The Effect of Sunspot Weighting

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Abstract:

Although Brunner began to weight sunspot counts (from 1926), whereby larger spots were counted more than once, he compensated for the weighting by not counting enough smaller spots such as to maintain the same reduction factor (0.6) as Wolfer to reduce the count to Wolf's original scale, so the weighting did not have any effect on the scale of the sunspot number. Waldmeier in 1947 formalized the weighting (on a scale from 1 to 5) of the sunspot count made at Zurich and its auxiliary station Locarno. This explicit counting method, when followed, inflates the relative sunspot number over that which corresponds to the scale set by Wolfer (and matched by Brunner). Re-counting some 60,000 sunspots on drawings from the reference station Locarno shows that the number of sunspots reported were 'over counted' by ~44% on average, leading to an inflation (measured by an effective weight factor) in excess of 1.2 for high solar activity. In a double-blind parallel counting by the Locarno observer Cagnotti, we determined that my count closely matches that of Cagnotti's, allowing us to determine from **direct** observation the daily weight factor for spots since 2003 (and sporadically before). The effective total inflation turns out to have two sources: a major one (15-18%) caused by weighting of spots and a minor one (4-5%) caused by the introduction of the Zurich classification of sunspot groups which increases the group count by 7-8% and the relative sunspot number by about half that. We find that a simple empirical equation (depending on the activity level) fits the observed factors well, and use that fit to estimate the weighting inflation factor for each month back to the introduction of effective inflation in 1947 and thus to be able to correct for the over-counts and to reduce sunspot counting to the Wolfer method in use from 1893 onwards.

Keywords: Sunspot weighting; Waldmeier sunspot weight factor; Correcting the Sunspot Number; Locarno sunspot drawings.

1. Introduction

In 1945 Max Waldmeier became Director of the Zürich Observatory. In 1961, Waldmeier published the definitive Zürich sunspot numbers up until 1960 (Waldmeier 1961). He noted that "Wolf counted each spot – independent of its size – but single. Moreover, he did not consider very small spots, which are visible only if the seeing is good. In about 1882 Wolf's successors changed the counting method, which since then has been in use up to the present. This new method counts also the smallest spots, and those with a penumbra are weighted according to their size and the structure of the umbra". Waldmeier (1968b, 1948) described the weighting scheme as follows "Später wurden den Flecken entsprechend ihrer Größe Gewichte erteilt: Ein punktförmiger Fleck wird einfach gezählt, ein größerer, jedoch nicht mit Penumbra versehener Fleck erhält das statistiche Gewicht 2, ein kleiner Hoffleck 3, ein größerer 5"[1]. However, Wolfer (1907) explicitly stated: "Notiert ein Beobachter mit seinem Instrumente an irgend einem Tage *g* Fleckengruppen mit insgesamt *f* Einzelflecken, ohne Rücksicht auf deren Grösse, so ist die daraus abgeleitete Relativzahl jenes Tages $r = k(10g+f)$"[2]. We can verify that Wolfer, contrary to Waldmeier's assertion that the Zürich observers began to use weighting "around 1882", did not weight the spots according to Waldmeier's scheme by comparing Wolfer's recorded count with sunspot drawings made elsewhere, e.g. Figure 1.

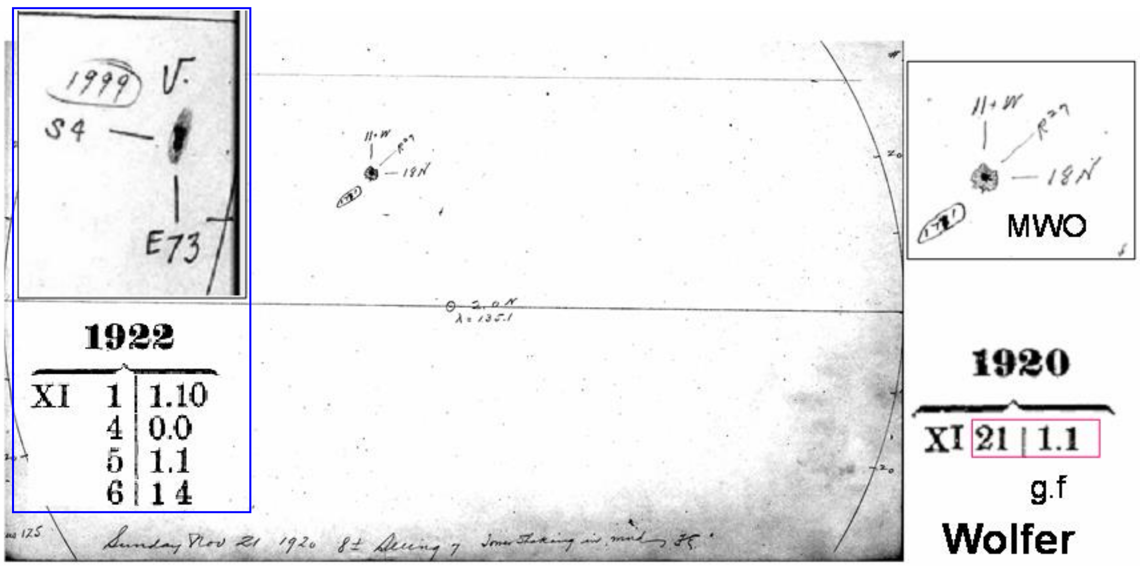

Figure 1: Drawing from Mount Wilson Observatory (MWO) of the single spot with penumbra on 21st Nov. 1920. The insert at the left shows a similar group observed at MWO on 5th Nov., 1922. For both groups, Wolfer should have recorded the observation as "*1.3*" if he had used the weighting scheme, but they were recorded as "*1.1*" (one group dot one spot), thus counting the large spot only once (*i.e.* with no weighting).

[1] A spot like a fine point is counted as one spot; a larger spot, but still without penumbra, gets the statistical weight 2, a smallish spot within a penumbra gets 3, and a larger one gets 5.

[2] When an observer at his instrument on any given day records *g* groups of spots with a total of *f* single spots, without regard to their size, then the derived relative sunspot number for that day is $r = k(10g+f)$.

There are many other such examples (*e.g.* 16th September, 1922 and 3rd March, 1924) for which MWO drawings are available at ftp://howard.astro.ucla.edu/pub/obs/drawings and even earlier *e.g.* June 20th-23rd, 1912 for which we have drawings from the Jesuit-run Haynald Observatory (Kalocsa, Hungary: http://fenyi.sci.klte.hu/deb_obs_en.html, see Slide 11 of http://www.leif.org/research/SSN-workshop1-Weighting.pdf). We can thus consider it established that Wolfer did not apply the weighting scheme. This is consistent with the fact that nowhere in Wolf's and Wolfer's otherwise meticulous yearly reports in *Mittheilungen über die Sonnenflecken* series is there any mention of a weighting scheme. We remind the reader about the format of Wolf's published observations, Figure 2:

Sonnenfleckenbeobachtungen im Jahre 1849.

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
1	9.31	3. 6	4. –	10.70	9.30	8.48	4.13	4 15	7.64	8.10	5.16	—
2	9.34	7.40	5. —	7. —	9.40	9.64	3. 3	6.18	5.35	7.10	7.41	8. 9
3	15. —	2. —	6.12	10.38	5.12	8.50	3. 6	6.15	4.27	3. 4	3.10	8.17
4	9.31	7.27	7.15	12.58	7.45	10.50	3 10	4.12	5.41	2. 3	4.31	—
5	9. –	9.22	2. —	8.20	8 50	8.45	7. —	5.20	1. 1	1. 2	—	9.47
6	8. —	10 34	7.24	10.60	7.38	7.45	4. 8	4.18	6.25	4. 6	—	2. 2
7	—	3. —	3. —	8.24	1. —	5. —	5.10	3.20	7.48	—	6.22	—
8	8.28	10.21	4. —	6.20	6.20	5.12	6.15	3.15	5.38	5.16	7.35	—
9	8.30	10.35	3. —	9.45	6.25	3. –	7.20	4.14	7.50	5.26	6.20	—

Figure 2: The number of groups g and the number of spots (Flecken) f for each day is recorded as '$g.f$', (Wolf, 1856). On days where the seeing was poor or when Wolf used a smaller telescope, the entries are in small type font or have no spot count.

To calculate the relative sunspot number, R, *e.g.* on April (IV) 4th, Wolf used the well-known formula $R = k{\cdot}(10{\cdot}12 + 58) = 178$ where the scale factor k is 1.00 for Wolf himself.

Clette *et al.* (2014) review the evidence from other solar indices for when the weighting was introduced a well as determining the magnitude of the effect. Svalgaard (2014) provided further details of the weighting issue. In the present article we shall further explore, quantify, and characterize how much the weighting of the sunspot count affects the Relative Sunspot Number.

2. Weighting at Locarno: The Reference Station

At the reference station 'Locarno' situated in the city of Locarno on the northern shore of Lago Maggiore in the Swiss canton of Ticino, weighting of the sunspot count has been employed since the beginning in 1957, closely following Waldmeier's prescription (Cortesi et al., 2016). To assess the magnitude of the increase due to weighting, Leif Svalgaard undertook to examine all the nearly 4000 drawings with individual counts of groups and the number of spots in each group made at Locarno (http://www.specola.ch/e/drawings.html) for the past decade (and some years before that) and to re-count the spots without weighting. An example of a drawing with the original weighted counts and the re-counted number of actual spots present is shown in Figure 3. As useful as the drawings are, the final count that is reported to the WDC is that which is performed visually at the telescope eyepiece and that in some cases differ occasionally from the count on the drawing; this is rare enough to not distort the result significantly.

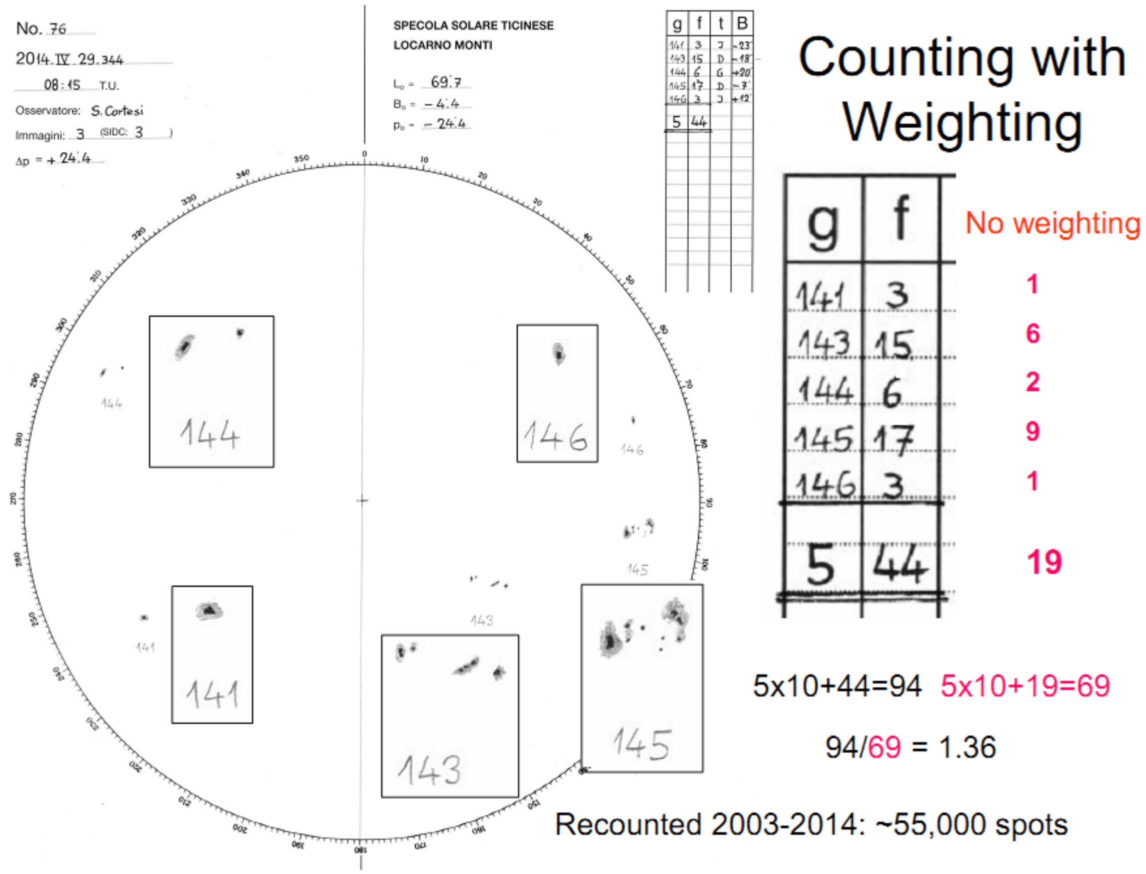

Figure 3: Drawing from Locarno showing the effect of weighting for the five groups present. Magnified views of the groups allow the reader to assess the weighting performed, *e.g.* to see that group 141 consists of one spot with a penumbra, which was assigned weight 3 according to Waldmeier's rule. For this drawing the weight factor of the day becomes 1.36.

At times, the observer did not count and report the very smallest spots even if they were included in the drawing, Figure 4:

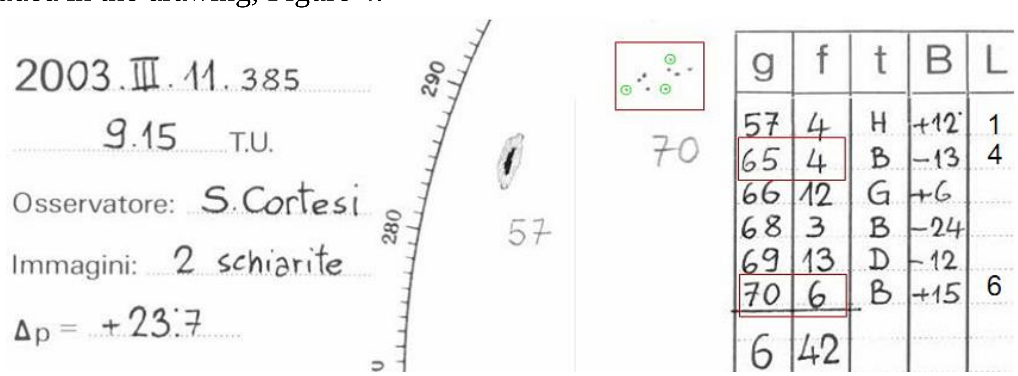

Figure 4: Drawing from Locarno showing tiny spots that were not counted (in green circles) for group number 70. Observers might differ on the 'rule' for omitting tiny spots, but the number of omitted spots is in any case small overall. A useful addition to the report would be the number of omitted spots, if not zero.

In case of the rare very large groups, it is quite a challenge to determine the actual spot count, Figure 5, especially if not all the weakest spots were counted. In this rather extreme case, the top drawing shows 74 spots, but the weighted count is only 58, so clearly many spots (at least 74 – 58 = 16) were not counted. One way to determine the number of un-counted spots would be to weight the large spots (none of which are omitted) according to Waldmeier's prescription, then subtract the sum of all the weighted values, and finally add in the number of spots that were weighted. The Figure shows how that would work. The shaky assumptions underscore the importance of recording the number of omitted spots, or what we could call the 'equivalent' number of omitted spots, if some tiny spots were 'lumped together'.

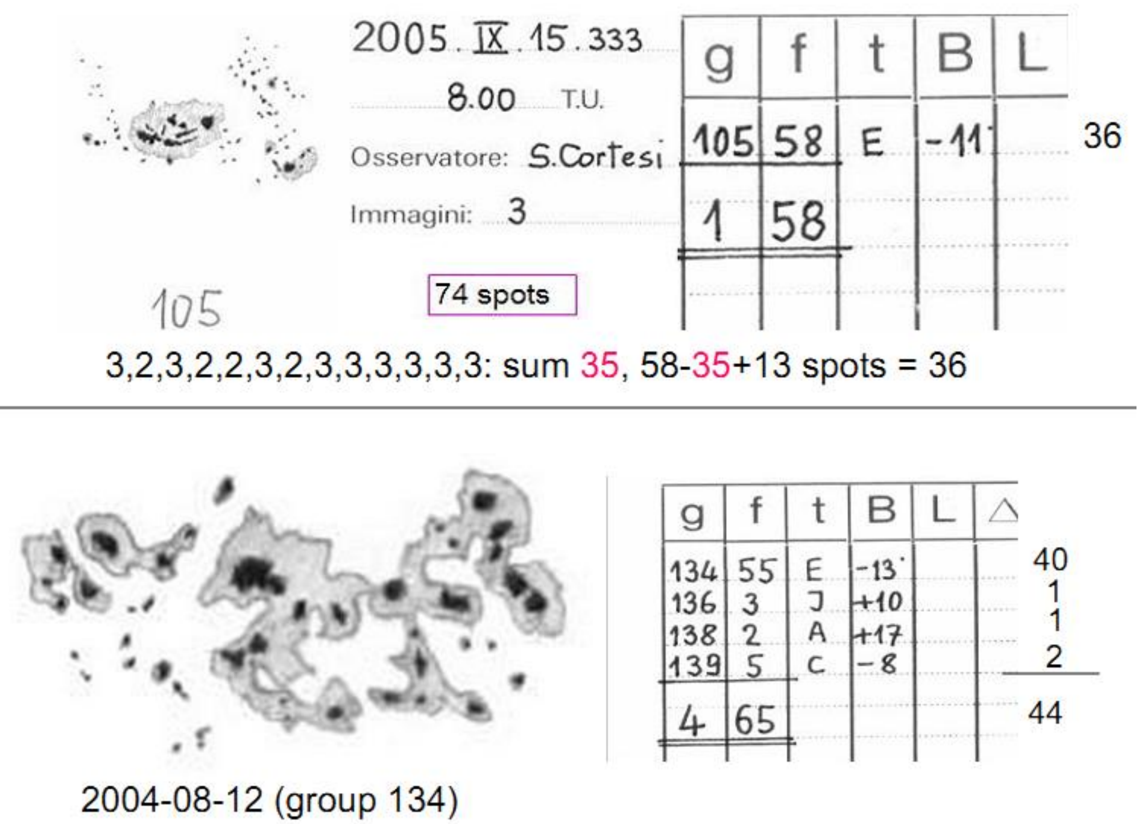

Figure 5: (Top) Drawing from Locarno showing a large, complicated group with many spots that were not counted. The number of spots according to the drawing was 74, but the weighted count was only 58. There were 13 spots (and umbrae) with weights of 3 and 2. The sum of the weighted spots was 35, so the number of spots with weight 1 must be 58 – 35 = 23 to which we must add 13 for a total of (actual?) spots of 36. This example is, admittedly, extreme, but such is the material we have to work with. (Bottom) Drawing of group 134 that on my count had 40 actual spots (and umbrae). The reader is invited to count as well.

To verify that the re-count is valid, i.e. that Svalgaard has understood and applied correctly the Waldmeier weighting scheme, the observer Marco Cagnotti in Locarno had agreed to maintain a (double-blind) parallel count of un-weighted spots at a continuing basis since January 1st, 2012, following a brief trial in August 2011, and the un-weighted count is now a part of the routine daily reports. Figure 6 shows that Svalgaard and Cagnotti very closely match each other in applying the weighting scheme, thus sufficiently validating the approach.

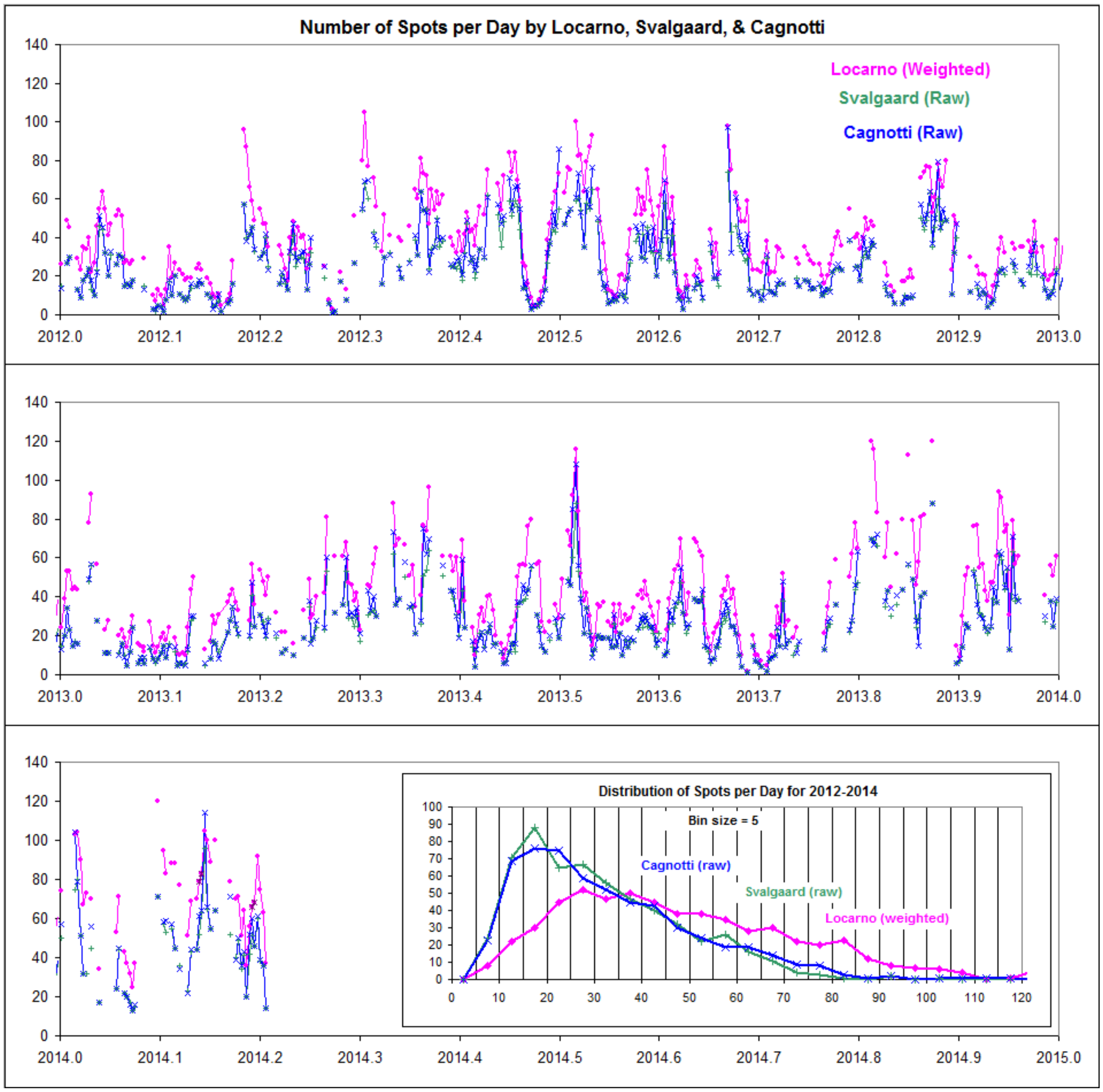

Figure 6: Comparison of the number of sunspots per day determined by Cagnotti (blue) and Svalgaard (green) without weighting, *i.e.* by counting each spot singly as prescribed by Wolfer with the number reported by Locarno (pink) employing the Waldmeier weighting scheme. The insert shows the nearly identical distribution of un-weighted counts in bins of five.

Is the weight factor observer dependent? With a novice one might be inclined to think so, but with training, observers tend to converge to agreement. We can compare the weighted counts and the number of groups reported by the veterans Cortesi and Bianda and the new observer Cagnotti from 2008 to the present (Figure 7): there does not seem to be much systematic difference with the possible exception of a very recent decline of Cortesi's weight factor. Observer Andrea Manna (AM) has a weight factor that is systematically about 0.04 lower than the other observers, in spite of seeing the same number of groups, so weighting does depend weakly on the observer.

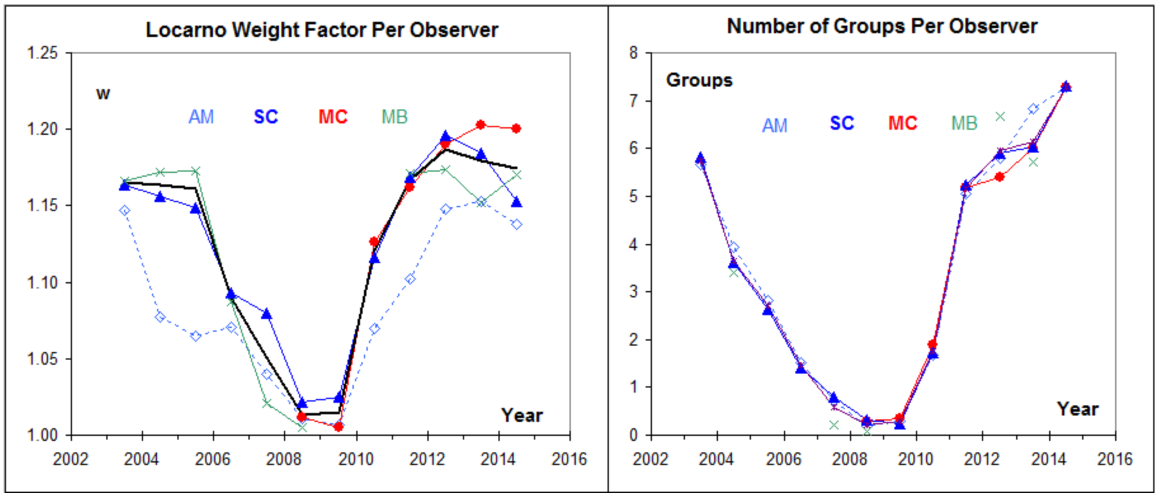

Figure 7: (Left) The weight factor for Locarno observers Cortesi (SC, blue, since 1957), Cagnotti (MC, red, since 2008), Manna (AM, open dashed blue, since 1991), and Bianda (MB, green, since 1983). (Right) The number of groups per day for each year reported by the same observers.

3. The Weighting Quantified by the Locarno Observers

Since August, 2014 the observers in Locarno have augmented their observations of the number of groups, *g*, and of weighted spots, *f*, with a count of actual, non-weighted spots, *s* (denoted '*LW*' at the right on the drawing – LW is the WDC SIDC/SILSO code designation for un-weighted Locarno counts), allowing us the calculate the weight factor as $w = (10g + f)/(10g + s)$, Figure 8:

Figure 8: The recent Locarno determination of both the weighted (*f*) and of the un-weighted number of sunspots (*LW*). For this particular day, the weight factor becomes $w = (30+17)/(30+8) = 47/38 = 1.237$.

Figure 9 shows the weight factors determined from the Locarno observations since August, 2014. The red curve shows the 27-day running average of the weight factor calculated using the relationship determined by Clette *et al.* (2014). It is clear that the Clette *et al.* (2014) expression for the weight factor agrees well with the observations for this level of solar activity. It is also clear that the value (1.116) marked by the blue line, as was suggested by Lockwood *et al.* (2014), is not a good fit to the observations and as such must be discarded.

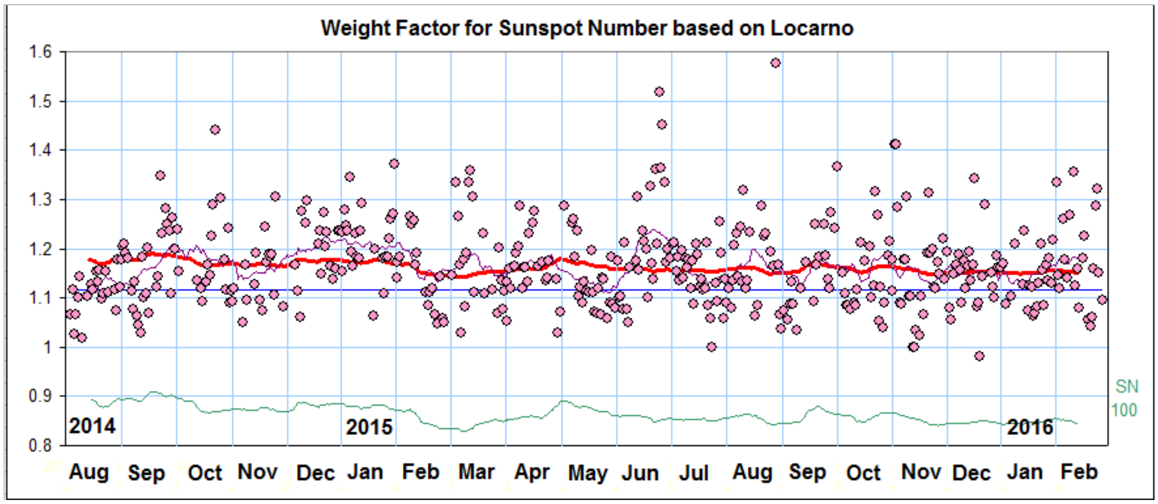

Figure 9: Weight factors (pink dots) computed from the recent Locarno daily data. The red curve shows the 27-day running average of the weight factor calculated using the relationship determined by Clette *et al.* (2014). The green curve at the bottom of the Figure shows the 27-day running average sunspot number (v2).

Figure 10 shows the Locarno weight factor as determined by Svalgaard (blue symbols) for both solar maximum and solar minimum conditions and continued (red symbols) by the Locarno observers until the present [and hopefully beyond]. The green dots show yearly averages.

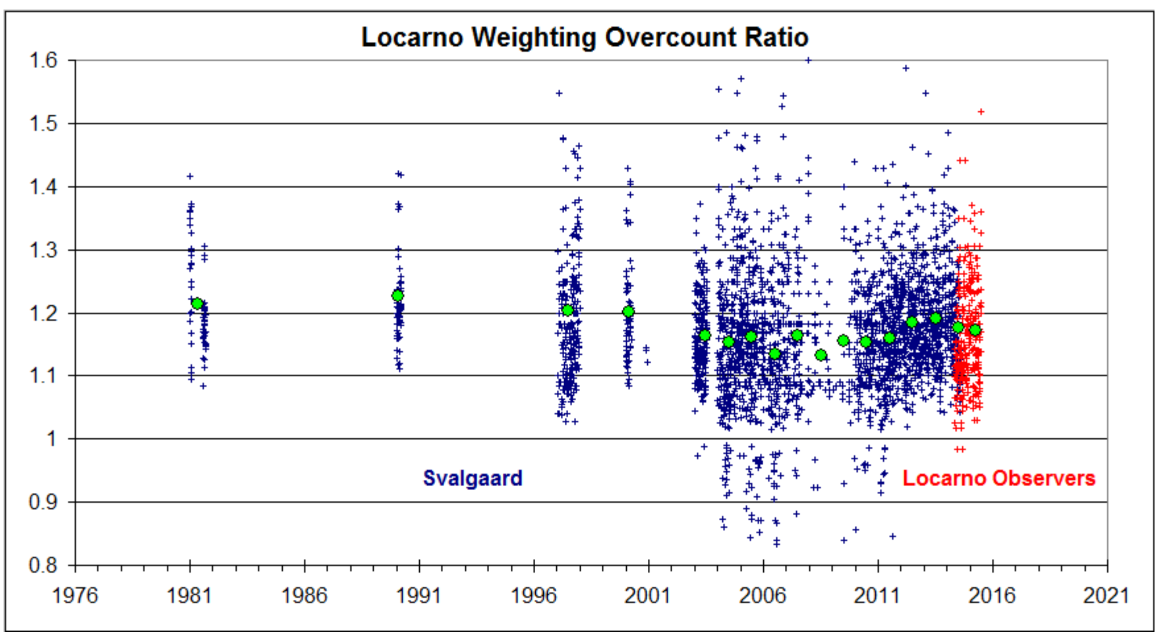

Figure 10: Locarno weight factor as determined by Svalgaard (blue symbols) for both solar maximum and solar minimum conditions and continued (red symbols)

by the Locarno observers until the present. The green dots show yearly averages of the weight factor when there were spots to count; note the weak solar cycle modulation.

The problem we are faced with is not really to calculate the weight factor for the current data. We don't need to; we know what the factor is for every day (with an observation). The problem is to determine the weight factor retroactively for the interval 1947-1980. For the Zürich data before 1980 we know the number of groups for each month and the relative sunspot number (encumbered by weighting because all observers were normalized to Zürich) for each day (and hence for each month). Can we from that correct the sunspot number for weighting? Before we attack that problem, we'll look closer at the data on a daily basis.

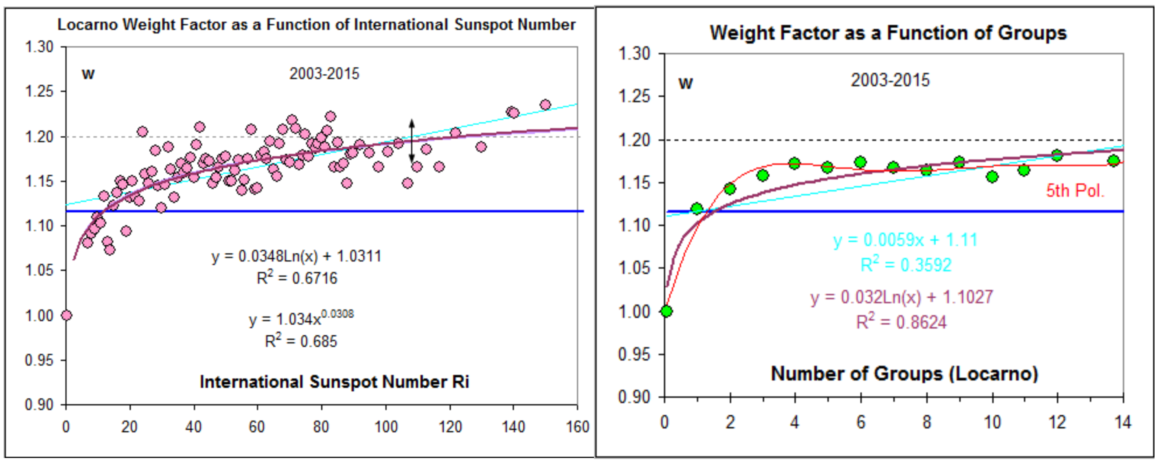

Figure 11: (Left) Locarno average weight factor for bins of the International Sunspot Number (*Ri*). Below *Ri* = 90, the bin size is unity, while above that, bins of progressively larger size are used to ensure enough values in each bin. A fiducial value of 0.3 has been used in lieu of a zero *Ri* to which a weight of 1 is assigned. The double-headed arrow shows an estimate of the error of the mean values. (Right) Average weight factor for unity-wide bins of the number of groups. A fiducial value of 0.1 has been used in lieu of a zero group number, to which a weight of 1 has been assigned.

On a daily basis, the dependences of the weight factor on *Ri* and on the number of groups are decidedly non-linear with a rapid drop-off towards low activity, but even a slightly wrong weight factor applied to a small value will have very little effect on the result. But it is clear that the daily weight factor is not just a simple function of the relative number *SSN* or of the group count alone, *GN*, but is a function of both (and of the observer as well): $w = F(SSN, GN, Obs)$. The situation is further complicated by *SSN* being also a function of *GN*, *Obs*, and of the number of spots, *SN*: $SSN = Q(GN, SN, Obs)$, so that we actually should write $w = F(Q(GN, SN, Obs), Obs)$. As the dependence on the Zürich observers is slight, we ignore the observer differences as furthermore also necessitated by the fact that we don't know who the observers were for each day during 1947-1980. To separate the influence of *GN* and *SN* we now plot the daily Locarno weight factor as a function of the reported (*i.e.* weighted) *SN* for bins of each group number, Figure 12:

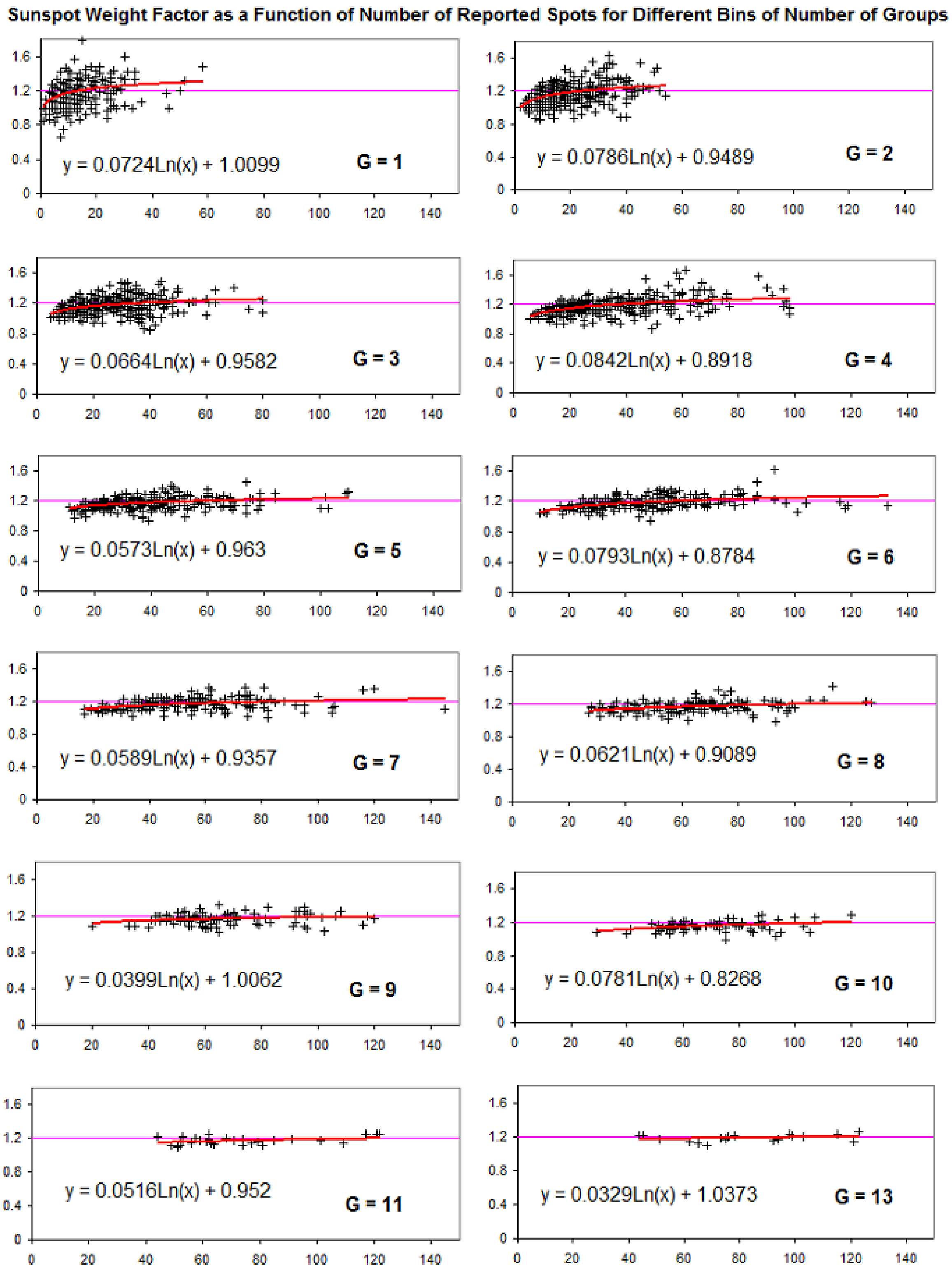

Figure 12: For each bin of group number G = 1, 2, 3, … the graphs show the Locarno weight factor for re-counted days of 1997-2015 as a function of the reported (thus weighted) number of sunspots (note: *not* the sunspot number). A fit to a logarithmic function of the sunspot count is derived for each group.

Using the functional fits derived from Figure 12 we calculate the weight factor on a grid of 1 unit of GN and 5 units of SN to obtain a visual representation of the weight factor 'landscape' function $w = F(Q(GN, SN))$, Figure 13 (left panel). The 'jagged' appearance could be improved by suitable smoothing, but the gain seems marginal. We can thus quantify the average effect of Weighting given the group and (reported) spot counts for daily values, should such values become available.

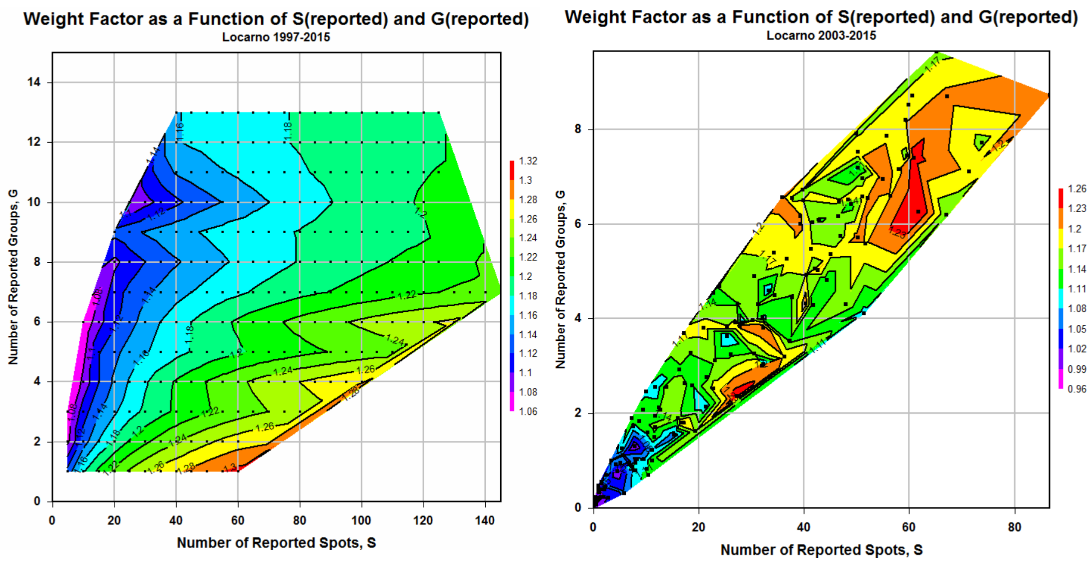

Figure 13: (Left) Contour map of the daily Locarno weight factor for 1997-2015 as a function jointly of the reported (thus weighted) number of sunspots, *S*, and of the number of groups, *G*. (Right) Contour map of the monthly Locarno weight factor for 2003-2015 as a function of both *S* and *G*.

It is also of interest to repeat the analysis for monthly values, *e.g.* as given in Waldmeier (1968b, 1978), as the scatter is much smaller, *c.f.* Figure 13 (right). The results are shown in Figure 14 and 13 (right panel).

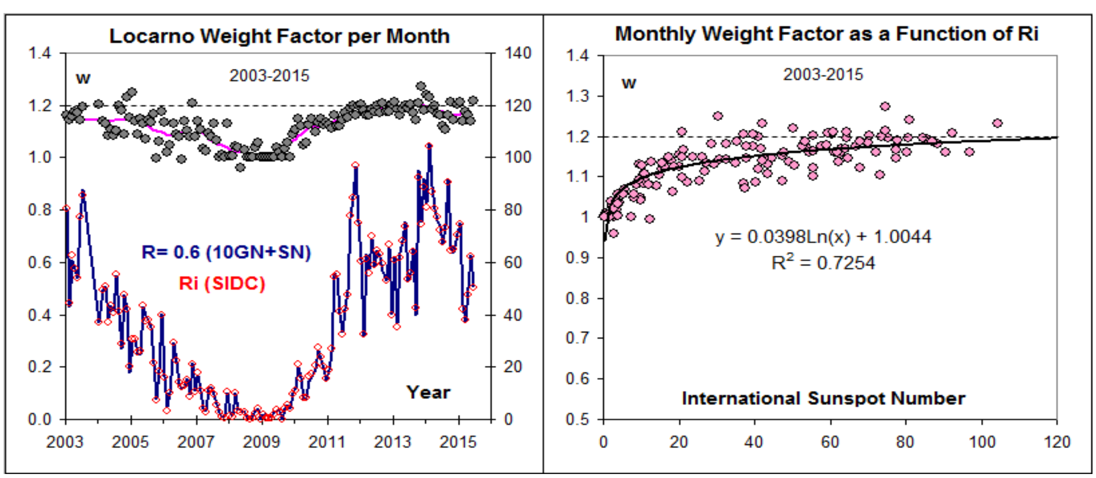

Figure 14: (Left) The Locarno weight factor for each month for 2003-2015 dipping down to unity for no activity and rising to 1.2 for the moderate activity

at the maximum of the weak solar cycle 24. At the bottom we show the time variation of the International Sunspot Number Version 1 (red circles) which is very closely the same as the Locarno relative number multiplied by the nominal *k*-factor of 0.60 (blue curve). (Right) The monthly weight factors as a function of the International Sunspot Number. The non-linear function shown is a decent fit to the weight factor data.

4. Correcting for Weighting

For monthly values, the group count and the spot count are constrained to a rather narrow diagonal band in Figure 13 (right) which suggests that a one-dimensional relationship with the relative sunspot number, such as given in Figure 14 (right), might be sufficient for correction of said number to an un-weighted value. We can test this assertion by calculating the weight factor using that formula ($w = 1.0044 + 0.0398 \ln(R_i)$; $R_i \geq 0.2$), dividing the International Sunspot Number since 2003 by the computed weight factor, and comparing the thus corrected number with the un-weighted relative number obtained by re-counting the spots without weighting on the Locarno drawings, Figure 15. The agreement is excellent, with a linear coefficient of determination $R^2 = 0.991$:

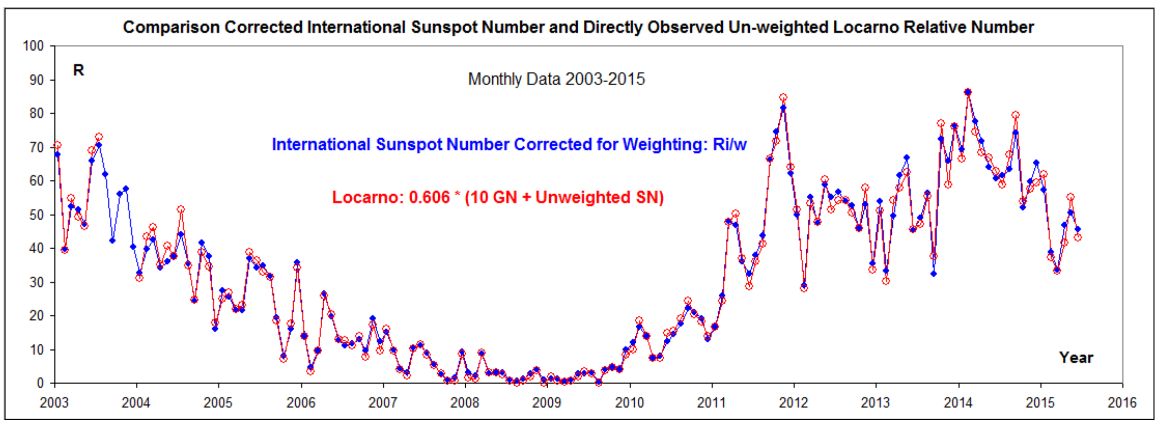

Figure 15: Comparison of monthly values of the International Sunspot Number as published by the WDC SILSO in Brussels (Version 1, pre-July-1st-2015) corrected for weighting (blue curve) and the Relative Number for Locarno calculated using the un-weighted number of sunspots (red curve) and a *k*-factor of 0.606.

Under the assumption that the weight factor function is also valid for the Waldmeier era at Zürich we can now correct the Zürich sunspot number for the inflation introduced by the weighting scheme, Figure 16 and Table 1.

In constructing Figure 16 (and in this paper generally) we used the pre-July-1st-2015 values of the International Sunspot Number without the corrections and reassessments introduced as of that date. It is important to take into account that the weight factor varies with the sunspot number itself, so one cannot (except as a first, crude approximation) use a constant weight factor throughout. The average yearly weight factors given in Table 1 are valid regardless of the sunspot numbers determined for each year and of the *k*-factors adopted. The factors were derived from the formula of Figure 14 using the nominal *k*-factor of 0.60, so its R_i-argument could be written $R_i = R_k$*0.6/k, where k is the k-factor for the relative sunspot number R_k. For R_k from the 'new' SILSO sunspot number series, k is equal to unity.

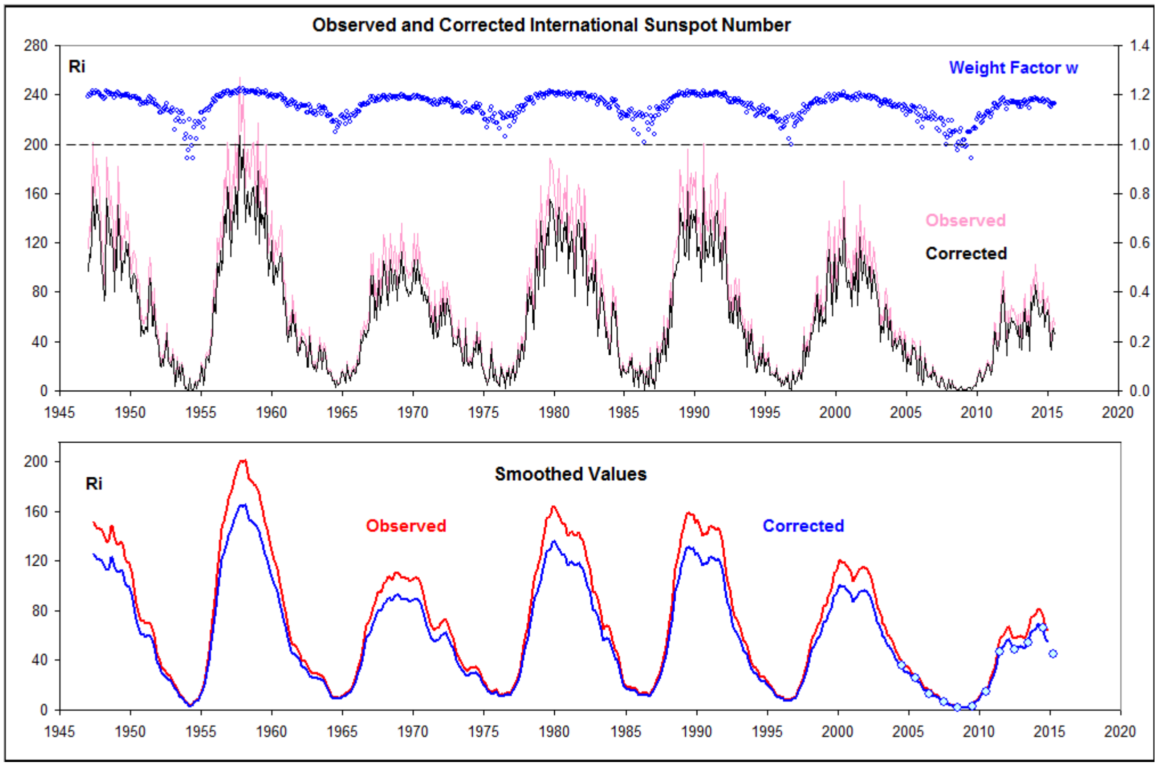

Figure 16: (Top) Comparison of monthly values of the International Sunspot Number as published by the WDC SILSO in Brussels (Version 1, pre-July-1st-2015), pink curve, and the values corrected for weighting (black curve) using the weight factors shown by the upper blue symbols. (Bottom) The monthly values smoothed (using the standard method introduced by Wolf). Light blue dots show yearly values of un-weighted counts from Locarno, *i.e.* not relying on the weight factor formula. Again, the agreement is excellent.

An interesting question is: how does this 'corrected New R_i' (which is simply SILSO V1 R_i freed from weighting and brought onto Wolfer's scale by removing the obsolete 0.6 k-value scale factor, call it V1.5) compare with WDC-SILSO V2 R_i released July 1st, 2015? Figure 17 provides a preliminary answer to that question:

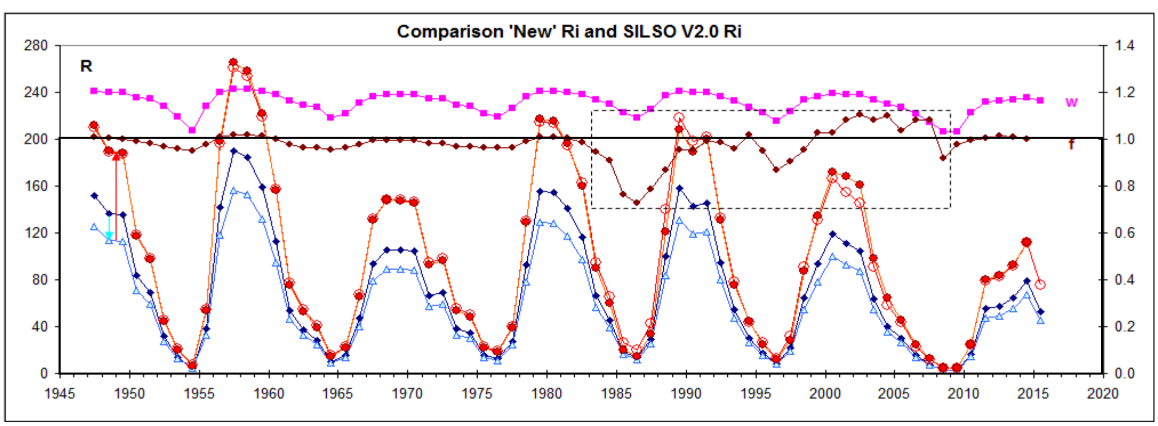

Figure 17: Dark blue diamonds (V1 Ri - old official Ri], scaled down to the ‘Corr. Ri’, light blue triangles [V1.5], by dividing by the weight factor, *w*, (upper pink squares). The 'Corr. Ri' is then scaled to the Wolfer scale (New Ri, red open circles) by dividing by the, no longer used *k*-value 0.60 and compared with SILSO V2 Ri (red filled circles). The ratio *f* = V2/New is shown by the brown dots.

The ratio *f* = V2/New (brown dots) is generally close to unity, although there is a weak solar cycle variation, probably due to an inadequate (constant) *w*-factor used for SILSO V2. The ratio varies irregularly for the years in the rectangle, possibly indicating some further adjustments (unexplained, but probably arising from issues with the data from Locarno). The irregularity is not serious near solar minima, as the sunspot number is small then, but the ~10% difference at the maximum and declining part of sunspot cycle 23 is a concern that should be addressed and explained.

5. Comparison with Sunspot Areas

Up to this point we have been concerned with direct *measurement* of the effect of weighting, which is, of course, the preferred and correct approach. Historically, the ‘discovery’ (Svalgaard, 2007, 2010, 2012, 2014) of the weighting came about by comparing the International Sunspot Number to other solar variables and activity indices and noticing (and quantifying) the Waldmeier ‘Discontinuity’ in 1947. Comparing with sunspot areas, Figure 18, shows the discontinuity clearly enough, as well as showing that there is no discontinuity prior to 1947, e.g. related to change of observers from Wolf to Wolfer (1894) and finally to Brunner (1926).

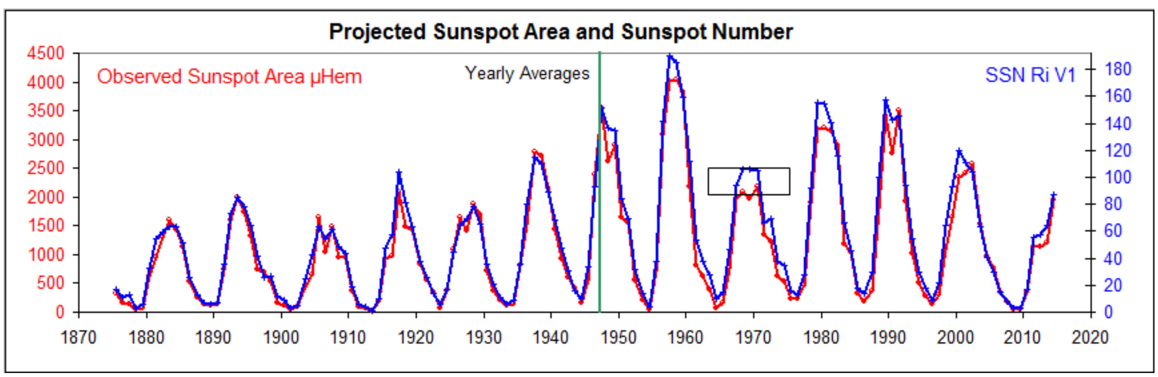

Figure 18: The yearly averaged projected (*i.e.* observed) area of the solar disk covered with sunspots in millionths of the area of the visible disk (Balmaceda et al. 2009; red curve with small dots and left-hand scale) compared to the International Sunspot Number Version 1 (blue curve with small plus-symbols and right-hand scale) scaled to match the areas before 1947. For yearly averages the non-linearity of the relationship between sunspot numbers and sunspot areas becomes small enough that simple linear scaling largely suffices to compare the two measures. The rectangle near year 1970 has a height of 20 sunspot units. The green vertical line at the year 1947 shows where we would place the Discontinuity.

In particular, Brunner and Wolfer seem to have the same calibration relative to the sunspot areas. Brunner also explicitly stresses (e.g. Brunner, 1945) that his reduction factor to Wolf's old unit is the same, 0.6, as Wolfer's. This is also clearly seen in Figure 19 comparing the number of spots reported by the Zürich (and Locarno) observers with the sunspot areas and the group number (Svalgaard and Schatten, 2016).

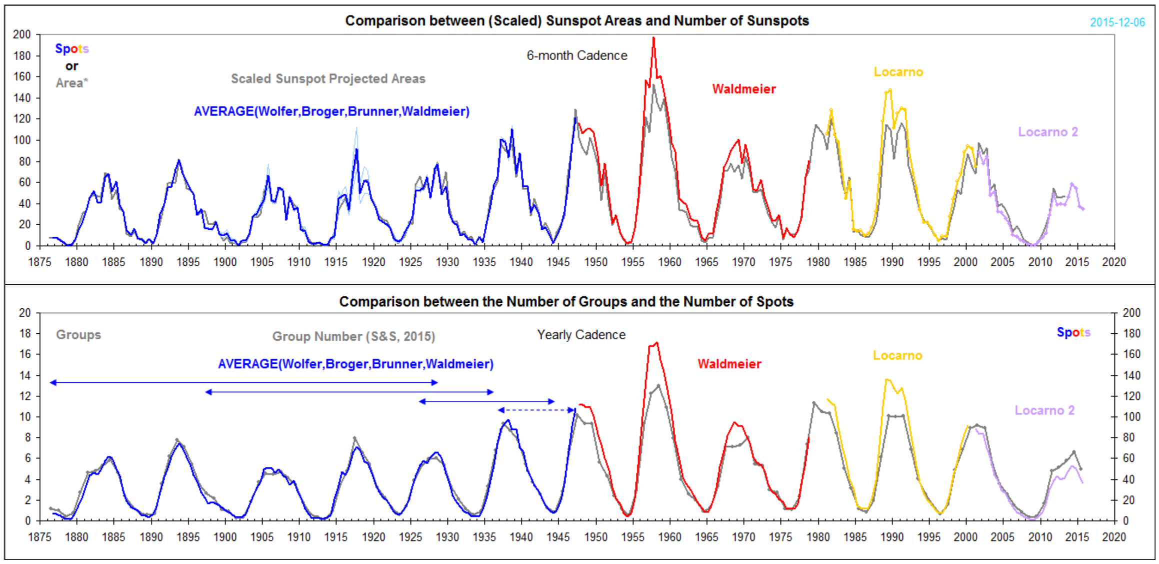

Figure 19: Comparing the number of sunspots (note: not the relative sunspot number) to the (scaled) sunspot areas (gray curve, upper panel) and the group number (gray curve, lower panel). The average of Wolfer, Broger, Brunner, and Waldmeier before 1947 is shown by a heavy, blue curve. The individual observers' data are shown by light, blue curves. After 1947, the data are color-coded (and labeled) by observer (Waldmeier, red; Locarno before 2000, yellow; recent Locarno, purple).

Incidentally, the good agreement between the several sunspot observers (before 1947) and the sunspot areas shows that the sunspot areas are likely to be correct as no systematic drift or difference is noticeable.

6. Weighting Before Waldmeier

William Brunner (1945), in his last contribution to the *Astronomische Mitteilungen*, wrote: „Die Grundlage der Zürcher Statistic für die Sonnenfleckenhäufigkeit bilden die aus Beobachtungen von g und f ermittelten täglichen Wolfschen Relativzahlen $r = k\,(10g + f)$, wobei g die Anzahl der beobachteten Fleckengruppen, f die Gesamtzahl der in diesen Gruppen vorhandenen Einzelflecken und k eine von Beobachter und instrument abhängige Konstante bedeuten.“[3]

Brunner thus stipulated that f is the *number* of all *single* spots, with no weighting at all, just simple counting. This is consistent with all previous *Mitteilungen*. Weighting is never

[3] The basis for the Zurich data about the frequency of sunspots is the daily Wolf Relative Sunspot Number $r = k\,(10g + f)$ computed from the observed g and f, where g is the number of sunspot groups, f is the total number of all the single spots present within those groups, and k is a constant depending on observer and instrument.

mentioned; on the contrary, it was always emphasized that counting was done ‘as always before’. On the other hand, weighting was clearly practiced by some Zürich observers, e.g. Max Broger. Our problem is to identify who and when and with what effect, if any. Brunner (1936) let slip a hint (“In large centers of activity one is inclined – and this perhaps rightly – to give some single spots according to their sizes a different weight”) that some weighting was likely performed. Figure 20 shows three drawings from Mount Wilson Observatory. The left-most is for a day where Wolfer reported observing 1 group with 1 spot (*1.1*). For the middle one, Wolfer reported 1 group with 2 spots (*1.2*). The weighted counts for these spots with penumbra would have been *1.3* and *1.6* (or *1.5*), respectively, attesting that Wolfer did not weight at those times. The rightmost drawing is of a sole, large spot reported as *1.4* by Brunner, showing that he counted the single spot with weight 4. Several other examples of such weighting by Brunner can be found, e.g. on 1930-08-16, 1931-03-05, 1932-02-05, 1932-03-29, and 1935-05-27.

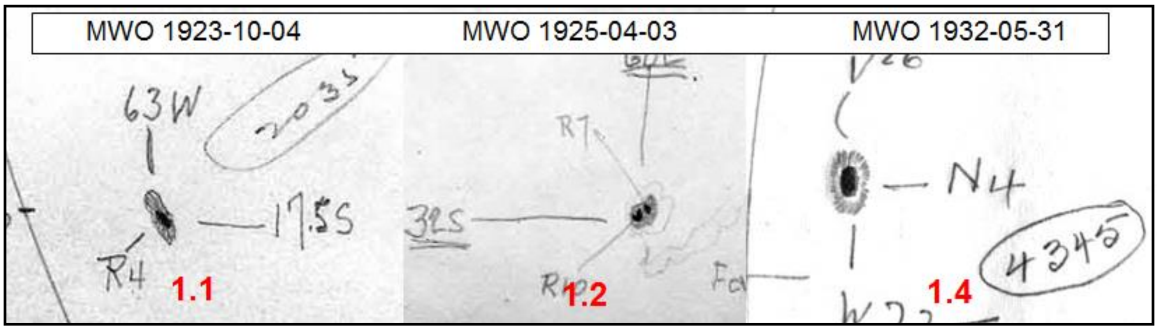

Figure 20: Mount Wilson Observatory drawings for the dates indicated where the Sun had only a single sunspot group on the disk. The two leftmost were also observed by Wolfer and given (red text) in the standard Wolf notation (*groups***.***spots*), indicating no weighting was performed. The rightmost group was observed by Brunner and reported as *1.4*, indicating that this single, large spot was counted with weight 4.

So, we must consider it established that Brunner weighted at least some of the spots, perhaps especially very large solitary spots, which would explain the dearth of 7’s for Brunner on Figure 33 of Clette et al. (2014). The questions are now how large the effect of this would be on the sunspot number and how consistently the weighting was performed. Because Brunner reports that his overall reduction factor is the same as Wolfer’s, the inflation caused by weighting large spots must be precisely compensated by an under-count of small spots, such as to leave no overall effect of the weighting. Figure 21 (right-hand panel) shows directly that *on average* Brunner and Wolfer reported the same number of spots (the slope of the linear fit though the origin is unity: 1.003±0.011) during the time (1926-1928) of their overlapping observations, but also shows that for low solar activity (number of spots less than, say, 75), Brunner reports more spots than Wolfer, while the opposite is the case for high activity with number of spots larger than ~75. A large number of spots means that there are many small spots; in fact, high sunspot numbers are dominated by the number of small spots which can run in the hundreds.

Brunner reminds us that „Wolf hat auch *größere Hofflecken* als 1 gezählt und nicht auf die structur und Auflösung des Kerns in Teilkerne geachtet und von den kleinsten Flecken nur mitgenommen, was bei genügend gutem Bild auf den ersten Blick su sehen ist“[4], as being the principal reason for the 0.6 reduction factor. In addition, Wolf could furthermore not even see the smallest spots anyway with his handheld portable small telescope in use after 1861.

If the Locarno observers faithfully followed Waldmeier’s prescription for weighting (presumably assured by Waldmeier’s ongoing quality control) and if Waldmeier just took over the procedure unchanged from Brunner (and as claimed by Waldmeier (1961) even from Wolfer, going all the way back to 1882) we would expect the distribution of the ratios of the weighted number of spots to the un-weighted as a function of activity to be the same for Brunner as for Locarno. Figure 21 (left) shows that it is not.

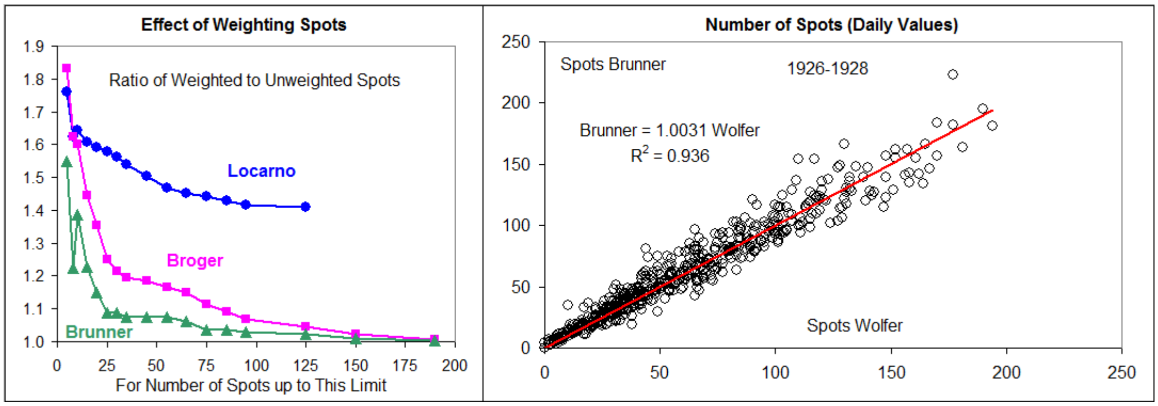

Figure 21: (Left) The slope of the correlation between weighted spots reported by Locarno (blue circles) and un-weighted spots at same for 2003-2015, between spots reported by Broger (pink squares) and un-weighted spots reported by Wolfer for 1897-1935, and between spots reported by Brunner (green triangles) and un-weighted spots reported by Wolfer 1926-1928, as a function of the maximum un-weighted sunspot count used for the correlation. (Right) Correlation between daily values of Brunner’s reported (with some weighting) spot count and Wolfer’s reported un-weighted count.

It is clear that the effect of (assumed) weighting by Brunner (and Broger) does not follow the same distribution as that for Locarno (and presumably Waldmeier), but that the effect is much smaller for high solar activity (with many spots) explaining why Brunner could maintain the same reduction factor as Wolfer. The effect of weighting for high solar activity is what essentially determines the amplitude or size of the sunspot cycles and thus heavily influences the reduction factor.

7. What is a Group?

Comparing the relative sunspot number with various other indices in order to assess the effect of weighting relies on the assumption that the ‘other half’ of the relative sunspot number – 10 times the number of groups – has had a constant calibration over time. Kopecký et al. (1980) cite the Zürich observer Zelenka drawing attention to the possible inflationary effect of the introduction of the Waldmeier Group Classification around 1940.

[4] Wolf also counted a *collection of spots within a common largish penumbra* as just a single spot and thus did not take the structure and splitting of the umbra into account, and only included the smallest spots if they were visible at first glance on a sufficiently good quality image.

We discussed the problem in Clette et al. (2014) and in Svalgaard and Schatten (2016), and show here just some examples, Figure 22:

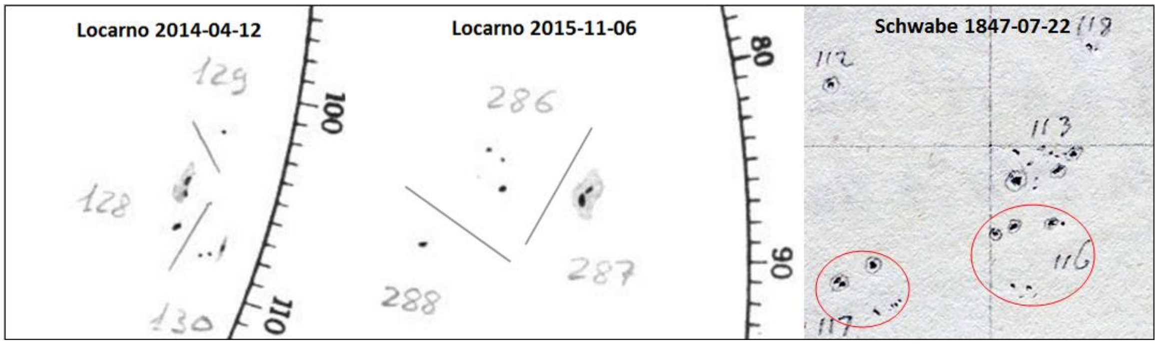

Figure 22: (Left) Group designations from Locarno drawings showing over-count compared to what simple proximity would dictate. (Right) Group designations from Schwabe's drawings (Adapted after Pavai et al. (2015)) showing under-count. The two groups in red ovals would today likely be counted as four groups.

Before the advent of magnetic measurements, a sunspot group was defined solely on the basis of its morphology and location relative to other groups. Sunspot groups were at first considered just to be spatially separate assemblies of sunspots. Beck (1984) and Friedli (2009) remind us that after the Waldmeier (1938) Classification was introduced, the *evolution* of a group became a determining factor in the very definition of a group which now, in addition to be a spatially isolated collection, also must evolve as an *independent* unit, going through (at least partly) the evolution sequence of the Waldmeier classification.

If Wolfer is to be the new standard it would seem that earlier groups are under-counted (e.g. very pronounced for the Staudach data (Svalgaard, 2016a)), while later groups are over-counted. This has been taken into account in the construction of the group number, but more research is needed to integrate that with the sunspot number. In Clette (2014) we found the over-count to be 7.5%. For the groups observed at Locarno since then, the over-count is 7.7%. This inflates the relative sunspot number by 4-5%.

8. The Weighting Effect Seen in the Ionosphere

Above ~250 km altitude the primary constituent of the atmosphere is atomic oxygen that can be ionized by EUV radiation with wavelength below 103 nm. The resulting conductive air is called the F-layer. Because the density is so low, recombination is so slow that the F-layer persists even during the night. During the day, the F-layer splits into two layers, with F2 being at the highest altitude. The F2 layer is a dependable reflector of radio signals as it reflects normal-incident frequencies at or below the (observable) critical frequency controlled by the EUV flux and hence by solar activity. Ostrow and PoKempner (1952) in a careful study of the critical frequency 1934-1952 observed at Washington D.C. found that the relationship with the sunspot cycle was not stable, but changed during the rise of Cycle 18 and concluded that 'the Zurich sunspot number is not an entirely satisfactory index of the solar activity responsible for ionospheric ionization',

Figure 23. We can see today that it is not the relationship that is at fault, but the sunspot number, due to the introduction of effective weighting.

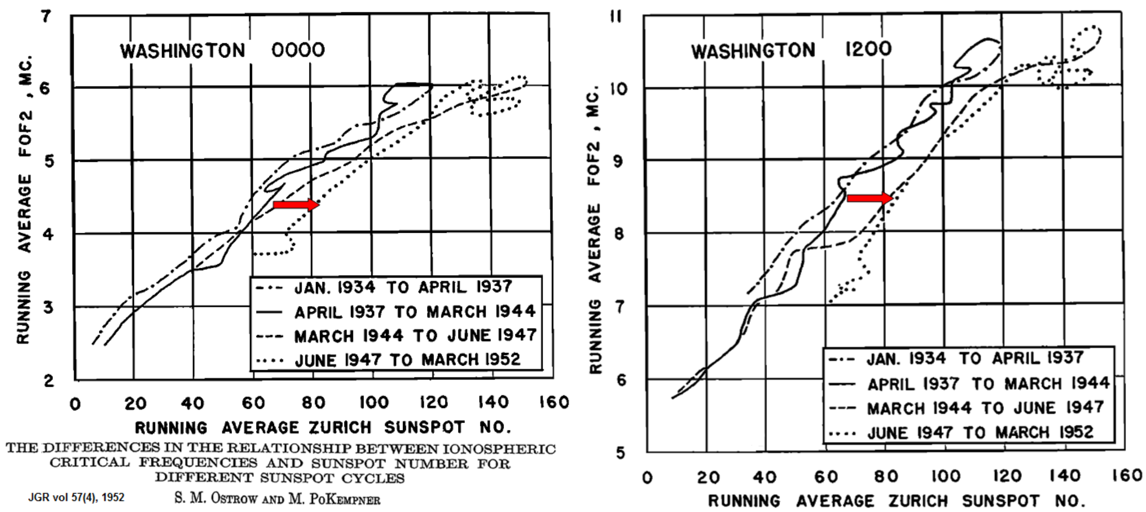

Figure 23: 12-month running averages of the monthly median critical frequency f^oF2 (MHz) versus 12-month running averages of monthly Zurich sunspot numbers for local night 00^h (left) and local day (12^h) at Washington D.C. (Adapted after Ostrow and PoKempner, 1952). The (red) arrows show that a 20% correction of the sunspot number during the rise of Cycle 18 restores the strong, uniform relationship between critical frequency and (corrected) sunspot number.

A dynamo current in the E-layer where the density is high enough produces a diurnal magnetic effect (discovered in 1722) observable on the ground also showing the same clear discontinuity in ~1947 (Svalgaard, 2016b).

9. Conclusions

Waldmeier in 1947 formalized the weighting (on a scale from 1 to 5) of the sunspot count made at Zürich and its auxiliary station Locarno, whereby larger spots were counted more than once. This counting method inflates the relative sunspot number over that which corresponds to the scale set by Wolfer and Brunner. Brunner had also weighted the largest spots, but evidently compensated by not counting enough small spots such that the overall effect on the sunspot number turned out to be nil. Svalgaard re-counted some 60,000 sunspots on drawings from the reference station Locarno and determined that the number of sunspots reported were ‘over counted’ by 44% on average, leading to an inflation (measured by a weight factor) in excess of 1.2 for high solar activity. In a double-blind parallel counting by the Locarno observer Cagnotti, we determined that Svalgaard’s count closely matches that of Cagnotti’s, allowing us to determine the daily weight factor since 2003 (and sporadically before). We find that a simple empirical equation fits the observed weight factors well, and use that fit to estimate the weight factor for each month back to the introduction of effective weighting in 1947 and thus to be able to correct for the over-count and to reduce sunspot counting to the Wolfer method in use from 1893 onwards. The Locarno observers have since August, 2014 counted spots both with and without weighting, and the un-weighted (real) spot count is now used in determining the official relative sunspot number.

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Table 1: Old R_i is the International Sunspot Number (version 1.0), Corr. R_i is Old R_i divided by the Weight Factor (calculation actually performed month-by-month, then averaged per year). 'New' R_i is Corr. R_i divided by 0.60, but does not quite match SILSO version 2.0 because of further (small) corrections to the latter.

The Year	Old R_i	Weight Factor	Corr. R_i	'New' R_i		The Year	Old R_i	Weight Factor	Corr. R_i	'New' R_i
1947.5	151.5	1.204	125.8	209.7		1982.5	116.3	1.193	97.4	162.3
1948.5	136.2	1.199	113.4	189.0		1983.5	66.6	1.169	56.8	94.7
1949.5	135.1	1.199	112.6	187.7		1984.5	45.9	1.149	39.4	65.7
1950.5	83.9	1.179	71.0	118.3		1985.5	17.9	1.115	16.0	26.7
1951.5	69.4	1.172	59.1	98.5		1986.5	13.4	1.093	12.0	20.0
1952.5	31.4	1.140	27.5	45.8		1987.5	29.2	1.129	25.5	42.5
1953.5	13.9	1.096	12.4	20.7		1988.5	100.0	1.185	84.0	140.0
1954.5	4.4	1.035	4.1	6.8		1989.5	157.8	1.205	130.8	218.0
1955.5	38.0	1.139	32.8	54.7		1990.5	142.3	1.201	118.9	198.2
1956.5	141.7	1.200	117.8	196.3		1991.5	145.8	1.202	121.2	202.0
1957.5	189.9	1.212	156.4	260.7		1992.5	94.5	1.184	79.6	132.7
1958.5	184.6	1.212	152.3	253.8		1993.5	54.7	1.162	47.0	78.3
1959.5	158.8	1.205	131.5	219.2		1994.5	29.9	1.137	26.1	43.5
1960.5	112.3	1.192	94.1	156.8		1995.5	17.5	1.115	15.6	26.0
1961.5	53.9	1.162	46.3	77.2		1996.5	8.6	1.079	7.9	13.2
1962.5	37.6	1.147	32.7	54.5		1997.5	21.5	1.118	18.9	31.5
1963.5	27.9	1.135	24.5	40.8		1998.5	64.2	1.168	54.8	91.3
1964.5	10.2	1.092	9.3	15.5		1999.5	93.2	1.183	78.5	130.8
1965.5	15.1	1.110	13.5	22.5		2000.5	119.5	1.194	100.0	166.7
1966.5	46.9	1.156	40.4	67.3		2001.5	110.9	1.191	93.0	155.0
1967.5	93.7	1.184	79.0	131.7		2002.5	104.1	1.189	87.5	145.8
1968.5	105.9	1.190	89.0	148.3		2003.5	63.6	1.169	54.3	90.5
1969.5	105.6	1.190	88.7	147.8		2004.5	40.4	1.150	35.1	58.5
1970.5	104.7	1.189	88.0	146.7		2005.5	29.8	1.136	26.1	43.5
1971.5	66.7	1.171	56.9	94.8		2006.5	15.2	1.109	13.5	22.5
1972.5	68.9	1.172	58.7	97.8		2007.5	7.5	1.073	6.9	11.5
1973.5	38.2	1.147	33.1	55.2		2008.5	2.9	1.034	2.7	4.5
1974.5	34.4	1.143	30.0	50.0		2009.5	3.1	1.033	2.9	4.8
1975.5	15.5	1.107	13.8	23.0		2010.5	16.5	1.114	14.8	24.7
1976.5	12.6	1.097	11.3	18.8		2011.5	55.6	1.161	47.6	79.3
1977.5	27.5	1.132	24.1	40.2		2012.5	57.6	1.165	49.4	82.3
1978.5	92.7	1.183	78.1	130.2		2013.5	64.7	1.169	55.2	92.0
1979.5	155.3	1.205	128.8	214.7		2014.5	79.1	1.178	67.1	111.8
1980.5	154.7	1.205	128.3	213.8		2015.5	48.6	1.162	41.8	69.7
1981.5	140.5	1.201	116.9	194.8						

Acknowledgements

We have benefited from participation in the four Sunspot Number Workshops (http://ssnworkshop.wikia.com/wiki/Home) and from discussions with the team at the WDC/SILSO. Sunspot data was supplied by WDC/SILSO, Royal Observatory of Belgium. We acknowledge with pleasure the use of drawings from Specola Solare Ticinese, Locarno (http://www.specola.ch/e/drawings.html). This study includes data from the synoptic program at the 150-Foot Solar Tower of the Mt. Wilson Observatory (ftp://howard.astro.ucla.edu/pub/obs/drawings). The Mt. Wilson 150-Foot Solar Tower is operated by UCLA, with funding from NASA, ONR and NSF, under agreement with the Mt. Wilson Institute. We thank the reviewer for prompting us to re-examine the contribution of William Brunner. LS thanks Stanford University for support.

References

Balmaceda, L. A., Solanki, S. K., Krivova, N. A., Foster, S.: 2009, A homogeneous database of sunspot areas covering more than 130 years, *J. Geophys. Res.* **114**, A07104, doi:10.1029/2009JA014299

Beck, R.: 1984, Zum Problem der Gruppeneinteilung von Sonnenflecken, *Sonne – Mitteilungsblatt der Amateursonnenbeobachter* **8**, 64 (http://www.leif.org/research/Beck-Rules-Groups-and-Spots.pdf)

Brunner, W.: 1936, Zürich Observatory, *Terr. Magn. Atmos. Electr.* **41**(2), 210, doi:10.1029/TE041i002p00210

Brunner, W.: 1945, Tabellen und Kurven zur Darstellung der Häufigkeit der Sonnenflecken in den Jahren 1749-1944, *Astr. Mitteil. Eidgn. Sternw. Zürich*, No. **145**, 135

Clette, F., Svalgaard, L., Vaquero, J.M., Cliver, E.W.: 2014, Revisiting the Sunspot Number – A 400–Year Perspective on the Solar Cycle, *Space Sci. Rev.* **186**, 35, doi:10.1007/s11214-014-0074-2

Cortesi, S., Cagnotti, M., Bianda M., Ramelli, R., Manna, A.: 2016, Sunspot Observations and Counting at Specola Solar Ticinese in Locarno since 1957, Solar Phys. (this issue)

Friedli, T. K.: 2009, Die Wolfsche Reihe der Sonnenfleckenrelativzahlen, *Verbesserte Likelihood Ratio Tests zur Homogenitätsprüfung in struturelle Zustandsraummodellen*, Südwestdeutcher Verlag für Hochschulschriften, Saarbrücken

Kopecký, M., Kuklin, G.V., Růžičková–Topolová, B.: 1980, On the relative inhomogeneity of long-term series of sunspot indices, *Bull. Astron. Inst. Czech.* **31**, 267

Lockwood, M., Owens, M.J., Barnard, L.: 2014, Centennial variations in sunspot number, open solar flux, and streamer belt width: 1. Correction of the sunspot number record since 1874, *J. Geophys. Res. Space Physics* **119**, 5172–5182, doi:10.1002/2014JA019970

Ostrow, S. M., PoKempner, M.: 1952, The difference in the relationship between ionospheric critical frequencies and sunspot number for different sunspot cycles, *J. Geophys. Res.* **57**(4), 473, doi:10.1029/JZ057i004p00473

Pavai, V. S, Arlt, R., Dasi-Espuig, M., Krivova, N. A., Solanki, S. K.: 2015, Sunspot areas and tilt angles for solar cycles 7-10, *Astron. & Astrophys.* **584**, A73, doi:10.1051/0004-6361/201527080

Svalgaard, L.: 2007, Calibrating the Sunspot Number using “the Magnetic Needle”, *CAWSES Newsletter*, **4**, issue 1, http://www.leif.org/research/CAWSES%20-%20Sunspots.pdf

Svalgaard, L.: 2010, Updating the Historical Sunspot Record, *SOHO-23: Understanding a Peculiar Solar Minimum* ASP Conference Series Vol. 428, edited by Steven R. Cranmer, J. Todd Hoeksema, and John L. Kohl. San Francisco: Astronomical Society of the Pacific, 2010, p.297

Svalgaard, L.: 2012, How well do we know the sunspot number? *Comparative Magnetic Minima: Characterizing quiet times in the Sun and Stars*, Proceedings IAU, Symposium No. **286**, 15, Eds.: C. H. Mandrini & D. F. Webb, doi:10.1017/S1743921312004590

Svalgaard, L.: 2014, http://www.leif.org/research/The-Effect-of-Weighting-in-Counting-Sunspots-and-More.pdf

Svalgaard, L.: 2016a, A Recount of Sunspot Groups on Staudach’s Drawings, *Solar Phys.*, this issue

Svalgaard, L.: 2016b, Reconstruction of Solar Extreme Ultraviolet Flux 1740-2015, *Solar Phys.*, this issue.

Svalgaard, L., Schatten, K. H.: 2016, Reconstruction of the Sunspot Group Number: The Backbone Method, *Solar Phys.*, this issue, doi:10.1007/s11207-015-0815-8

Waldmeier, M.: 1948, 100 Jahre Sonnenfleckenstatistik, *Astron. Mitteil. Eidgn. Sternw. Zürich*, No. **152**, 1

Waldmeier, M.: 1961, The sunspot-activity in the years 1610-1960, *Schulthess & Co., Swiss Federal Observatory, Zürich*

Waldmeier, M.: 1968a, Solar Activity 1944-1964 (Cycles No. 18 and 19), *Astr. Mitteil. Eidgn. Sternw. Zürich*, No. **284**, 1

Waldmeier, M.: 1968b, Die Beziehung zwischen der Sonnenflecken-relativzahl und der Gruppenzahl, *Astr. Mitteil. Eidgn. Sternw. Zürich*, No. **285**, 1

Waldmeier, M.: 1978, Solar Activity 1964-1976 (Cycle No. 20), *Astr. Mitteil. Eidgn. Sternw. Zürich*, No. 368, 1

Wolf, R.: 1856, Beobachtungen der Sonnenflecken in den Jahren 1849-1855, *Mittheil. über die Sonnenflecken* **I**, 3

Wolfer, A.: 1907, Die Häufigkeit und heliographische Verteilung der Sonnenflecken im Jahre 1906, *Astron. Mitteil. Eidgn. Sternw. Zürich*, **XCVIII**, 10, 251