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# Eighty-Eight Year Periodicity in Solar-Terrestrial Phenomena Confirmed

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The existence of a 60–100 year periodic variation in solar and/or solar-terrestrial phenomena has been a matter of dispute for many years. A wide variety of data sets previously have been analyzed, and the results of the analyses have been interpreted as showing evidence either for or against such a variation. However, all data sets that are proxy for solar wind in the ecliptic at 1 AU show variations consistent with a period of about 88 years. Here we report that a maximum entropy spectral analysis of the number of aurora reported per decade in Europe and the Orient from 450 A.D. to 1450 A.D. shows a strong stable line at a 88.4 ± 0.7 years. Since the data set contains 11 cycles, this analysis establishes the reality of the "long cycle" for 1000 years. The mean amplitude and phase are then estimated from a superposed epoch analysis. The mean amplitude was 2.2 auroral reports per decade and the last minimum phase in these data occurred between 1403 A.D. and 1413 A.D.

## INTRODUCTION

Firmly establishing whether or not long-period variations exist in solar output and in solar terrestrial relations is of fundamental importance in understanding the sun and solar wind and in our ability to predict variations in interplanetary and magnetospheric environments. However, there has been a great deal of interest and controversy concerning the reality of systematic changes in solar output in the frequency range of one or two cycles per century. This type of possible variation has been called, among other things, the Gleissberg variation, the "long cycle," the "87-year cycle," and the secular variation. The frequency range is particularly intriguing because on the one hand there are hints of such changes in the historical record and on the other hand the period is so long that it has been difficult to accumulate data adequately to test for such a variation. Furthermore, tests made on different data sets have resulted in conflicting conclusions.

## STATUS OF THE PROBLEM

Recently, *Feynman* [1983] reviewed six data sets which had been used previously as proxy data for solar outputs to test for periodic changes in the 60–100 year period range. These data sets were chosen because in each case the relationship between the observed quantities and the solar wind was understood at least in principle. For this reason, data pertaining to solar-weather relationships were not included. The six data sets were the sunspot numbers since 1720 as an expression of the solar cycle, the *aa* index of geomagnetic activity, a post–Maunder minimum auroral data set, a pre–Maunder minimum auroral data set, and two sets of $^{14}C$ data. The different data sets are, of course, proxy data for different aspects of solar variability. The sunspot number is related to the photospheric magnetic field. The $^{14}C$ is produced indirectly by cosmic rays that have propagated through broad reaches of the heliosphere and so refers to properties of the solar wind throughout the heliosphere. Geomagnetic variations and auroral counts refer to solar output in a very restricted region of space, i.e., the ecliptic plane at 1 AU. Since the data sets are proxy for at least three different aspects of solar activity and solar wind, a periodicity in any one of these quantities does not necessarily imply a periodicity in the others, and disagreements exist among the results only when two analyses of proxy data for the same aspect of the sun and/or solar wind yield conflicting results.

The post-1720 11-year sunspot number cycles had relatively small amplitudes at the beginning of the 19th and of the 20th centuries, and these small amplitude cycles have been widely interpreted as being due to minimums in an 87-year periodic variation. *Sonett* [1982] has recently run a maximum entropy power spectrum on these data and, as expected, found a line at an 87-year period. However, there are only $2\frac{1}{2}$ cycles of data at that period, and in our opinion that is not a long enough time series to establish a variation as periodic. The post-1720 frequency of auroral sightings in Sweden [*Rubenson*, 1882] and the geomagnetic variations as measured by *aa* [*Mayaud*, 1973] are also consistent with an 87-year cycle. Although neither of these latter two sets of solar terrestrial data shows a yearly average value that is proportional to the yearly average sunspot number [*Feynman and Crooker*, 1978; *Legrand and Simon*, 1981; *Feynman*, 1983], they both have minimum values at about the same time as the sunspot cycle amplitude minimums, i.e., Swedish auroral reports minimized in the early 19th and 20th centuries [*Silverman and Feynman*, 1980] and *aa* minimized in the early 20th century.

The situation is different with the two $^{14}C$ data sets. *Stuiver* [1980] carried out a power spectral analysis of the post–700 A.D. rates of production of $^{14}C$ and did not find any increase in power in the 60- to 100–year frequency range. However, in an independent study, *Lin et al.* [1975] did a covariance function analysis of 8000 years of $^{14}C$ anomaly data and found a broad increase in power in the region of 80 years, which they interpreted as evidence for the Gleissberg cycle. This conflict of results is not yet resolved.

The final data set discussed by *Feynman* [1983] is the me-



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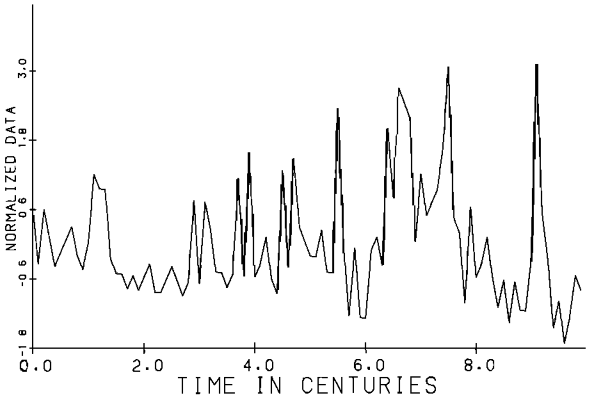


Fig. 1. The normalized and detrended number of auroral observations per decade. Zero corresponds to 450 A.D. The data before normalization and detrending is from *Siscoe* [1980].

dieval auroral set which was reviewed by *Siscoe* [1980] and which will be further analyzed here. The data set is derived from reports of auroras seen in Europe and in the Orient from 450 A.D. to 1450 A.D. Siscoe, following *Keimatsu* [1976], investigated the accuracy of the data by comparing the number of auroras per century reported from China and Europe separately. Not only were the envelopes of the two frequency distributions remarkably close, but the actual number of reports from the two areas were surprisingly similar. In addition, features corresponding to the 7th century medieval auroral minimum and the 12th century medieval auroral maximum are seen in the $^{14}C$ data. Siscoe also presented the number of auroras seen per decade in the combined European-Oriental data set. In Figure 1, we show the data to be used here, i.e., the data taken from *Siscoe* [1980] and then normalized and detrended. As Siscoe pointed out and as had been often suggested before (see review by *Siscoe* [1980]), the data set appeared to show a periodic variation with a mean period of about 87 years. In order to test the validity of this observation in more objective way, *Feynman* [1983] carried out a modified superposed epoch analysis for the 87-year signal and found what appeared to be a statistically significant result. However, this method of analysis is unsatisfactory because the period must be chosen ahead of time and so cannot be accurately determined and because the evaluation of the statistical significance of the results is subjective.

In this paper we carry the objectivity of the analysis of the medieval auroras further by using a maximum entropy method (MEM) spectral analysis. This method has been shown to be superior to other methods in certain cases in that the period of the variation can be determined very accurately [*Radoski et al.*, 1975]. Once the period is known from MEM, it can be used in a superposed epoch analysis to determine the amplitude and phase, as shown below.

## Determination of the Period

The maximum entropy method (MEM) of power spectral analysis consists of two independent steps, invented by John Burg in 1967 and 1968. See Burg's Ph.D. thesis [*Burg*, 1975], as well as papers by *Radoski et al.* [1975], *Smylie et al.* [1973], and *Ulrych and Bishop* [1975] for detailed reviews and discussions of the method. See also the excellent review of modern spectrum estimation by *Kay and Marple* [1981], as well as a set of important reprints, including the original Burg papers, edited by *Childers* [1978].

The first of these two steps is the so-called Burg technique which determines a prediction error filter with *m* weights which, when run in both time directions over the *M* data points, minimizes the mean square prediction error. Since the data are being used to predict later (or earlier) values of the data, the method is also called autoregressive.

The second step is called MEM and uses the *m* prediction error coefficients in an expression derived by Burg to estimate the power spectral density by maximizing entropy subject to constraints. This two step method will be referred to as Burg-MEM. This technique has been shown to produce smooth, accurate spectra with enhanced resolution relative to the older Blackman-Tukey [*Blackman and Tukey*, 1959] or Cooley-Tukey [*Cooley and Tukey*, 1965] methods. *Fougere et al.* [1976] have shown that in certain cases, however, Burg-MEM has some undesirable properties, namely, line splitting in the low-noise cases and line shifting in moderate- and high-noise cases more characteristic of real data sets. These problems were solved by *Fougere* [1977] by using a nonlinear generalization of the Burg technique. This new two-step procedure, which we will call nonlinear MEM, also has been applied to our 100-point data set. The results, although in general agreement with Burg-MEM results, should be considered to be quantitatively superior.

In order to test the stability of the spectral lines, both Burg and nonlinear MEM spectrums were calculated using a variety of weights. The result of the nonlinear analysis for 35 filter weights is shown in Figure 2*a* and the Burg result for 50 filter weights is shown in Figure 2*b*. In both cases the spectrums show a dominant narrow line at a frequency corresponding to

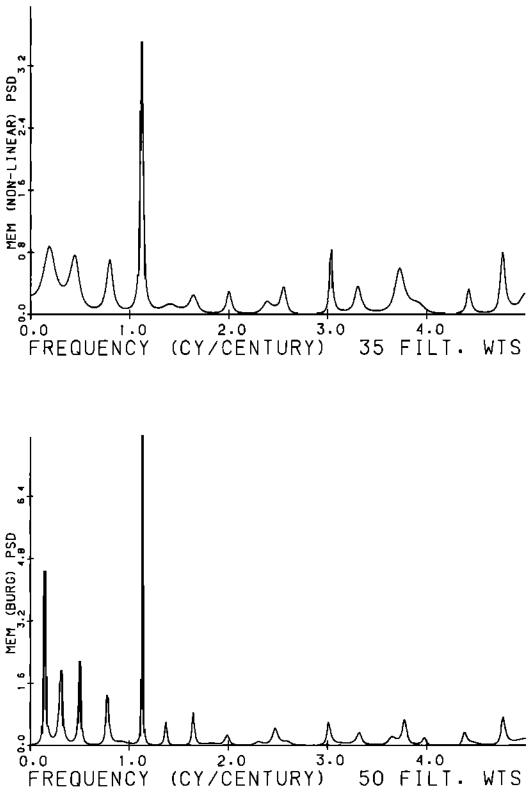


Fig. 2. Typical maximum entropy spectrums of the data shown in Figure 1. The top panel shows a nonlinear spectrum calculated using 35 filter weights, and the lower spectrum uses the Burg method with 50 filter weights. The lines at 1.13 cycles/century (~88 years) are stable and dominant.

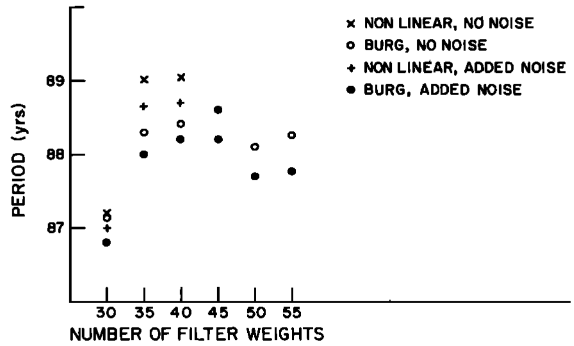


Fig. 3. Estimates of the period of the long cycle from 16 spectrums of the data in Figure 1. Our best estimate of the period is 88.4 ±0.7 years.

one cycle per 88 years. The effect of uncertainty in the data base was tested by adding noise of amplitude one to the data. This corresponds to an uncertainty of about three major mid-latitude aurora per decade, since the range of values in the original data was from 0 to 15 auroral observations. The power spectrums were almost indistinguishable from those run on the data without noise.

Although the spectrums are very stable, there are slight differences in the periods of the peak. In Figure 3 we have plotted the peak periods for 19 spectrums. Except for the four spectrums with 30 filter weights, all of the periods are longer than 87.5 years. Since the frequency determination is probably less accurate for 30 weights than for the higher weights, these four spectrums have not been included in the determination of the period. All of the 15 other spectrums show peaks, the periods of which lie in the range 88.4 ± 0.7 years, and that value is adopted as our best estimate. There can be no doubt that the line found in our spectrums is real, and since there are 1000 years of data covering 11 cycles, the reality of the periodicity during these 1,000 years is firmly established.

## Amplitude and Phase

As stated above, MEM power spectral analysis cannot determine the amplitude and phase of a variation but, given the period, these quantities can be found from a superposed epoch analysis of the data. The zero times of the epochs were chosen to make the average interval length 88 years. Since the data consists of the number of auroras per decade, it is not possible to use intervals of exactly 88 years each. Instead, each interval consists of 9 decades of data, but the zero time is adjusted so that the last decades of two intervals (750 A.D. and 1190 A.D.) are also used as the first decades of the next intervals. The results are shown in Figure 4, where the closed circles represent the individual data points, the open circles the bin averages, and the open triangles the fit given by

$$H(X) = 4.9 + 2.2 \cos\left[\frac{2\pi}{8.8}(X-1) - 22.6°\right]$$

where $X$ is the bin number. This function was determined by a least squares fit to the bin averages and gives a good estimate to the amplitude and phase of the long cycle variation. The amplitude is given in recorded auroral sightings per decade.

Because of the importance of these results, it is worthwhile to review the superposed epoch method briefly to evaluate their validity. Any function can be decomposed into the sum of periodic terms,

$$f(t) = \Sigma_i A_i \cos(\omega_i t + \alpha_i)$$

If we assume that the period of one of the terms is known, then we can write

$$f(t) = A\cos(\omega t + \alpha) + \Sigma_i B_i \cos(\omega_i t + \beta_i)$$

where $\omega$ is known but $A$, $\alpha$, and the $B_i$, $\omega_i$, and $\beta_i$ are unknown. The purpose of the superposed epoch analysis is to find $A$ and $\alpha$. We construct a time series by sampling $f(t)$ at intervals $\Delta t$. The $g$th term of the series is

$$f(g\,\Delta t) = A\cos(\omega g\,\Delta t + \alpha) + \Sigma_i B_i \cos(\omega_i g\,\Delta t + \beta_i) \quad (1)$$

However, since we know $\omega$, we can choose $\Delta t$ so that $\omega L\,\Delta t = 2\pi$, where $L$ is an integer. Using this choice of $\Delta t$ we perform a superposed epoch analysis of the time series by binning the data in $L$ bins. That is, we group the data samples so that in the first bin we have

$$f(0), f[(L)\,\Delta t], ---, f[(sL)\Delta t], ---, f[(m-1)L\,\Delta t]$$

where the data set is $mL$ periods long. In the $(n+1)$th bin we have

$$f[(n)\,\Delta t], f[(n+L)\,\Delta t], ---, f[(n+sL)\,\Delta t],$$
$$---, f\{[n+(m-1)]L\,\Delta t\}$$

where $s$ is an integer.

The average value of the members of the $(n+1)$th bin is

$$G_{n+1} = \frac{1}{m}\sum_{p=0}^{m-1} f[(n+pL)\,\Delta t] \quad (2)$$

or, more explicitly,

$$G_{n+1} = \frac{1}{m}\sum_{p=0}^{m-1} A\cos[\omega(n+pL)\,\Delta t + \alpha]$$
$$+ \frac{1}{m}\sum_{p=0}^{m-1} \Sigma_i B_i \cos[\omega_i(n+pL)\,\Delta t + \beta_i] \quad (3)$$

but we have chosen $\Delta t$ so that

$$\cos[\omega(n)\,\Delta t + \alpha] = \cos[\omega(n+1)\,\Delta t + \alpha] = ---$$
$$= \cos[\omega n + sL)\,\Delta t + \alpha] = ---$$

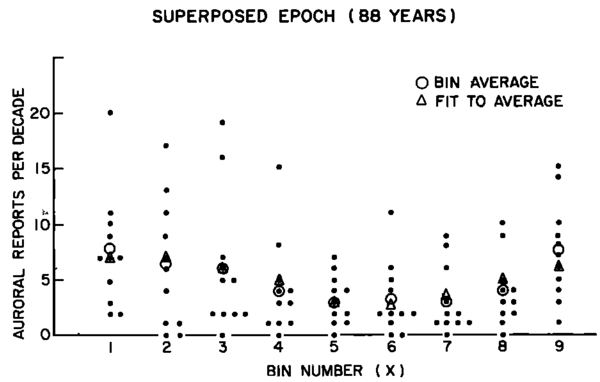


Fig. 4. Superposed epoch analysis of data from *Siscoe* [1980]. Solid dots show data points, open circles give bin averages, and open triangles are calculated from the best fit estimates. If the same number of aurora was reported in the same bin in more than one cycle, the points are plotted next to one another. For example, the figure shows that two auroras were reported in the third decade during four cycles. The average length of the period used in the superposition was 88 years.

and the $m + 1$ terms in the first sum are all equal, and the sum is given by

$$mA \cos [\omega(n)\, \Delta t + \alpha]$$

However, for a given $\omega_i$ the terms in the second summation in (3) will not have a predetermined phase relationship to one another. If $\omega_i \ll \omega$ and not commensurate, this quasirandom sampling will cause the term to vanish (provided that $B_i$ is not too large). Even if $\omega_i \gg \omega$ and/or $B_i \gg A$, the process of summation will decrease the relative importance of the second summation. The $G_{n+1}$ will then be of the form

$$G_{n+1} = A \cos [\omega(n)\, \Delta t + \alpha] + E_{n+1}$$

where $E_{n+1}$ is the contribution to the bin average of the summations over $p$ and $i$ in the second term of expression (3).

The size of the error in determining $A$ and $\alpha$ due to the $n$ dependence of $E_n$ can be estimated by fitting the bin average with a cosine function (thus determining the best values of $A$ and $\alpha$) and comparing the fitted values $H_{n+1}$ with the actual values of $G_{n+1}$ from the data.

In Figure 3 the $G_{n+1}$ are given by open circles and $H_{n+1}$ by triangles. It is clear that the contribution of the $n$ dependence of $E_n$ is small. The errors in $A$ and $\alpha$ are therefore small, although we do not here attempt to put a formal estimate of limits on them.

We conclude that the mean amplitude of the long cycle was 2.2 recorded auroral sightings per decade and the phase was such that the last minimum in these data occurred in the decade between 1403 A.D. and 1413 A.D. A long extrapolation of the results to modern times would produce a minimum circa 1850, whereas the observed minimum in auroral frequency occurred circa 1815. Such a long extrapolation may be unwarranted because the phase is a rather poorly determined function of quite noisy data. The entire phase problem requires further study.

## Discussion and Conclusion

We have shown that the number of recorded auroral reports/decade in midlatitude Europe and Asia was periodic with a period of 88.4 ±0.7 years from 450 A.D. to 1450 A.D. Although it appears to be difficult, if not impossible, to avoid concluding that solar output must also have displayed periodic behavior, it is not possible to identify the parameters that changed. The problems that occur in such an identification have been discussed elsewhere [*Feynman*, 1982] and will not be treated at length here. Suffice it to say that almost no studies have been done relating midlatitude auroral observations to solar wind parameters. We have only the general observation that high levels of geomagnetic activity are related to low-latitude auroral appearances. We also know, of course, that geomagnetic activity is associated with solar wind velocity and with southward interplanetary magnetic field, but the empirical relationship is still a matter of dispute [*Holzer and Slavin*, 1983; *Kamide*, 1983].

There is evidence that the changes in solar outputs that take place in connection with the long cycle are quite different from those taking place because of the 11-year cycle. This was inferred from a study of auroral frequency from 1890 to 1935 at high geomagnetic latitude in Sweden [*Silverman and Feynman*, 1980]. The data from Karesuando (geomagnetic latitude 65.2°) shows both a well-marked variation with the long cycle and an equally clear 11-year variation. But whereas the number of auroras reported reaches relative minimums at each 11-year cycle minimum, it reaches a relative maximum at the minimum of the long cycle. Since the frequency, position, and intensity of auroras is determined by solar outputs such as interplanetary wind velocity and magnetic field intensity, it follows that the long cycle variation cannot be dominated by changes in the same parameters as those determining the 11-year cycle.

Although we cannot definitively identify which solar outputs change during the long-cycle variation, we have shown that the long cycle in solar terrestrial relations is real and periodic, that it is present in 1000 years of auroral data, and that the period is 88.4 ±0.7 years.

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