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J. R. Wolf and H. A. Wolfer: An Historical Note on the Zurich Sunspot Relative Numbers

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SUMMARY

There has been a substantial amount of confusion in the statistical literature concerning the history of the classical time series of Zurich sunspot relative numbers. In this note, we briefly review the lives and work of J. R. Wolf (1816–93) and H. A. Wolfer (1854–1931), both of whom have been independently credited with the sunspot data, and assess the contribution made by each to the development of that data.

Keywords: HISTORY OF STATISTICS; TIME SERIES; SUNSPOT RELATIVE NUMBERS

1. INTRODUCTION

The Zurich series of sunspot relative numbers has been analyzed in almost every textbook on time series, and it has been used in numerous papers in the statistical literature to illustrate the finer points of both the time domain and the frequency domain methodology. Various forms of this series can be found in the book by Max Waldmeier (1961), the fourth Director of the Swiss Federal Observatory in Zurich; in that book, yearly means of the series are given for the period 1700-1960, monthly means for 1749-1960, and daily numbers for 1818-1960. From 1961 to 1980, daily, monthly, and yearly sunspot numbers have been tabulated, with accompanying detailed discussion in regular yearly articles by Waldmeier in the journal *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich*. Despite the obvious importance and attention accorded the series of sunspot relative numbers in all its forms, however, there appears to be substantial confusion in the statistical literature as to the history of the series and to the discoveries of sunspot behaviour related to that history. There are three main points at issue here:

I. A small number of books and papers in the statistical literature attribute the sunspot series of relative numbers solely to Rudolf Wolf (see, for example, Brillinger and Rosenblatt, 1967; Bloomfield, 1976; Morris, 1977; and Tong and Lim, 1980), while a much larger group attributes these same sunspot numbers instead to Alfred Wolfer (see, for example, Yule, 1927; Bartlett, 1946, 1966; Moran, 1954; Bailey, 1965; Anderson, 1971; Koopmans, 1974; Phadke and Wu, 1974; Box and Jenkins, 1978; Woodward and Gray, 1978, and Bingham, 1978). It is worth noting here that the astronomy and astrophysics literature, on the other hand, almost unanimously attribute the sunspot numbers to Wolf, with hardly any mention of Wolfer. Furthermore, most of the above-mentioned statistical references specify Wolf or Wolfer without first names, initials, or indeed any reference to their original works. Much of the confusion certainly stems from the similarity of their names, yet whether Wolf or Wolfer is actually given the credit seems to depend almost exclusively on whether the author cites Waldmeier (1961) or Yule (1927) respectively as his primary source for the data. An important feature of the Yule article and the Waldmeier book that no-one seems to have noticed so far is that the lists of annual mean sunspot relative numbers presented in those two references are not identical; indeed, during their common period of 1749-1924, there are a total of 21 years in which discrepancies occur between the two lists, and for ready comparison these are tabulated in Table 1 of this paper. This observation has

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TABLE 1
Comparison of Yule's sunspot numbers with those of Waldmeier

Year	*Yule*	*Waldmeier*
1820	15.7	15.6
1827	49.7	49.6
1828	62.5	64.2
1830	71.0	70.9
1839	85.8	85.7
1840	63.2	64.6
1841	36.8	36.7
1848	124.3	124.7
1849	95.9	96.3
1850	66.5	66.6
1852	54.2	54.1
1857	22.8	22.7
1860	95.7	95.8
1868	37.3	37.6
1869	73.9	74.0
1870	139.1	139.0
1872	101.7	101.6
1873	66.3	66.2
1875	17.1	17.0
1877	12.3	12.4
1893	84.9	85.1

Sources: 1. Yule (1927), Table A, pp. 419–420. 2. Waldmeier (1961), pp. 20–21.

obvious implications for those papers which attempt to compare published models of the sunspot series drawn from different data sources (see, for example, Woodward and Gray, 1978).

II. There also appears to be some confusion as to the actual year in which the sunspot relative numbers were first introduced. Some authors follow Waldmeier (1961) and say 1848 (see, for example, Woodward and Gray, 1978; Bray and Loughhead, 1964; and Link, 1978), while others say 1849 (see, for example, Morris, 1977; Jagger, 1977; and Keipenheuer, 1953). The fact of the matter, however, is that both these dates are incorrect. The formula for the sunspot relative number was actually defined and redefined several times over a period of a decade from 1851 to 1861, when it finally appeared close to the form that is used today.

III. Finally, several authors also claim as historical fact that Heinrich Schwabe showed, in 1843, that sunspot appearance varied according to a cycle of approximately 11 years (see, for example, Stetson, 1937, p. 7; Newton, 1958, p. 56; Abetti, 1963, p. 37; Brillinger and Rosenblatt, 1967; Tandberg-Haussen, 1967; p. 181; Brillinger, 1975, p. 138; and Wilson *et al.*, 1981). Culver (1979, p. 132) quotes instead the year 1838. Yet, both 1838 and 1843 are incorrect, as is the association of Schwabe with the discovery of an 11-year sunspot cycle. Although Schwabe was the first to publish extensive data on sunspot occurrences, starting with 12 years of his own sunspot observations for 1826–37 (Schwabe, 1838), he offered neither conclusions nor comments. These observations of Schwabe continued to be published from 1841 on an annual basis with similar articles in the same journal. Then, in Schwabe (1844), he published his complete set of sunspot observations for the 18 years from 1826 to 1843, and concluded—*for the first time*—that: "From my earlier observations . . . it appears that there is a certain periodicity in the appearance of sunspots, and this theory seems more and more probable from the results of this year". He then went on to conclude that: "If one compares the number of (sunspot) groups with the number of days free from spots, one will find that sunspots have a period of about 10 years . . . The future will tell whether this period persists, whether the time of least activity of the sun in producing sunspots lasts one or two years, and whether this activity increases more rapidly than it decreases." Thus, Schwabe did recognize a periodic component in his sunspot data, but this conjecture was published in 1844 (the paper was actually dated 31 December 1843) and the conjectured periodicity was 10 years, not 11. It was not until 1852 that Wolf was able to pin down this periodicity more precisely as 11.11 years.

The purpose of this note, then, is to point out the origins of the Zurich sunspot relative numbers, and especially to set straight the record concerning the respective roles that both Wolf and Wolfer played in that history. This paper is a condensed version of a much longer technical report (Izenman, 1983), which is available upon request from the author.

2. DEVELOPMENT OF THE SUNSPOT RELATIVE NUMBERS

Johann Rudolf Wolf (7 July 1816–6 December 1893) described himself on the occasion of his seventieth birthday in the following terms: "I have always consoled myself that he such as I who is not a genius, can still achieve much that is useful when he does his work right and chooses his work to suit his talents". His contemporaries would probably have agreed with his self-assessment. At the memorial service following his funeral, he was described as possessing "a fine historical sense", "an unusual memory", "a compulsion for collection", and "a good sense for organization"; he was also said to have led "a quiet life of concentration", and that he was "an honest, impartial, and honoured colleague", and "a Swiss confederate of the old school". He certainly led a very full life. Educated at the University of Zurich in higher mathematics, physics and astronomy, he spent three years travelling through Europe and attending classes at the University of Vienna, the University of Berlin, and the Sorbonne. He met, among others, Encke, Dirichelet, Poggendorff, Steiner, Crelle, Gauss, Libri and Poisson. Among the lectures he attended were ones on the calculus of probability (by Libri) and on least squares (by Dirichelet). These surely made a great impact on him since he wrote over a dozen papers having probabilistic and statistical content, including lengthy discussions on topics related to what we now call Monte Carlo simulations and trimmed means. He also seems to have been one of the first (if not the first) to smooth an empirical time series by use of something other than a simple moving average. He wrote several papers during the early 1840's on prime number theory and certain aspects of geometry. But his main professional interests were in astronomy, and he published hundreds of papers on his astronomical discoveries, many books, and founded two journals which are still in existence today. He was associated, at various times in his career, with the Berne Realschule, the University of Berne, the Gymnasium in Zurich and the Eidgenossische Technische Hochschule. He was also appointed Director of the Berne Observatory (in 1847), and later (in 1856) Director of the Swiss Federal Observatory and (in 1864) Director of the Central Meteorological Office, both in Zurich. His many honours included being elected a Foreign Associate of the Royal Astronomical Society (11 November 1864).

Wolf's interest in sunspot observation started when he became Director of the Berne Observatory, just prior to a sunspot maximum. After corresponding with Schwabe and other sunspot observers, he started his own sunspot viewing on 4 December 1847 (Wolf, 1848), and spent the entire year of 1848 familiarizing himself with their observation. It was only in January 1849 that he actually began to record in a systematic fashion his sunspot sightings, the first of which were published in August 1849. In his articles, he recorded for each day he observed a number of quantities including (i) the total number of visible groups, g, of spots, including isolated spots; (ii) the total number of individual spots, f, within each of those groups; (iii) the sky conditions, which he classified as "free of clouds", "spots could be seen through clouds", or "spots could not be seen at all". and (iv) which of two telescopes he used for his viewing that day, a large telescope, or a smaller portable field-glass. Wolf published these daily collections of numbers twice a year from 1849 to 1855.

His first attempt at computing what he called "relative numbers" for sunspots appeared on p. 94 of Wolf (1852a). By adjusting the number of sunspot groups he observed each day, adding to that number one-tenth the number of (individual) spots for that day, he arrived at a daily "relative number" of $g + \frac{1}{10} f$; to obtain a "monthly relative number", he then averaged the appropriate daily relative numbers by dividing their sum by the number of days, n, he had observed that particular month. It is important to note here that for this first version of his "relative numbers" index of sunspot behaviour, the three components, g, f and n, were taken for only those days for which *both* the larger telescope was used and when the sky was judged to be "free of clouds"; thus, the number of usable observation days which went into computation of the index proved to be extremely variable each month.

In 1851, Alexander von Humboldt published his huge three-volume work *Cosmos*, and in the third volume he included 25 years of Schwabe's sunspot data from 1826 to 1850. Humboldt also quoted comments by Schwabe to the effect that older observations than his might well be consistent with his conjecture of a ten-year sunspot period. Upon reading Schwabe's comment, Wolf set out to verify Schwabe's conjecture by extending the data base as far back as was possible to go. To do this, he searched through several hundred volumes in the libraries of Basel, Berne and Zurich for old sunspot records. Then, in an article dated 6 November 1852 and entitled "New investigations into the sunspot period and its meaning" (Wolf, 1852b), he set out carefully all the research he had undertaken to provide a solid historical foundation for sunspot periodicity. The major result of that paper was his establishment of "the exact duration of the sunspot period". It is worth showing here the method by which he arrived at his conclusion since it did not utilize in any way the first version of his "sunspot relative numbers", but was based instead on what he considered were six clear minima and six clear maxima years of sunspot activity, namely,

minima	*maxima*
1645.0 ± 1.0	1626.0 ± 1.0
1755.5 ± 0.5	1717.5 ± 1.0
1810.5 ± 1.0	1816.3 ± 1.0
1823.2 ± 0.5	1829.5 ± 1.0
1833.6 ± 0.5	1837.5 ± 0.5
1844.0 ± 0.5	1848.6 ± 0.5

From the years of minima, he computed eight estimates of the sunspot period:

$$(1810.5 \pm 1.0) - (1755.5 \pm 0.5) = 55.0 \pm 1.12 = 5(11.00 \pm 0.22)$$
$$(1823.2 \pm 0.5) - (1755.5 \pm 0.5) = 67.7 \pm 0.71 = 6(11.28 \pm 0.12)$$
$$(1833.6 \pm 0.5) - (1755.5 \pm 0.5) = 78.1 \pm 0.71 = 7(11.16 \pm 0.10)$$
$$(1844.0 \pm 0.5) - (1755.5 \pm 0.5) = 88.5 \pm 0.71 = 8(11.06 \pm 0.09)$$

$$(1810.5 \pm 1.0) - (1645.0 \pm 1.0) = 165.5 \pm 1.41 = 15(11.03 \pm 0.09)$$
$$(1823.2 \pm 0.5) - (1645.0 \pm 1.0) = 178.2 \pm 1.12 = 16(11.14 \pm 0.07)$$
$$(1833.6 \pm 0.5) - (1645.0 \pm 1.0) = 188.6 \pm 1.12 = 17(11.09 \pm 0.07)$$
$$(1844.0 \pm 0.5) - (1645.0 \pm 1.0) = 199.0 \pm 1.12 = 18(11.06 \pm 0.06)$$

and from the years of maxima, he computed eight further estimates of the sunspot period:

$$(1816.3 \pm 1.0) - (1717.5 \pm 1.0) = 98.8 \pm 1.41 = 9(10.98 \pm 0.16)$$
$$(1829.5 \pm 1.0) - (1717.5 \pm 1.0) = 112.0 \pm 1.41 = 10(11.20 \pm 0.14)$$
$$(1837.5 \pm 0.5) - (1717.5 \pm 1.0) = 120.0 \pm 1.12 = 11(10.91 \pm 0.10)$$
$$(1848.6 \pm 0.5) - (1717.5 \pm 1.0) = 131.1 \pm 1.12 = 12(10.93 \pm 0.09)$$

$$(1816.3 \pm 1.0) - (1626.0 \pm 1.0) = 190.3 \pm 1.41 = 17(11.19 \pm 0.08)$$
$$(1829.5 \pm 1.0) - (1626.0 \pm 1.0) = 203.5 \pm 1.41 = 18(11.31 \pm 0.08)$$
$$(1837.5 \pm 0.5) - (1626.0 \pm 1.0) = 211.5 \pm 1.12 = 19(11.13 \pm 0.06)$$
$$(1848.6 \pm 0.5) - (1626.0 \pm 1.0) = 222.6 \pm 1.12 = 20(11.13 \pm 0.06).$$

Each of the expressions on the right-hand-side is of the general form

$$\xi_i \pm \delta_i = c_i(p_i \pm u_i),$$

where c_i is an integer estimate of the number of cycles in ξ_i years, each cycle lasting a period of p_i years with probable error u_i years. From the above estimates of the sunspot period, $p_1, p_2, \ldots, p_{16}$, Wolf computed an overall estimate of period length by a weighted average of the 16 p_i values each having weight $w_i = 1/u_i^2$:

$$\hat{p} = \left(\sum_{i=1}^{16} w_i p_i \right) \Big/ \left(\sum_{i=1}^{16} w_i \right) = 11.11 \text{ years.}$$

(The computations presented here are corrected versions of those published by Wolf. The overall estimate of period length, $\hat{p}$, as computed—incorrectly—by Wolf fortunately, however, gave the correct value.) Wolf never explained why he decided that p_i values of around 11 were more appropriate than 10 or 12, for example, and, therefore, why he factored the number of years from minimum to minimum or from maximum to maximum as he did; he did indicate in his paper that alternative factorizations were certainly possible, but for some reason he opted for the above as the "correct" one, probably based on the historical data he had retrieved. It is also worth noting that the method by which he factorized the $\xi_i \pm \delta_i$ values above meant that two minima (or maxima) far apart would yield a smaller value of u_i than would two closer minima (or maxima), an assumption that seems somewhat questionable. The standard error of $\hat{p}$ can be derived, first, by expressing $\hat{p}$ as a linear combination of the years of maxima and minima, and then by assuming these assessed years of maxima and minima are uncorrelated; see Izenman (1983) for details. The resulting standard error of $\hat{p}$ is 0.0374 years, or, 13.65 days; Wolf's value of this standard error was 0.038 years, or 13.87 days, but was obtained from a formula that was not given in the paper.

In 1855, Wolf changed his definition of the daily sunspot relative number. By multiplying his 1851 definition by 10, Wolf now defined it as $10g + f$, where g is the number of groups and f the number of spots (Wolf, 1856). He explained his reasoning behind his relative number as follows: "I chose the number 10, on the one hand, because it seemed to work well for a large number of instances which I studied for this purpose and, on the other hand, because it was simpler to deal with than any other number close to it". He also enlarged his data base to take account of the numerous gaps in his daily records. Now, he (a) incorporated daily data from Schwabe for those days in which gaps existed in his own records but for which Schwabe had data, and (b) relaxed his 1851 definition so that more of his own data could be used, particularly those observations which fell into his "spots could be seen through clouds" category, plus *all* data from his portable field-glass. Wolf then recomputed the monthly and yearly mean sunspot relative numbers for the years 1849-55 in his 1856 paper. In Wolf (1858), he again explained his formula:

> "Since each group must contain at least one spot, I subtract the number of groups g from the number of spots f, so that the remainder is the number of extra spots $f-g$; this I again break up into groups by multiplying by a fraction q, to whose number I add the earlier number of groups for a number of different groups. I get, then, for the daily state of the spots, the following:
>
> $$\begin{aligned} t &= p\,r = p\{g + q(f-g)\} \\ &= p(1-q)\,g + pqf \\ &= mg + nf, \end{aligned}$$
>
> where I multiply by some factor p, since only relative numbers concern me. The numbers m and n are to a certain extent the relative weights to be applied to the number of groups and the spots for this determination. I believe now that if a new place on the sun is attacked by a spot-building action, then this is more important than if in an already-present group a new spot arises through a small change, and, therefore, I make m much larger than n. I will

not be much mistaken if I make $m = 10$ and $n = 1$, because 10 and 1 lie in the neighbourhood of 9 and 1, 11 and 1, etc., if only for the sake of greater comfort. Therefore, I always calculate my daily relative number by the formula $t = 10g + f$. Naturally, the monthly average of these daily relative numbers gives a truer picture of the month the more numerous the daily numbers are."

Wolf published his tables of daily sunspot numbers and their monthly and yearly averages on an annual basis from 1856, and, until 1859, he included daily data of Schwabe without identifying which was his data and which was Schwabe's. It is important to note that the data of Schwabe was being assigned the same weight of importance as was Wolf's own data in determining the overall daily, monthly, and yearly sunspot numbers.

A few years later, Wolf again revised the definition of his sunspot relative number (Wolf, 1861, p. 186), this time by multiplying his 1856 formula by what he called a "reduction factor" A, so that the daily relative number was now defined as $A(10g + f)$. The value of A was to depend on the observer, his telescope, and the viewing conditions. In this way, Wolf was able to incorporate data from other observers to fill the remaining gaps in his tables and to provide a check on the already-present values. As a base level, Wolf assigned the value $A = 1$ to his own data which he obtained with his larger telescope, and $A = 1.5$ for data he recorded with either of two smaller telescopes. The first time that these reduction factors were used and published was in the 1861 paper. Clearly, there would be no reason to assume that the value of A for a particular observer remain constant year after year, or even within a year. In 1867, Wolf published a list of the reduction factors that he had determined for the historical records.

In 1875, Wolf introduced a major innovation into his study of the sunspot numbers. He published a smoothed version of the sunspot relative numbers for the years 1836–1873 by first computing a moving average of length 12 over the monthly means, and then computing a second moving average, this time of length 2, over the previously smoothed series; this is clearly equivalent to the formula

$$sm(r_t) = \left\{ r_{t-6} + 2 \sum_{k=-5}^{5} r_{t-k} + r_{t+6} \right\} /24,$$

where $\{r_t\}$ is the monthly mean sunspot numbers. From this smoothing procedure, six values are lost at each end. This method is still followed today (Waldmeier, 1961, p. 9).

It was also in 1875 that Alfred Wolfer began working as an assistant to Wolf, and his first contribution to the recording of sunspot behaviour appeared in an acknowledgement by Wolf in a paper published in 1876. By this time, Wolf's listings of the sunspot numbers in daily, monthly, or yearly form, for the period 1749–1876, are almost identical to those appearing in Waldmeier (1961). From this point on, the main work on these series dealt with the improvement of data quality through additional sources. For some cases, these sources filled gaps, and for others, they confirmed or modified existing data. From 1877, Wolf started a new policy of assigning reduction factors A for each observer more frequently than once a year. Thus, in 1877, a value of A was determined *twice* a year for the two six-month periods January–June and July–December. For example, the series of Wolfer's half-yearly values for A for 1877–88 were:

1877:	0.86, 0.82	1883:	0.64, 0.54
1878:	0.67, 0.78	1884:	0.54, 0.52
1879:	0.65, 0.69	1885:	0.54, 0.56
1880:	0.76, 0.74	1886:	0.58, 0.54
1881:	0.67, 0.69	1887:	0.51, 0.51
1882:	0.62, 0.67	1888:	0.49, 0.43

Then, in 1889, a value of A was assigned *four* times a year for each observer. Wolfer's set of values for 1889-1893 were:

1889: 0.64, 0.72, 0.61, 0.52
1890: 0.66, 0.48, 0.44, 0.48
1891: 0.54, 0.52, 0.53, 0.58
1892: 0.64, 0.62, 0.60, 0.64
1893: 0.56, 0.52, 0.55, 0.54

When Wolf died at the end of 1893, Wolfer took over the compilation of sunspot numbers, and in 1895 he published a survey of his and other observers' reduction factors through the years as assessed by Wolf. The problem was that Wolf had set himself up as the base level to which other sunspot sightings could be compared, and now a new base level had to be determined. Wolfer's solution to this problem was to average his own values of A from 1877 to 1893, thereby arriving at the new base level for Zurich to be 0.60. This value of A has remained the same until today. There are a few points to be made concerning this averaging procedure. First, Wolfer mistakenly used the value 0.89 for the second half of 1878 instead of the correct value 0.78. It is also clear from a cursory inspection of Wolfer's values of A over the entire 17 years that an overall decline in those numbers can be seen, so that a simple average might not yield the most appropriate new base level for Zurich. One can possibly argue from this that there is a slight discontinuity at 1894 present in the series.

There is very little biographical information available on Alfred Wolfer, probably reflecting the fact that his impact on astronomy or on science in general was nowhere near that of his mentor Wolf. Indeed, Wolfer's main preoccupation was that of an administrator and teacher than as an innovative researcher. Born on 27 January 1854, he studied mathematical sciences at the E.T.H. in Zurich, where he attended Wolf's lectures and exercises. He graduated in August 1875, with a teaching diploma for his work on the division errors of the meridian circle, and became Wolf's official assistant in 1876, a position he held on to for the next 15 years. Following the death of Wolf, Wolfer was named his successor in 1894 as Director of the Swiss Federal Observatory, and was also appointed to a position at the University of Zurich, being promoted to Professor there in 1922. Wolfer was elected a Foreign Associate of the Royal Astronomical Society on 13 June 1913.

In regular yearly articles he updated Wolf's series of sunspot numbers, even correcting Wolf's own published numbers when numerical inaccuracies were discovered, either computational or typographical. Then, a complete revision of all sunspot data from 1749 to 1924 was published in Wolfer (1925). It was this very article that caught the attention of Yule, and which in turn led to the first application of the autoregressive time series model (Yule, 1927). Yule, unfortunately, referred to the data as "Wolfer's sunspot series" and thereby started the confusion in the statistical literature that persists till today. Wolfer died on October 8th 1931.

3. CONCLUDING REMARKS

It is clear from the above study that full credit for the origin and subsequent development of the series of sunspot relative numbers must go to Rudolf Wolf. Wolfer played a very minimal part in the whole story, his primary contribution being that of changing the value of the Zurich reduction factor from 1.0 to 0.6, and of continuing to compile and publish the values of that series beyond the death of Wolf. The association of the sunspot numbers with Wolfer was the result of a mistake by Yule; hopefully, this note will set the record straight. The tradition of compiling the daily, monthly, and yearly numbers has since been continued by W. Brunner and Waldmeier, the latter of whom recomputed all the data available and published the revised lists in Waldmeier (1961); this is the reason that the numbers which appeared in Wolfer (1925), and hence in Yule (1927), and those that appear in Waldmeier (1961) are not identical with each other. Clearly, Waldmeier's numbers should be used when investigating sunspot activity rather than Yule's, which are out of date.

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