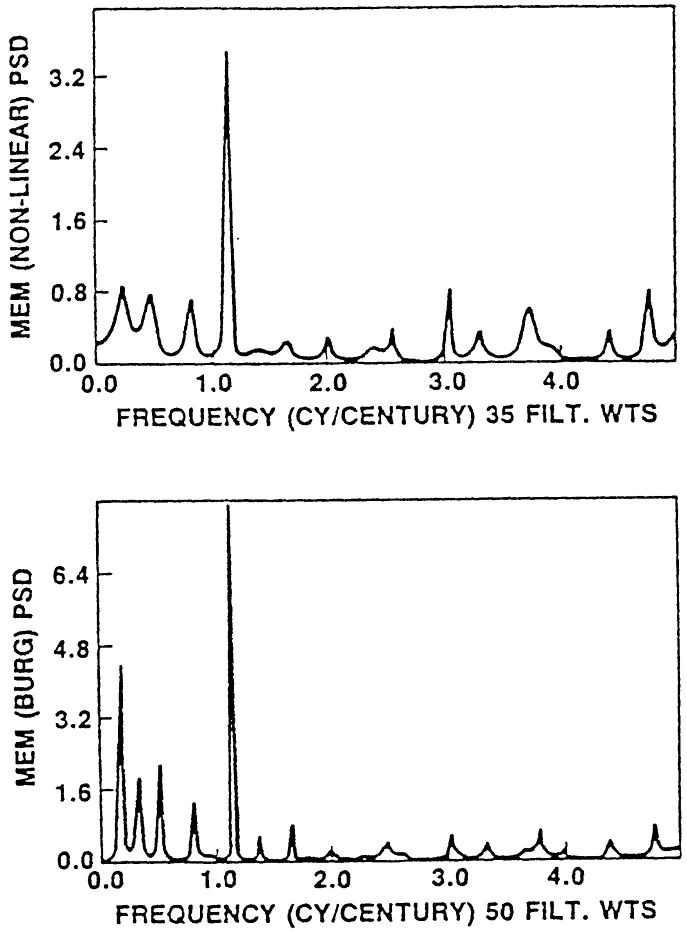

Fig. 1. Spectrums of auroral data from 450 AD to 1450 AD from Feynman and Fougere (1984). Note ordinate scales are linear.

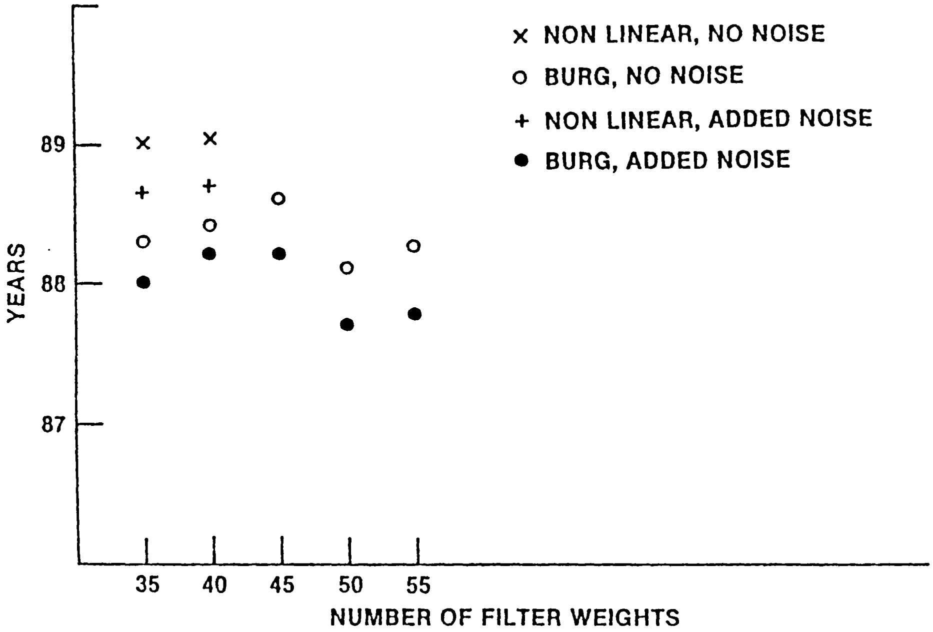

Fig. 2. Period of the Gleissberg cycle from Feynman and Fougere (1984).

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period is associated with a change in the asymmetry of the sunspot number and flaring activity in the northern and southern solar hemispheres (Vitinskii, 1965).

A comparison of the period of the 11.2 plus or minus 0.25 with the 88.4 plus or minus 0.7 shows that the periods are easily consistent with the 88-yr period being the third subharmonic of the 11-yr fundamental.

3. Phase Shift of the 88-Year Cycle

The second test discussed here distinguishes the behavior of a quasi-periodic variation from a chaotic variation. A quasi-periodic system can exhibit periods when the variations are small and have much of the appearance of the suppressed activity associated with chaos. However, when the chaotic suppressed activity interval is over and periodic behavior returns the phase of the periodic variation will be maintained across the interval if the system was in a periodic state but the phase will be lost if the system was chaotic (Weiss, 1988; Thompson and Stewart, 1986). The phase of the 88-yr variation was determined both before and after the Maunder minimum. It was found that the phase between 450 AD and 1450 AD was such that the last minimum before 1450 AD occurred between 1403 and 1413 (Feynman and Fougere, 1984). Extrapolating these results to modern times and taking account of both the uncertainty in the period and the time of phase minimum the modern minimums are expected to come between 1753 and 1768, between 1841 and 1858, and between 1929 and 1946. These dates are shown as shaded bands in Figures 3, 4, and 5 and their relation to the post-Maunder minimum phase of the 88-yr variation is discussed below.

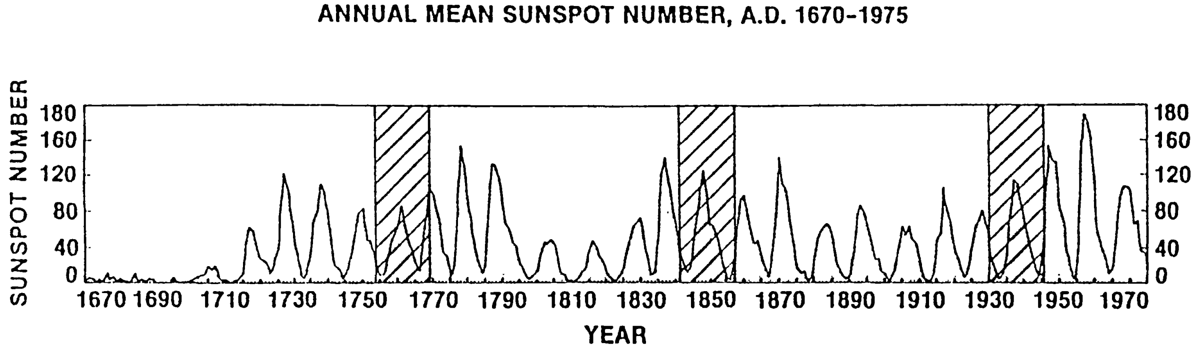

Fig. 3. Sunspot data (Eddy, personal communication). The striped regions show expected times of 88-year cycle minima calculated from pre-Maunder minimum phase.

Evidence for the reappearance of the Gleissberg cycle after the Maunder minimum is strong (Sonett, 1982) and the dates of the most recent minimums can be determined to about 10 to 20 yr. Figure 3 shows the sunspot numbers since 1600. The actual minimums in the 88-yr cycle in sunspot number cycle amplitude occurred during the first decades of the 19th and 20th centuries in marked contrast to the predicted minimums. For the 19th century, auroral data in Figure 4 also show a deep minimum between 1810

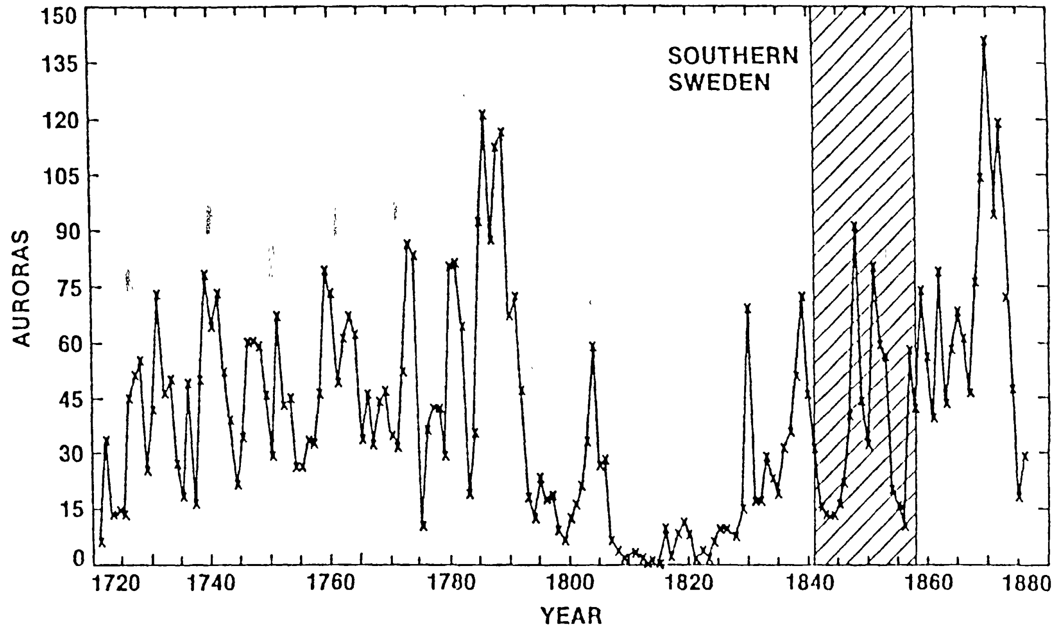

Fig. 4. Aurora in Sweden, south of Uppsala. From Silverman (personal communication). The data was collected by Rubenson (1897). Striped regions as in Figure 3.

and 1825 (Silverman and Feynman, 1980) which is also seen in 14C (Damon, 1988) and has been ascribed to the 88-yr cycle. The 20th century minimum appears as a minimum in the geomagnetic data in 1904 (Feynman and Crooker, 1978) as seen in Figure 5. A slightly different date is derived from studies of the average lifetime of sunspots (Kopecký, 1980) which show a minimum lifetime during the 15th solar cycle (cycle minimum 1913, maximum 1917). The 14C data is not available for dating the early 20th

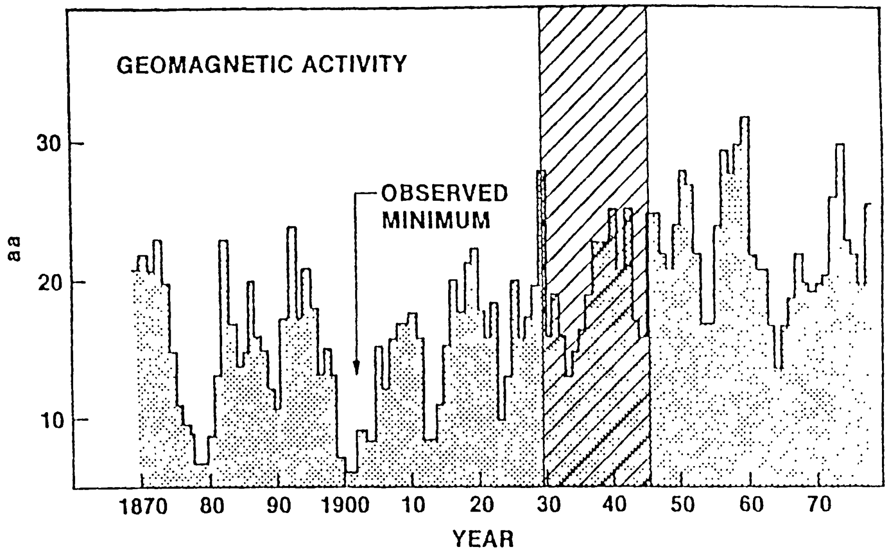

Fig. 5. Geomagnetic activity adapted from Mayaud (1975). The aa is a three-hour index measuring the range of geomagnetic variation (in units of 2 nT) and is equivalent to the arm index used for current data. Annual averages are shown. Striped regions as in Figure 3.

century minimum because the increase in the carbon content of the atmosphere from human activity was so large by the beginning of this century that 14C would not be expected to distinguish changes as small as those associated with the minimums of the 88-yr cycle (Kopecký, 1979).

A comparison of the data in Figures 3, 4, and 5 with the expected times of minimum shown in each figure as a darkened band, shows beyond doubt that there was a change in phase of the 88-yr cycle. The phase change is estimated to be about 35 yr. The accuracies with which the period and the phase of the pre-Maunder minimum variation have been determined are sufficient to rule out the possibility that the pre- and post-Maunder minimum 88-yr cycles are in phase, thus demonstrating that the Maunder minimum behaved as expected for a chaotic solar dynamo and not for a periodic dynamo.

4. Discussion

Although there are some reports of a 44-yr cycle in the literature the weakness of the evidence for a 44-yr cycle needs to be discussed since it would be the 2nd period doubling of the 11-yr cycle (or first doubling of a 22-yr cycle). Sonett (1982) reported a 44-yr line in a power spectral analysis of sunspots and it has also been suggested that the 44-yr cycle can be seen in changes of the form of the 11-yr cycle (Vitinskii, 1965). As part of this study the post-Maunder minimum sunspot data was re-examined using a superposed epoch analysis to search for a 44-yr cycle. The search was unsuccessful.

A very plausible explanation for the lack of a clearly observed strong 44-yr cycle presents itself when the parameters in which the 11, 22, and 88-yr cycles are observed are compared to the changes in the magnetic field of the Sun. It is the solar magnetic field which is actually the basic physical system whose behavior is in question (Weiss, 1988). In computer model studies of period doubling behavior it has been shown that the amplitudes of the variations have well-determined relationships. However, there is no reason to expect that the amplitudes of the quantities studied here should reflect the amplitudes of the solar dynamo changes. They should, however, have the same periods. The parameters that change in the Sun have a complex relationship with the solar or solar-terrestrial quantities that can be observed. In general, all cause and effect relationships are such that the relative amplitudes of affected periodic signals can be much different than the relative amplitudes of causative variations but the period of the effect remains the same as the period of the cause. For example, for solar variability, the amplitude and phase of the 11-yr sunspot number cycle is very different from that of the 11-yr cosmic-ray flux variation. The cosmic-ray flux variation is only a factor of two whereas the sunspot number varies by two orders of magnitude. In addition, they are 180 degrees out of phase. In a less familiar example, the 11-yr cycle is clearly visible in sunspot areas but much less so in the lifetime of sunspots whereas sunspot lifetimes show a very clear 88-yr cycle and a surprisingly weak 11-yr cycle (Kopecký, 1979). Historically the 22-yr cycle was discovered quite early by noting that the sunspot number and geomagnetic activity tended to be relatively low every other cycle (Chernowsky, 1966). The discovery that the 22-yr cycle corresponded to a switch in the polarity of the Sun's magnetic field at the poles was an entirely separate finding. Although suggestions have been made (Sonett, 1982), there is no generally accepted theory that explains why the change in polarity should result in a change in the amplitude of the sunspot cycle or in the behavior of aurora or geomagnetic activity. It would seem that these parameters would not depend on which pole of the Sun exhibited a north-pointing magnetic field. In a like manner, the 88-yr cycle is observed through sunspot numbers and solar terrestrial activity but corresponds to a shift in the dominant hemisphere of the Sun (Waldmeier, 1957; Kopecký, 1979). For example, before the early 1900's, the southern hemisphere of the Sun had more spots and active regions than the northern hemisphere, whereas after the 88-yr minimum the opposite was true (Waldmeier, 1957). Again, there is no general agreement on concepts that would lead to the expectation that change in hemispheric dominance would result in the changes seen in total sunspots or solar terrestrial phenomena. It is possible that a change in the magnetic field of the Sun with a 44-yr period expresses itself only weakly in the solar and solar-terrestrial parameters tested thus far.

5. Conclusions

The observations reported here can be described as consisting of period doubled oscillations which are modulated by a slower variation. The Sun exhibits intervals of suppressed activity during which the phase of the periodic variations is not conserved. We interpret these data in terms of a chaotic solar dynamo. In the simplest model the Sun has a dynamo number such that it is in a chaotic regime but is very close to the bifurcation between the doubled period regime and chaotic regime. The argument that the dynamo is chaotic rests on two observations, that the intervals between the periods of suppressed activity (Maunder and Spörer type minima) are irregular, and that the phase of the 88-yr cycle is lost on exit from the Maunder minimum. The argument that the dynamo remains close to the bifurcation between period doubling and chaos is that the 11, 22, and 88 yr variations form part of a period doubled set.

There are several somewhat different explanations of this behavior suggested by the highly simplified model studied by Weiss, Cattaneo, and Jones (1984).

5.1. CONSTANT DYNAMO NUMBER

In this view the solar dynamo is imagined to have a constant control parameter which is at the value so that the system is chaotic but very close to the bifurcation between period doubling behavior and chaos. The simplified model discussed by Weiss, Cattaneo, and Jones is followed remarkably well. If the behavior of a system is studied as a function of control parameter it is found that when the system bifurcates to chaos the result is a broadening of the spectral lines. That is, near the bifurcation value the lines still appear but they are broadened and noise is present. This behavior is completely consistent with what is seen in solar activity. This explanation may be expanded to explain why the dynamo number has this value as is argued below.

5.2. SELF-REGULATING DYNAMO NUMBER

In this view the dynamo number is imagined to vary by a very small amount, just enough to take the system from the region of period doubling to chaos and back. In other words, we imagine that the self-consistent dynamo is such that the dynamo number is held at the bifurcation value by the actions of the dynamo itself. That is, one might imagine that as the solar dynamo developed in time the dynamo number changed until the system was in the period doubled regime. Here the motions produced a field that changed the dynamo number by a small amount so that the motion went into chaos which is characterized by times of suppressed activity with loss of phase coherence. During the suppressed activity the motions would be sufficiently suppressed to decrease the dynamo number by a small amount again so that the system went back out of chaos to the period-doubling regime. This sequence of events would then be repeated again and again so the control parameter would be held very close to the bifurcation. The changes of the control parameter need not be large enough to actually move the system in and out of the chaotic regime, but only large enough to keep the control parameter confined to values in the chaotic regime but close to the period-doubled-chaotic bifurcation. Modeling will be required in order to estimate the actual size of the dynamo number changes due to feedback.

Regardless of the detailed mechanism the data show the Sun is hanging up near the region of bifurcation to and from chaos, i.e., the solar dynamo is stalling. We suggest this is a fundamental characteristic of stars such as the Sun and is due to the nonlinear interaction between the fluid motions and the magnetic field strength.

5.3. NOTE

During the last day of the final drafting of this report we were informed that a 175-yr variation and a variation at about 350 yr have now been seen in power spectrum of 7000 yr of 14C data (Damon and Sonett, private communication). These correspond to the 4th and 5th period doublings of the 11-yr cycle. These new findings lend very strong support to our interpretation of the data.

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