A NEW LOOK AT WOLF SUNSPOT NUMBERS IN THE LATE 1700's

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Abstract. Long-term homogeneous observations of solar activity or many solar cycles are essential for investigating many problems in solar physics and climatology. The one key parameter used in most long-term studies is the Wolf sunspot number, which is susceptible to observer bias, particularly because it is highly sensitive to the observer's ability to see the smallest sunspots. In this paper we show how the Wolf sunspot number can be derived from the number of sunspot groups alone. We utilize this approach to obtain a 'Group Wolf number'. This technique has advantages over the classical method of determining the Wolf number because corrections for observer differences are reduced and long-term self-consistent time series can be developed. The level of activity can be calculated to an accuracy of ± 5% using this method. Applying the technique to Christian Horrebow's observations of solar cycles 1, 2, and 3 (1761–1777), we find that the standard Wolf numbers are nearly homogeneous with sunspot numbers measured from 1875 to 1976 except the peak of solar cycle 2 is too low by 30%. This result suggests that further analyses of early sunspot observations could lead to significant improvements in the uniformity of the measurements of solar activity. Such improvements could have important impacts upon our understanding of long-term variations in solar activity, such as the Gleissberg cycle, or secular variations in the Earth's climate.

1. Introduction

The Wolf sunspot number is the only solar index which has a record spanning several centuries of observations. As such, it is important in many solar studies requiring a long data record. Examples of such studies include determining the chaotic nature of solar activity (Mundt *et al.*, 1991), the possible existence of an internal solar chronometer (Sonnet, 1983; Dicke, 1978), and the potential solar influences on climate (e.g., Reid, 1991). All these studies require measurements which are homogeneous; i.e., internally self-consistent so that differences over time represent true changes in the Sun and not artifacts in the compilation or changes caused by improvements in instrumentation as technology advanced. Compilers of the Wolf sunspot number (e.g., Waldmeier, 1961; McKinnon, 1987) describe the Wolf sunspot numbers as being poor for 1700–1748, questionable for 1749–1817, good for 1818–1847, and reliable after 1848. These qualitative judgments unfortunately do not allow researchers a quantitative method of accessing the quality of the historic solar observations. In this paper, Section 2 describes a technique which can be used to make earlier observations more self-consistent with modern observations. Traditionally, the Wolf sunspot number is sensitive to the number of individual sunspots, many of which are on the borderline of visibility. The new

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technique of deriving sunspot numbers avoids that problem and still gives sunspot numbers very similar to the Wolf sunspot numbers published in recent years. It is a robust technique which is insensitive to observer differences. In Section 3, the technique, as pilot study for a larger study, is applied to Christian Horrebow's sunspot observations from 1761 to 1777. These results are compared to the standard compilation of the Wolf sunspot numbers and an assessment of their reliability given. The results of the pilot study indicate that significant improvements in the homogeneity of the sunspot record can be achieved by this technique. In the concluding section, we outline an approach for applying the technique to all observations before 1848.

Many researchers have commented upon the reconstruction of solar activity by Wolf and have questioned his reconstruction for the earlier periods. For example, Broun (1876) disputed his timings of sunspot maxima and minima. Loomis (1870) wondered whether a solar cycle peaking in 1794 existed which was missed by Wolf. Newcomb (1901) contended there are imperfections in the sunspot record around 1790. Schuster (1906) pointed out that the sunspot record before 1826 is characterized by two frequencies of oscillations, although after this date one frequency is sufficient. McNish and Lincoln (1949) noted that the sunspot record before 1834 belongs to a different statistical population than the data afterwards. These studies may lead one to think that the early sunspot record is 'poor' as compilers now judge it. On the other hand, it may be that the Sun itself was behaving differently in the more distant past than it is now. Eddy (1976), in his documentation of the Maunder Minimum, has shown that the Sun can behave quite differently over time scales of several decades. Baliunas and Jastrow (1990) have noted that solar type stars appear to have behavior that could be Mauder Minimum type events. It is therefore not inconceivable that in the 1700's the Sun was behaving in an unusual fashion intermediate between a prolonged minimum and modern type behavior. This paper provides a test of a procedure which can be used to resolve whether the unusual behavior noted by the authors cited above is due to a faulty reconstruction of solar activity or due to some anomalous behavior in the Sun.

2. Method and Data

The Zürich Wolf sunspot number (R_z) is defined by the following equation:

$$R_z = k(10g + n)\,, \tag{1}$$

where g is the number of sunspot groups, n is the number of individual sunspots and k is constant correction factor which brings each observer to a common scale. In the classical reconstruction of sunspot numbers, a value of k must be deduced or assumed to be equal to 1. The observations by each observer must then be read and for each day a value for the number of sunspot groups and individual sunspot numbers must be extracted. Unfortunately, individual observers differed in their instruments, interests, and diligence of recording their observations. Some observers (e.g., Flaugergues, active around 1800) may record for many days but only state whether some sunspots were present or not. Such limited recordings make a reconstruction of the Wolf sunspot

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number difficult, if not impossible. Other observers (e.g., Herschel, most active from 1800 to 1807) may diligently record group numbers and individual sunspot numbers, but only do so consistently for a few years. Even diligent observers such as Herschel may record only the number of sunspot groups without giving the number of individual sunspots (n). In fact, a difficulty in using Equation (1) is that it is dependent on knowledge of n, which may be absent or, if present, is highly dependent upon the observer's telescope and seeing conditions. Using different values of k for each observer can mitigate this problem somewhat but not remove it. Uncertainties in n and hence in k may be the primary reason why reconstructions of solar activity before 1848 are uncertain.

Recently (Hoyt and Schatten, 1992), Herschel's sunspot observations were examined for the period 1800–1806. It was found in that study that Herschel's observations could be made homogeneous with modern observations. It is probable that nearly all observers recognize sunspot groups as distinct entities which can be counted and recorded. Using the yearly mean Wolf sunspot numbers and Greenwich Observatory groups for 1875 to 1976, a regression equation relating the Wolf sunspot number of sunspot groups is:

$$R_g = (1.67 \pm 9.53 + (9.67 \pm 0.224)g + (0.215 \pm 0.017)g^2. \qquad (2)$$

Here we refer to numbers derived from Equation (2) as 'Group Wolf numbers' (R_g) to distinguish them from the standard or Zürich Wolf sunspot numbers (R_z) found using Equation (1). The quadratic term in Equation (2) is introduced to approximate the sunspot number properly for high solar activity. When less than twelve sunspot groups are present, a linear equation suffices. When solar activity rises above twelve groups, more complex sunspot groups (many with numerous individual sunspots) appear and a non-linear term is required to have R_g match the R_z values. Equation (2) has a major advantage compared to Equation (1) because only the number of sunspot groups is required to estimate sunspot number. Uncertainties in k and n will not affect the results if one uses Equation (2). Early solar observers often only recorded the number of sunspot groups and not the number of individual sunspots, so Equation (2) can be applied to more observations. On the other hand, the disadvantage of Equation (2) relative to Equation (1) is that one gives up information on the individual sunspot counts, when that data is available. We will show, however, that with many observers, this represents a large amount of noise and little signal. Provided all observers agree that sunspots groups are distinct entities, Equation (2) provides a robust measure of the sunspot number independent of the observer, his seeing conditions, and the telescope used. Most importantly, Equation (2) makes earlier observations consistent with sunspot numbers from 1875 to 1976 and provides error bars on the newly reconstructed sunspot numbers. A long-term homogeneous Group sunspot number time series appears feasible with this approach, which provides nearly identical numbers to the familiar Wolf sunspot number measured in recent years. Although this paper's intent is to describe a technique to make the commonly used sunspot numbers homogeneous over long periods of time, it should be noted that using the number of sunspot groups alone provide a valid index of solar activity. However, it is not our goal here to introduce new

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and unfamiliar measures of solar activity. Thus, we transform the group number to sunspot number.

To test how well Equation (2) generates reasonable sunspot numbers, the monthly mean sunspot groups ere extracted from the Greenwich Observatory measurements. Equation (2) was applied to this data set and the Group Wolf numbers compared to the Zürich standard Wolf numbers for the period 1875 to 1976. The correlation between the group and standard Wolf sunspot numbers is 0.950. Since the Wolf sunspot number was designed to provide a proxy measure of variations in the projected sunspot area, it is worthwhile comparing the two sunspot numbers to sunspot areas. The Zürich sunspot numbers have a correlation of 0.949 with area and the group sunspot numbers have a correlation of 0.918. Although the Zürich numbers are a better model for projected sunspot areas, it is not much better than using Equation (2). Equation (1) explains only 6% more of the variance although it requires three input parameters rather than just one and not all three parameters may always be available in the early records. These correlations support the view stated earlier that very lettle signal is contained in these two extra parameters. In Figure 1, the difference between the Group and Zürich Wolf sunspot numbers is plotted. In general, the difference in the numbers are less than 20. A residual 11 year cycle remains in the differences which tells us that Equation (2) sometimes underestimates the strength of the stronger solar cycles but can either slightly overestimate or underestimate the less intense cycles. The average absolute error in the

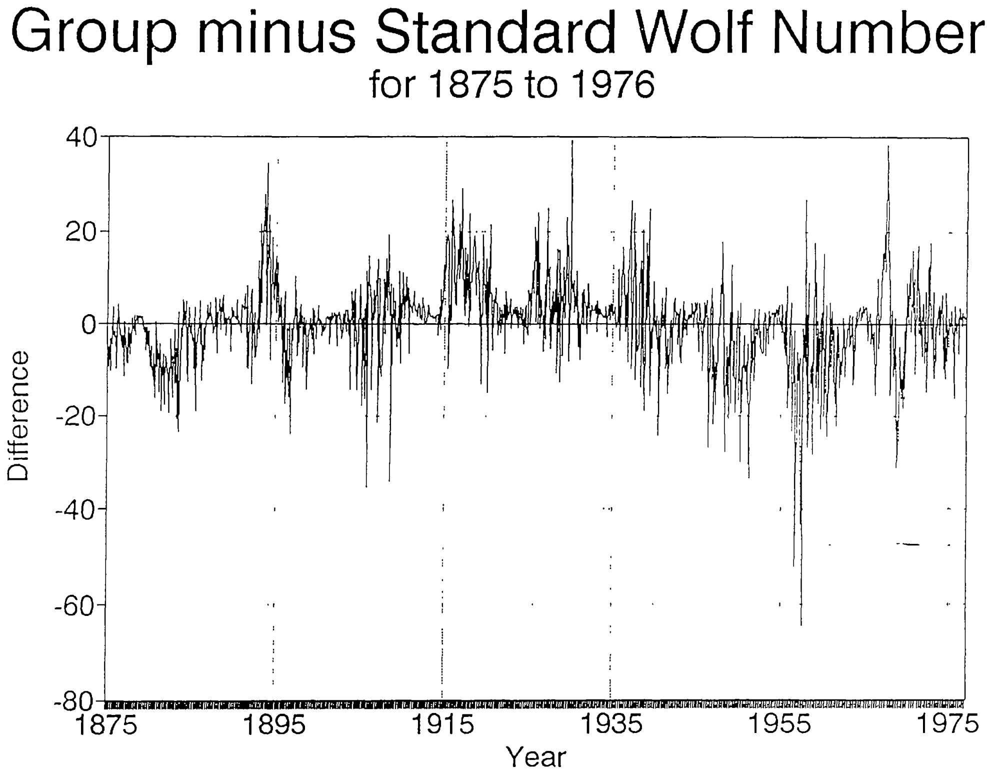

Fig. 1. The Group Wolf sunspot numbers minus the Zürich Wolf sunspot numbers for 1875 to 1976. The average difference for any one month is about 7. Extremely strong cycles will be underestimated, but weaker cycles are usually slightly overestimated.

sunspot number using Equation (2) is 6.57. For Zürich sunspot numbers greater than 100, the group sunspot numbers differ by 9.84% on average for any one month. Choosing all months in a cycle when the Zürich sunspot number is above 100 to represent the peaks in activity, the mean error from Equation (2) is + 2.96% for cycle 17, − 4.93% for cycle 18, and − 6.75% for cycle 19. Thus, applying Equation (2) allows Wolf sunspot numbers to be generated so that the peak in solar activity from cycle to cycle can be found with an accuracy of ± 5%.

3. Application to Christian Horrebow's Observations

As an illustration of the technique, it is applied to Christian Horrebow's solar observations at the Hafnia Observatory in Copenhagen from 1761 to 1777 and compared to Wolf's reconstructuion for the same years. We chose this period because Thiele (1859) tabulates the monthly mean number of sunspot groups observed by Horrebow. Horrebow's time series of monthly mean group numbers is reproduced in Table I. Of the 204 months potentially available, observations for 161 months exist, or 79% of the time. Wolf in reconstructing his monthly means also used Horrebow's observations. It is evident, for example, that when there are extensive gaps in Horrebow's observations (e.g., the first half of 1765), the Wolf sunspot numbers are linearly filled interpolated values. Wolf had some data from other observers at this time such as Mallet and perhaps Zucconi.

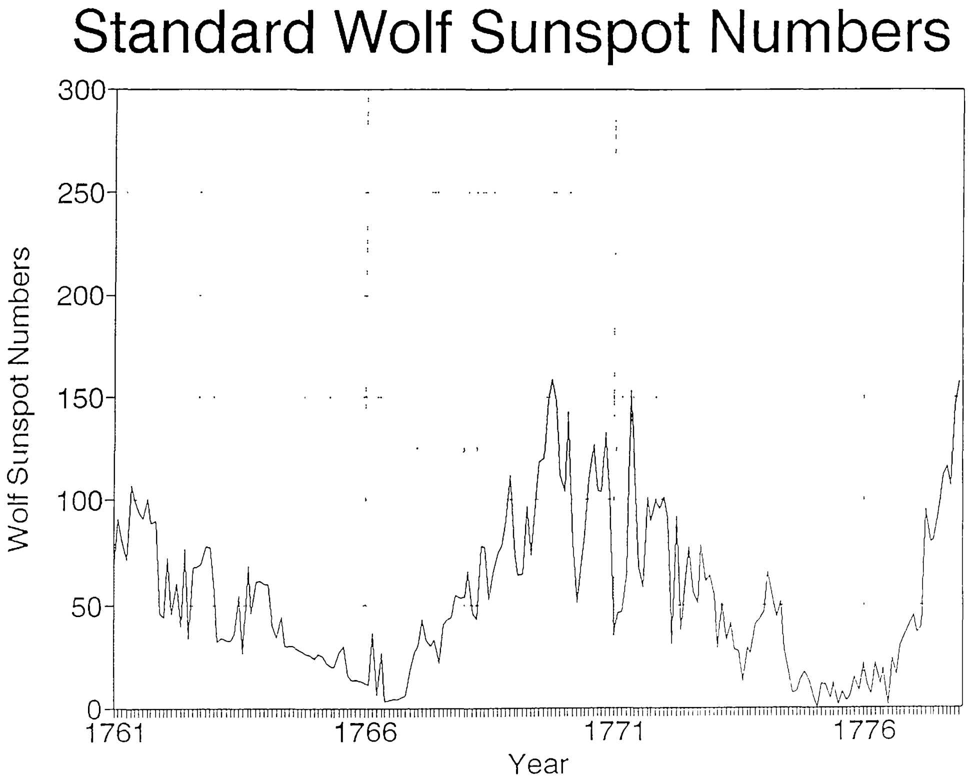

Fig. 2. The standard monthly mean Wolf sunspot numbers for the period 1761 to 1777. These are the commonly used values derived by Wolf and tabulated by Waldmeier (1961) and McKinnon (1987). The scaling on this plot is set to be the same as for Figure 3.

TABLE I

Monthly mean Wolf sunspot numbers as reconstructed by Wolf and as reconstructed using Horrebow's observations. The columns give the year, month, the standard or Zürich Wolf sunspot number, Horrebow monthly mean group number, and the Group Wolf sunspot number reconstructed from the mean group numbers.

Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.	Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.	Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.
1761	1	70.0	5.0	55.4	1766	9	4.3	0.0	1.7	1772	5	38.0	2.6	28.3
1761	2	91.0	6.2	69.9	1766	10	5.0	0.0	1.7	1772	6	57.0	6.2	69.9
1761	3	80.7	3.0	32.6	1766	11	5.7	0.0	1.7	1772	7	77.3	5.5	61.4
1761	4	71.7	4.5	49.6	1766	12	19.2	0.6	7.6	1772	8	56.2	3.2	34.8
1761	5	107.2	9.6	114.4	1767	1	27.4	2.0	21.9	1772	9	50.5	3.5	38.2
1761	6	99.3	6.1	68.7	1767	2	30.0	0.0	1.7	1772	10	78.6	5.6	62.6
1761	7	94.1	6.7	76.1	1767	3	43.0	1.4	15.6	1772	11	61.3	4.6	50.7
1761	8	91.1			1767	4	32.9	1.6	17.7	1772	12	64.0	5.2	57.8
1761	9	100.7			1767	5	29.8	2.0	21.9	1773	1	54.6	4.9	54.2
1761	10	88.7			1767	6	33.3	3.6	39.3	1773	2	29.0	1.2	13.6
1761	11	89.7	4.7	51.9	1767	7	21.9	2.0	21.9	1773	3	51.2	2.8	30.4
1761	12	46.0			1767	8	40.8	3.5	38.2	1773	4	32.9	2.3	25.1
1762	1	43.8			1767	9	42.7	3.5	38.2	1773	5	41.1	2.6	28.3
1762	2	72.8			1767	10	44.1	3.4	37.0	1773	6	28.4	1.8	19.8
1762	3	45.7			1767	11	54.7	6.8	77.4	1773	7	27.7	0.9	10.5
1762	4	60.2			1767	12	53.3	5.4	60.2	1773	8	12.7	1.2	13.6
1762	5	39.9			1768	1	53.5	4.4	48.4	1773	9	29.3	2.0	21.9
1762	6	77.1			1768	2	66.1	5.5	61.4	1773	10	26.3	2.2	24.0
1762	7	33.8			1768	3	46.3	4.7	51.9	1773	11	40.9	3.0	32.6
1762	8	67.7			1768	4	42.7	3.2	34.8	1773	12	43.2	2.9	31.5
1762	9	68.5			1768	5	77.7	7.1	81.2	1774	1	46.8	3.3	35.9
1762	10	69.3			1768	6	77.4	6.4	72.4	1774	2	65.4	5.5	61.4
1762	11	77.8			1768	7	52.6	4.6	50.7	1774	3	55.7	4.0	43.8
1762	12	77.2			1768	8	66.8	4.9	54.2	1774	4	43.8	2.8	30.4
1763	1	56.5			1768	9	74.8	6.1	68.7	1774	5	51.3	3.1	33.7
1763	2	31.9			1768	10	77.8	5.9	66.2	1774	6	28.5	1.6	17.7

Table I (continued)

Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.	Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.	Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.
1763	3	34.2			1768	11	90.6	4.6	50.7	1774	7	17.5	0.8	9.5
1763	4	32.9			1768	12	111.8	9.0	106.2	1774	8	6.6	0.1	2.6
1763	5	32.7			1769	1	73.9	6.3	71.1	1774	9	7.9	0.3	4.6
1763	6	35.8			1769	2	64.2	7.0	79.9	1774	10	14.0	0.6	7.6
1763	7	54.2			1769	3	64.3	5.0	55.4	1774	11	17.7	1.4	15.6
1763	8	26.5			1769	4	96.7	8.0	92.8	1774	12	12.2	0.5	6.6
1763	9	68.1			1769	5	73.6	5.8	65.0	1775	1	4.4	0.1	2.6
1763	10	46.3			1769	6	94.4	7.9	91.5	1775	2	0.0	0.0	1.7
1763	11	60.9			1769	7	118.6	12.4	154.7	1775	3	11.6	0.4	5.6
1763	12	61.4			1769	8	120.3	10.4	125.5	1775	4	11.2	0.5	6.6
1764	1	59.7	2.0	21.9	1769	9	148.8	15.8	208.2	1775	5	3.9	0.3	4.6
1764	2	59.7	2.6	28.3	1769	10	158.2	18.7	257.8	1775	6	12.3	1.0	11.6
1764	3	40.2	3.7	40.4	1769	11	148.1	17.2	231.7	1775	7	1.0	0.2	3.6
1764	4	34.4	1.0	11.6	1769	12	112.0	17.5	236.9	1775	8	7.9	0.6	7.6
1764	5	44.3	2.0	21.9	1770	1	104.0	10.9	132.7	1775	9	3.2	0.2	3.6
1764	6	30.0	0.7	8.5	1770	2	142.5	14.1	180.8	1775	10	5.6	0.4	5.6
1764	7	30.0	0.0	1.7	1770	3	80.1	7.7	88.9	1775	11	15.1	0.7	8.5
1764	8	30.0	1.4	15.6	1770	4	51.0	3.9	42.7	1775	12	7.9	0.9	10.5
1764	9	28.2	1.0	11.6	1770	5	70.1	5.3	59.0	1776	1	21.7	2.3	25.1
1764	10	28.0	1.0	11.6	1770	6	83.3	8.0	92.8	1776	2	11.6	1.1	12.6
1764	11	26.0	2.0	21.9	1770	7	109.8	12.6	157.7	1776	3	6.3	0.4	5.6
1764	12	25.7			1770	8	126.3	10.8	131.2	1776	4	21.8	1.5	16.7
1765	1	24.0			1770	9	104.4	8.8	103.5	1776	5	11.2	0.7	8.5
1765	2	26.0			1770	10	103.6	6.5	73.6	1776	6	19.0	1.3	14.6
1765	3	25.0			1770	11	132.2	12.8	160.7	1776	7	1.0	0.1	2.6
1765	4	22.0			1770	12	102.3	8.2	95.5	1776	8	24.2	1.5	16.7
1765	5	20.2			1771	1	36.0	1.5	16.7	1776	9	16.0	1.5	16.7

Table I (continued)

Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.	Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.	Year	Month	Stand. Wolf No.	Group No.	Group Wolf No.
1765	6	20.0			1771	2	46.2	3.3	35.9	1776	10	30.0	2.2	24.0
1765	7	27.0			1771	3	46.7	2.3	25.1	1776	11	35.0	2.0	21.9
1765	8	29.7	0.5	6.6	1771	4	64.9	5.9	66.2	1776	12	40.0	3.0	32.6
1765	9	16.0	0.0	1.7	1771	5	152.7	13.7	174.6	1777	1	45.0	3.4	37.0
1765	10	14.0	0.0	1.7	1771	6	119.5	9.4	111.6	1777	2	36.5	2.0	21.9
1765	11	14.0	0.0	1.7	1771	7	67.7	6.3	71.1	1777	3	39.0	2.9	31.5
1765	12	13.0			1771	8	58.5	5.3	59.0	1777	4	95.5	3.0	32.6
1766	1	12.0			1771	9	101.4	9.4	111.6	1777	5	80.3		
1766	2	11.0	0.0	1.7	1771	10	90.0	2.5	27.2	1777	6	80.7		
1766	3	36.6	0.4	5.6	1771	11	99.7	6.9	78.7	1777	7	95.0		
1766	4	6.0	0.0	1.7	1771	12	95.7	10.0	119.9	1777	8	112.0	4.2	46.1
1766	5	26.8	0.0	1.7	1772	1	100.9	8.7	102.1	1777	9	116.2	6.2	69.9
1766	6	3.0	0.0	1.7	1772	2	90.8	7.3	83.7	1777	10	106.5	11.0	134.1
1766	7	3.3			1772	3	31.1	3.0	32.6	1777	11	146.0	11.9	147.2
1766	8	4.0			1772	4	92.2	6.9	78.7	1777	12	157.3	9.7	115.7

In Figure 2, the standard monthly mean Wolf sunspot numbers are plotted. The peak value is 158.2 in October, 1769. Using Equation (2), Wolf sunspot numbers are derived from Horrebow's observations. The monthly means are tabulated in Table I and plotted in Figure 3. From this analysis, the peak monthly mean also occurs in October, 1769 but equals 257.8 ± 20. Thus, the standard Wolf sunspot numbers are not always homogeneous with modern observations for the peak of solar cycle 2, but rather are low by about 30% for the calendar year 1769 (Zürich Wolf number = 106; Group Wolf number = 139). This inhomogeneity in the standard sunspot number record is the largest error for the period 1761 to 1777. Errors in solar activity of this magnitude may be occurring throughout the solar record for the 1700's. The correction of errors could affect our interpretation of solar activity and have important bearings on questions such as the Gleissberg cycle, the nature of solar chaotic behavior, and the presence of absence of a solar chronometer. An improved reconstruction of solar activity for this period may lead to better models of solar irradiance variations, which would have consequences upon climate change. In general, the standard Wolf sunspot numbers are only self-consistent with modern observations to within about 20 (see Figure 4). For much of the time from 1761 to 1777, Wolf slightly overestimated the level of solar activity.

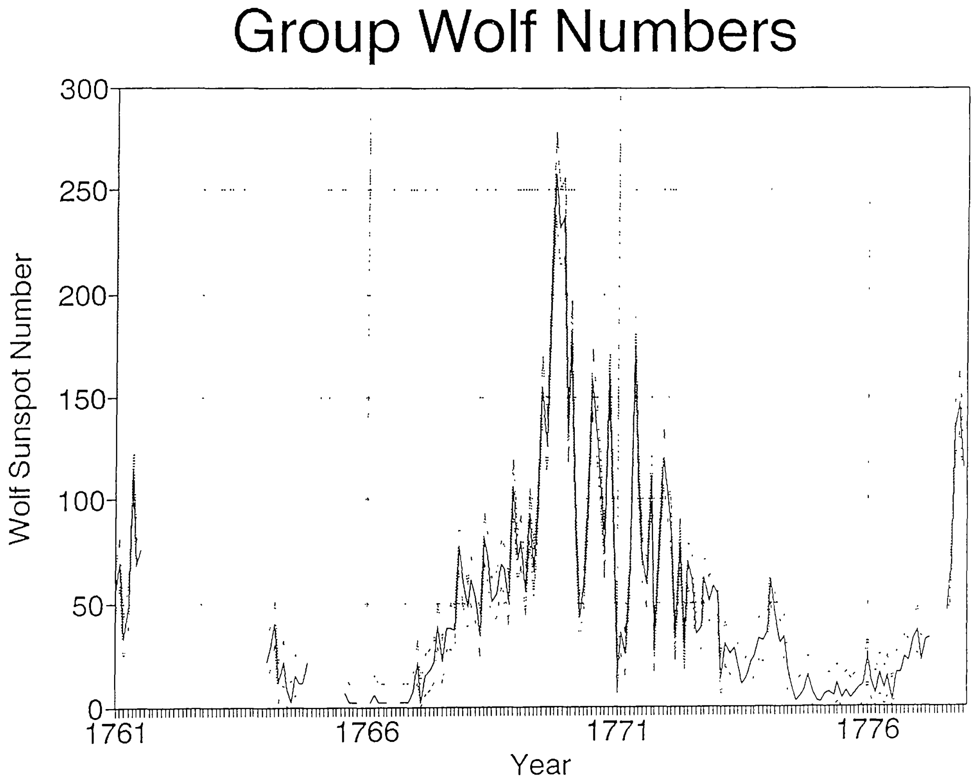

Fig. 3. The monthly mean Wolf sunspot numbers derived in this paper using Horrebow's observations and Equation (2). Parts of solar cycles 1, 2, and 3 are covered. Solar cyle 2 is more intense than that given in Figure 1. Dotted lines are the one standard deviation uncertainties in the derived sunspot numbers.

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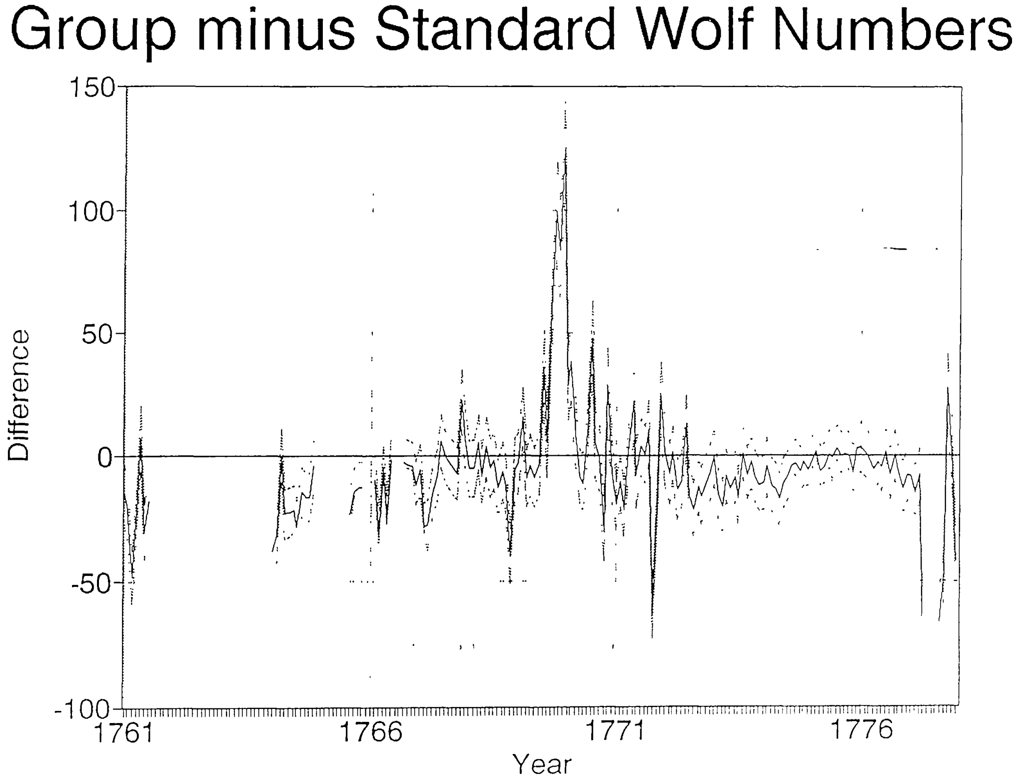

Fig. 4. The Group sunspot numbers minus the standard sunspot numbers. A positive value indicates the Group sunspot numbers are higher than the standard values and, in particular, indicate solar cycle 2 is more intense than generally assumed. Most often solar activity for this time was overestimated. Dotted lines give the one standard deviation uncertainty in the differences.

4. Conclusions

The early sunspot observations thus can be made homogeneous and self-consistent with modern observations provided one has a record of the number of sunspot groups observed. Using the Greenwich Observatory measurements of sunspot groups and the modern values of the Wolf sunspot numbers, an Equation (2) can be derived relating the two variables. This equation allows all earlier observations of the number of observed groups alone to be converted to Wolf sunspot numbers which are consistent with modern observations. The average error in the monthly mean Wolf sunspot number will be about 7, or less than 14% of the long-term mean. Problems with observer bias arising from differences in equipment and viewing conditions are reduced by this technique.

Applying the method to Horrebow's observations from 1761 to 1777, a time series of Wolf sunspot number is derived. Comparing these results to the standard Wolf sunspot numbers, we deduce that Wolf overestimated the level of acctivity during this period, although the peak of solar cycle 2 in 1769 is significantly underestimated. The yearly mean for 1769 should probably be about 139 rather than 106. The 13 month running mean peak equals 146.9 and occurs in October, 1769. Thus, solar cycle 2 is much like solar cycle 8 with a smoothed peak of 146.9 in 1837, solar cycle 11 with a smoothed peak of 140.5 in 1870, or solar cycle 18 with smoothed peak of 151.8 in 1947. Of the 27 cycles since the Maunder Minimum, solar cycle 2 ties cycle 8 as the sixth strongest cycle rather the eleventh as has been accepted.

The technique described and applied here is promising. It is a major effort to collect additional earlier observations of the number of sunspot groups. Not all the earlier observations are still extant, but re-examining the remaining available observations promises to provide an improved and fully self-consistent time series of solar activity from 1700 to the present.

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