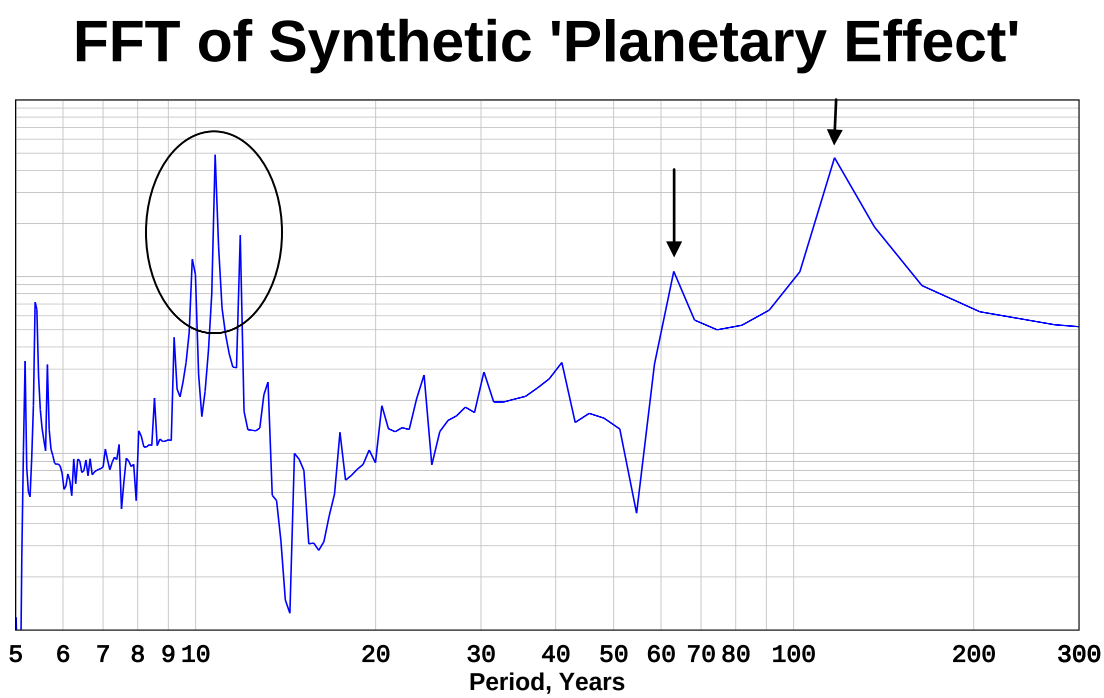
We shall construct a synthetic 'sunspot cycle' as a function of time, t, as the sum of two cosines with different periods, P_1 and P_2. Since the sunspot number is always positive, we take the absolute value of the sum. Since the real sunspot number is not necessarily a linear function of the underlying 'real activity measure' [e.g. sunspot number ~ (sunspot area)^0.73) we'll approximate the synthetic sunspot number, for simplicity, as the square root of the absolute value of the sum of the two cosines [this assumption does not change the timing of the maxima and minima, but only makes the amplitudes more 'realistic']:

$$\text{'Sunspot Number'} = \mathrm{SQRT}(\mathrm{ABS}(k*\cos(\pi/P_1*t) + \cos(\pi/P_2*t))) \qquad (1)$$

The constant k shall be assumed initially to be unity, giving the two cosines equal weight. We now set P_1 = the period of Jupiter = P_J = 11.86199 years, and P_2 = half the time between conjunctions of Jupiter and Saturn = ½ $(P_S*P_J)/(P_S-P_J)$ = 9.92945 years, where the period of Saturn is P_S = 29.45713 years. Because of the ABS operator, a full cycle is just π and not 2π.

The power spectrum [as computed with the FFT, which is good enough for those purely periodic terms] looks like this [where the ordinate three-decade logarithmic scale is in arbitrary units]



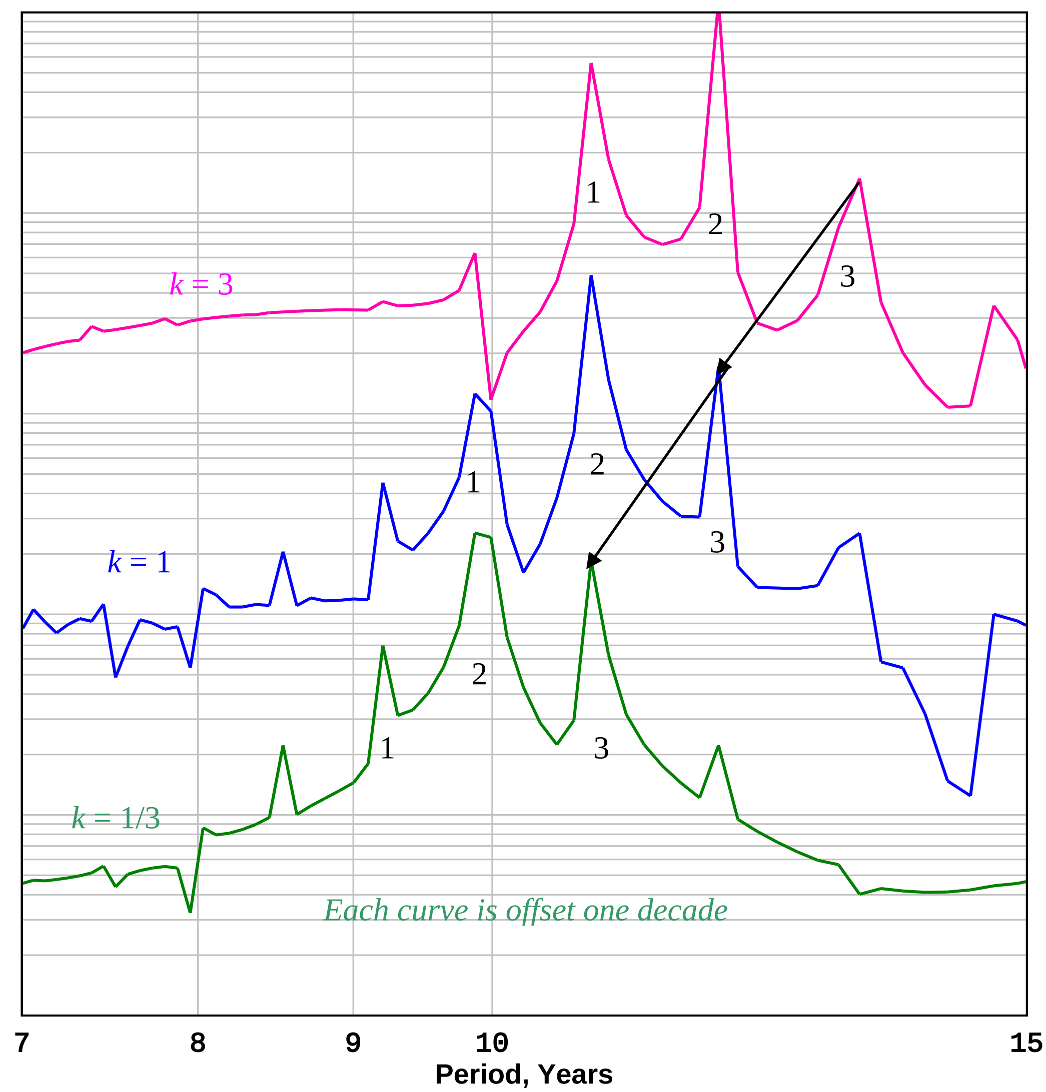
Peaks at Years	Scafetti (Years)
9.91	9.99 Figure 3
10.78	10.91 Figure 3
11.87	11.87 Figure 3
117.03	120-130 Figure 8
63.02	60-64 Figure 7

We see three peaks near 11 years, with periods that within the errors are identical to the peaks in Scafetti Figure 3 [even with the same relative size of the 'side peaks', ~0.4], plus two peaks at ~63 and ~117 years which are also claimed by Scafetti [P_{13} and P_{12} on page 12].

So instead of assuming that the activity record is the sum of three elementary waves: 0.4 {9.99} + 1.0 {10.91} + 0.4 {11.87}, where the 10.91 year wave is the ‘real’ Schwabe period produced by the solar dynamo and the ‘side peaks’ represent tidally induced modulations, we see that only *two* cycles are needed to be produced by astrological means; the middle [highest] peak follows naturally without further assumptions. So it is not necessary to postulate *three* waves, when two will do.

The close correspondence between Scafetta’s peaks and mine is only for $k = 1$. Other [significantly different] values of k move the peaks out of correspondence:

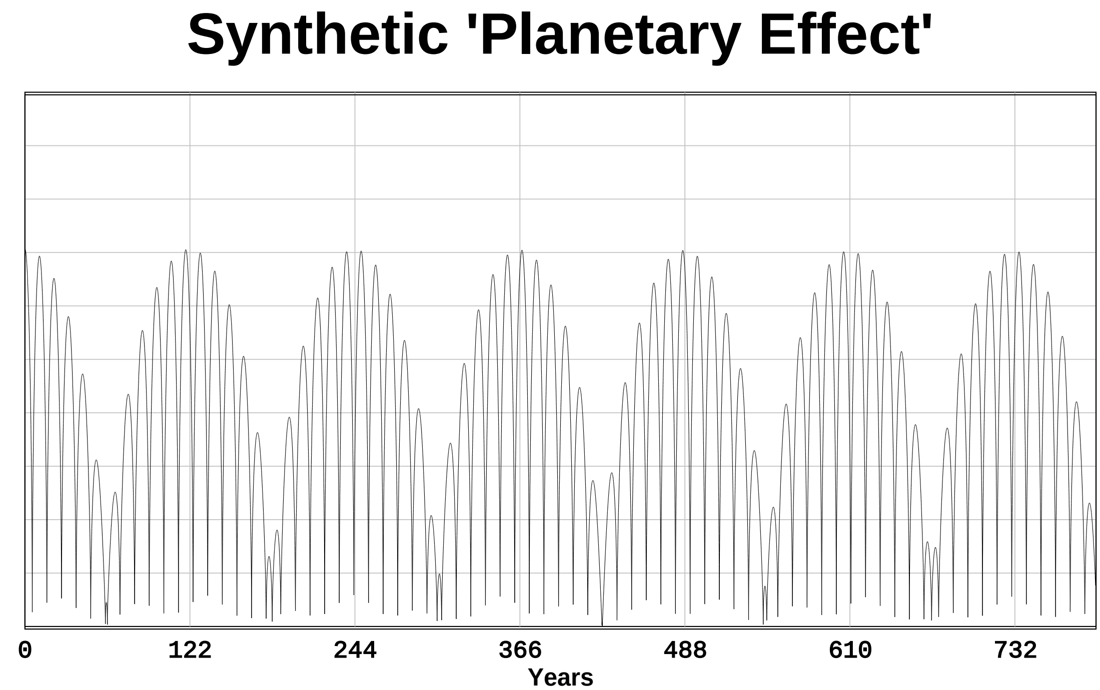
FFTs of Synthetic 'Planetary Effect'



Peak #	k = 1/3	k = **1**	k = 3
1	9.20	**9.91**	10.78
2	9.92	**10.78**	11.87
3	10.78	**11.87**	13.21

At first blush, the foregoing analysis seems to confirm that astronomical factors are important. But if you look at the resulting ‘sunspot curve’ it is also clear that just a long-term modulation of the amplitude of the solar cycle is also a good description of the data. This is, of course, not so strange, because in general we have:

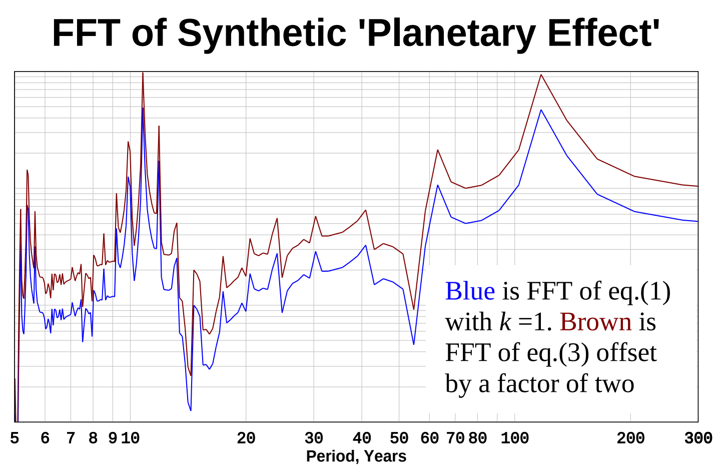
$$\cos\alpha + \cos\beta = 2\cos[(\alpha+\beta)/2]\cos[(\alpha-\beta)/2] \qquad (2)$$



In fact,

$$\text{‘Sunspot Number’} = \mathrm{SQRT}(\mathrm{ABS}(\cos(\pi/P_0*t)*\cos(\pi/P_3*t))) \qquad (3)$$

produces exactly the same curve when P_0 = 10.81 years and P_3 = 121.8944 years as eq.(1) with k = 1. And, of course, exactly the same FFT power spectrum:



So, the sum of two cosines can be written as the product [‘amplitude modulation’] of two cosines. This means that the astronomical cycles can mimic a basic solar dynamo with period 10.81 years which is amplitude modulated by a ~120-year cycle. Because of eq.(2) we can, of course, not say which is which, based on the numerology alone.

For that we have to look at the physics. Scafetta assumes that tides caused by the planets are responsible, so we need to look at a bit of [standard] tidal theory. The gravitational potential Φ at distance $\mathbf{r}$ around a central body with mass modified by a body of mass M_o, orbiting at a distance d, is to good approximation given by:

$$\Phi(\mathbf{r}) = -GM_c/r - GM_o r^2/d^3\ [3 \sin^2\theta \cos^2\varphi - 1]/2 \qquad (4)$$

where θ is the polar angle and φ is the azimuthal angle. Since the potential on an equipotential surface can be set equal to any constant, we may set it equal to $-GM_c/r_c$, where r_c is the radius of the (undistorted) central body, giving

$$-GM_c/r_c = -GM_c/r - GM_o r^2/d^3\ [3 \sin^2\theta \cos^2\varphi - 1]/2 \qquad (5)$$

Let $h(\theta, \varphi) = r - r_c$ be the height of the displacement due to the tide, then rearrangement of eq.(5) gives (after division through by $-GM_c$):

$$h(\theta, \varphi) = (M_o/M_c)(r_c^{\ 4}/d^3)[3 \sin^2\theta \cos^2\varphi - 1]/2 \qquad (6)$$

where we approximate $r_c r^3$ by $r_c^{\ 4}$, since, by definition, $r = r_c + h$ and h is very small compared to r_c.

For simplicity [and still to good approximation as most planetary orbits are close to a common plane] we consider the 2D case where $\theta = 90^o$ (looking 'down' on the orbital plane). The tidal height as a function of longitude (φ) is then

$$h(\varphi) = (M_o/M_c)(r_c^{\ 4}/d^3)[3 \cos^2\varphi - 1]/2 \qquad (7)$$

We can define the tidal *range* to be the difference between high tide (h>0) where $\varphi = 0^o$ or 180º and low tide (h<0) perpendicular to the line connecting the centers of the two bodies, at $\varphi = 90^o$ or 270º. The tidal range is thus

$$T = h(0^o) - h(90^o) = 3/2\ (M_o/M_c)(r_c/d)^3 r_c \qquad (8)$$

If we take the region in the Sun where solar magnetic fields are thought to originate to be the radius of the tachocline: $r_c = 0.713\ R_☼ = 496{,}248{,}000$ m and express masses in units of the Earth, we get for the maximal tidal range ('bulge') generated by each planet:

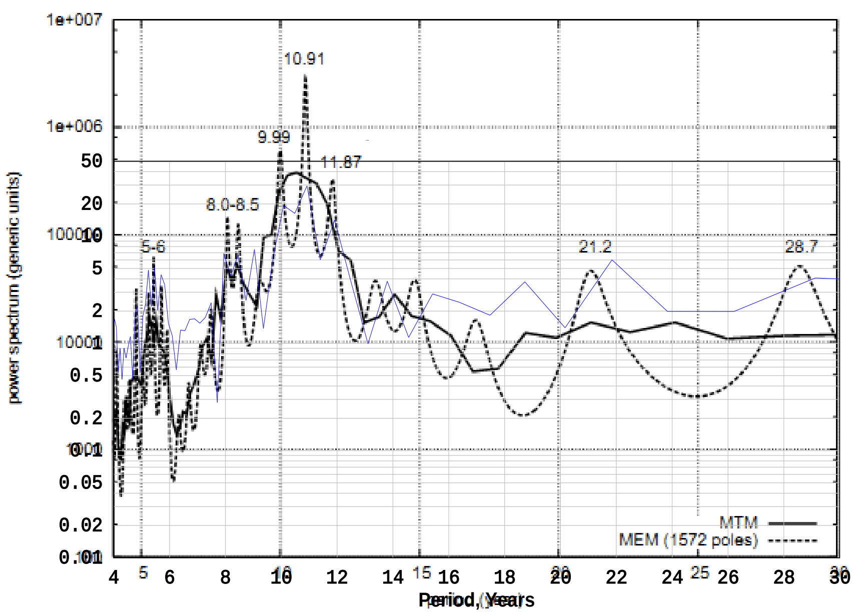
Planet	M_o	M_c	r_c m	d m	d AU	T mm
Mercury	0.0553	332946	496248000	5.7909E+10	0.3871	0.07776
Venus	0.8150	332946	496248000	1.0820E+11	0.7233	0.17577
Earth+Moon	1.0123	332946	496248000	1.4960E+11	1.0000	0.08261
Mars	0.1074	332946	496248000	2.2794E+11	1.5237	0.00248
Jupiter	317.8281	332946	496248000	7.7828E+11	5.2025	0.**18420**
Saturn	95.1609	332946	496248000	1.4274E+12	9.5415	0.00894
Uranus	14.5358	332946	496248000	2.8705E+12	19.1880	0.00017
Neptune	17.1478	332946	496248000	4.4983E+12	30.0695	0.00005

It is not clear how the tidal bulge of 0.1842 *millimeter* raised by Jupiter could have any effect. To make things worse, if Jupiter's orbit were circular, the tidal forcing would be constant and would seemingly not lead to a generation of solar activity tied to the Jupiter period. So, we have to rely on the fact that the orbit is not circular.

Jupiter	M_o	M_c	r_c m	d m	d AU	T mm
Aphelion	317.8281	332946	496248000	8.1652E+11	5.4581	0.15952
Perihelion	317.8281	332946	496248000	7.4057E+11	4.9504	0.21380
Difference						**0.05428**

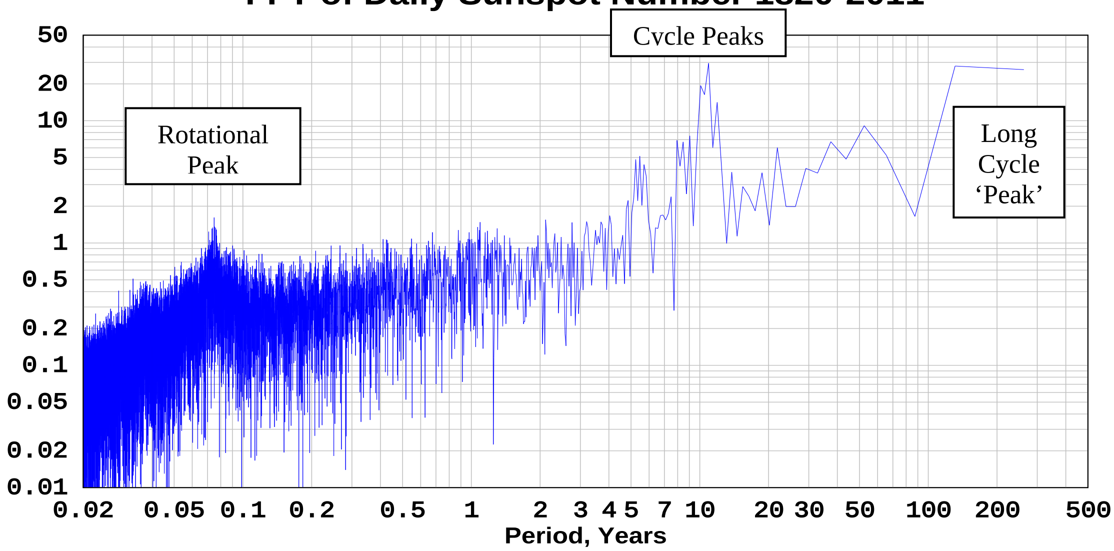
The difference in range between perihelion and aphelion is 0.05428 mm which is a factor 0.05428/0.00894 = 6.07 larger than the tide generated by Saturn, so it is hard to explain why *k* = 1, i.e. that the two forcings must be of about equal magnitude to explain the triple 11-year peak in the power spectrum where it is observed. It is, of course, possible that there are unknown processes that magnify the tidal effects by *many* orders of magnitude and, in addition, work differently on tides from different planets to ensure that *k* = 1. This reviewer does not subscribe to such special pleading.

Since tides due to Venus are almost as large as those due to Jupiter, the tidal theory would predict a strong signal at the conjunctions Venus-Jupiter at 0.649 years. The Scafetta Figure 3:

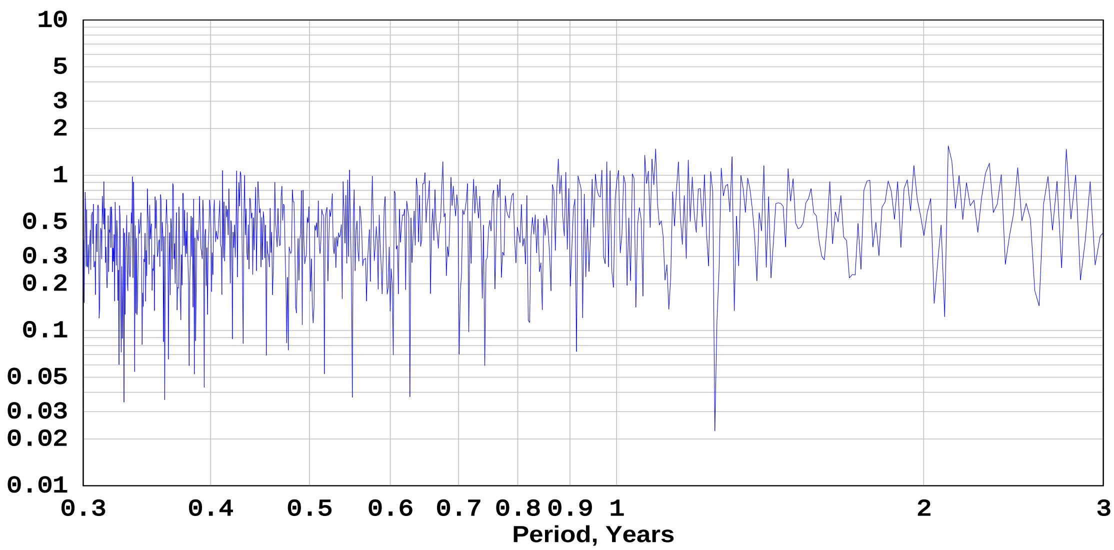


does not cover enough frequency bandwidth to show such a signal [or any other signals near one year, e.g. the 1.599 years between conjunctions of Venus and Earth], so I calculated the FFT power spectrum using the ~67000 *daily* values of the sunspot number from the years 1820-2011 as shown on the following Figure(s) [and overlain the Scafetta Figure 3 above, the thin blue curve]. The only peaks rising above the noise at periods less than three years are at the 27.34 day synodic rotation period and at half that [13.8 days]. There are no hints of tidal influence of any of the inner planets:

FFT of Daily Sunspot Number 1820-2011



FFT of Daily Sunspot Number 1820-2011

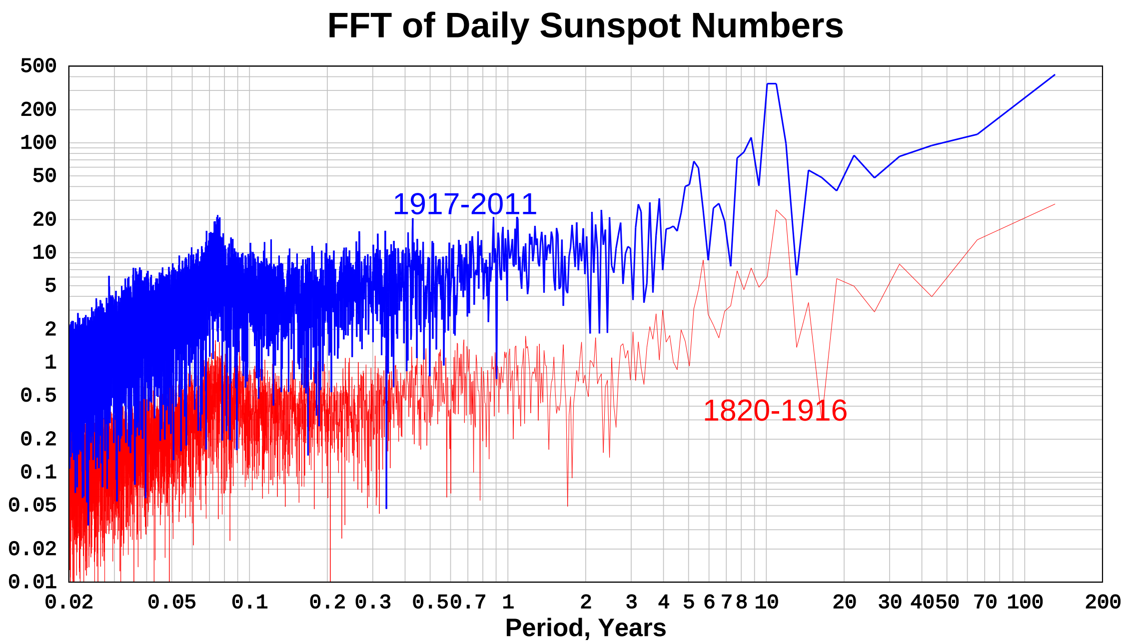


Again, special pleading might be invoked, e.g. that the force has to be impressed over long enough time to have effect, and the inner planets move too fast. But then the Sun is rotating in 25-27 days, so a tidal bulge due to any planet moves rapidly through the Sun, perhaps hardly staying in one place long enough to have effect by the above special pleading argument.

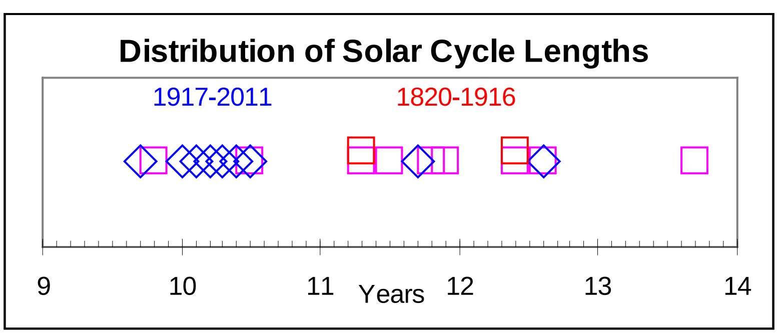
Peaks at Years	Scafetti (Years)
10.04	9.99 Figure 3
10.92	10.91 Figure 3
11.92	11.87 Figure 3
~120-130	120-130 Figure 8
52	60-64 Figure 7

I find [within the error bars] the same periods as Scafetta, although the long cycle is less distinct because of the shorter dataset.

A good way to investigate the stability of a correlation is to perform the analysis on the first half and then on the last half of the data [this is really just ‘due dilligence’ that every respectable researcher should do]. So we calculate the power spectrum separately for the first ~100 years and the last ~100 years [and display them offset one decade]:



It is clear that there are no stable periods between the rotational signal and ~4 years, arguing against significant modulation by the inner planets. But more importantly, the 100 years is not enough to resolve any splitting of the ‘11-year’ peaks due to the 120-year modulation. In the three-wave harmonic composite advocated by Scafetta, these peaks should have been split. Instead we observe single, broad peaks at periods 11.3 years for 1820-1916 and at 10.6 years for the later interval 1917-2011. This shows that the ‘bi-modal’ solar cycle is not a permanent feature of solar activtivy, but rather comes about by lumping together two different intervals with different solar cycle lengths:



From my analysis I must conclude that the empirical model proposed by Scafetta based on planetary tides is numerology rooted in coincidences.