# Toy Solar Cycles

The sum of two cosine waves is equal to [twice] the product of two other cosine waves:

$$\cos\alpha + \cos\beta = 2\cos[(\alpha+\beta)/2]\cos[(\alpha-\beta)/2]$$

This means that it is impossible to distinguish between two times series that are sums of waves or products of [related] waves, and hence impossible to decide what the underlying processes are: sums of waves or products of waves. We'll illustrate this with a toy example – which looks, however, quite similar to the sunspot record.

Take Vukcevic's numerological sunspot formula. It is a poor fit to the actual record, but will do as a toy example:

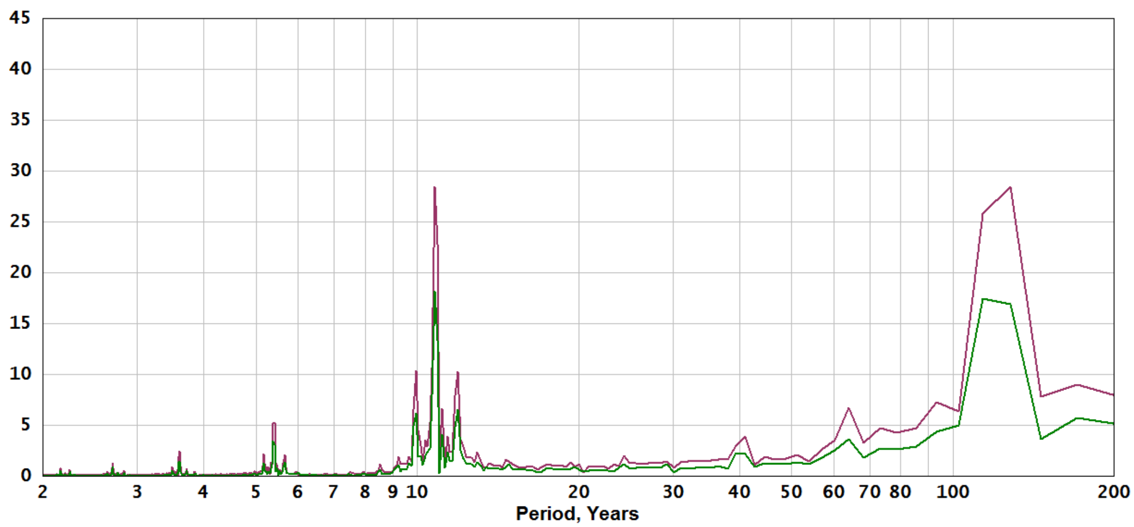
$$\boldsymbol{SSN}_{\text{Vuk}} = \text{abs}[\cos(2\pi/4+2\pi/A\,(t-t_o))+\cos(2\pi/B\,(t-t_o))]$$

Where the two constants $A$ = 2 * Jupiter's orbital period = 23.7 years, and $B$ = time between conjunctions of Jupiter with Saturn = 19.9 years, and $t_o$ is a suitable epoch.

We can construct another toy series that has real connection to the solar cycle and uses the product of two SIN waves [makes no difference if SIN or COS]:

$$\boldsymbol{SSN}_{\text{Real}} = \text{abs}[\sin(2\pi/C\,(t-t_o)/2)\sin(2\pi/D\,(t-t_o)/2)]$$

Where the two constants $C$ = 10.8 years and $D$ = 120 years describe the length of an average sunspot cycle and of the 'long' cycle. These two values were chosen to match the Vukcevic series. In both formulae the absolute value is used because the sunspot number is always positive. The FFTs of both series are identical to many decimal places [and with a very small amount of random noise become truly indistinguishable]. We have calculated them for yearly values for the interval 1600-2150 AD:



The two curves have been multiplied by arbitrary factors to avoid falling on top of each other [they are virtually identical].

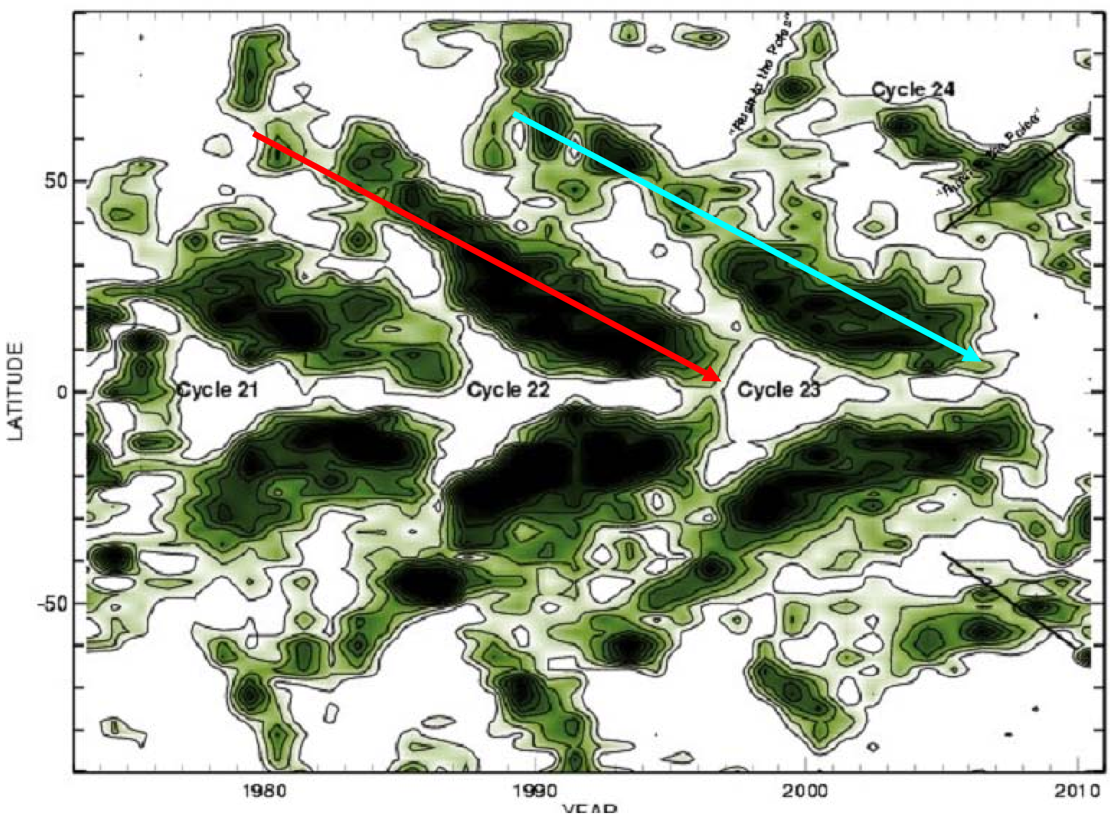
The ~120 year ‘long cycle’ peak is prominent, as well as the ‘solar cycle’ line at 10.78 years, which is split into two side peaks at 9.94 and 11.91 years. Several [uninteresting] harmonics are also present.

From the power spectrum we cannot distinguish between the two cases. Either we have two waves [of 19.9 and 23.7 years] added together or we have one principal cycle [of 10.8 years duration] amplitude-modulated by a 120 year wave. In the case of the Sun, which is it? From the time series analysis alone we cannot tell. You can, of course, *choose* one or the other, but then it is just your [arbitrary] choice. It comes down to finding a *model* for the processes.

In signal processing an often used model is that of modal excitation, where eigenmodes are excited by some external force or even by noise in the system. The eigenmodes are usually superposed on each other and excitation of one mode does not influence a different mode. For our Vukcevic toy-example this model would have two modes with periods 19.9 and 23.7 years, that are strictly cyclic [a real system may not observe strict cycles].

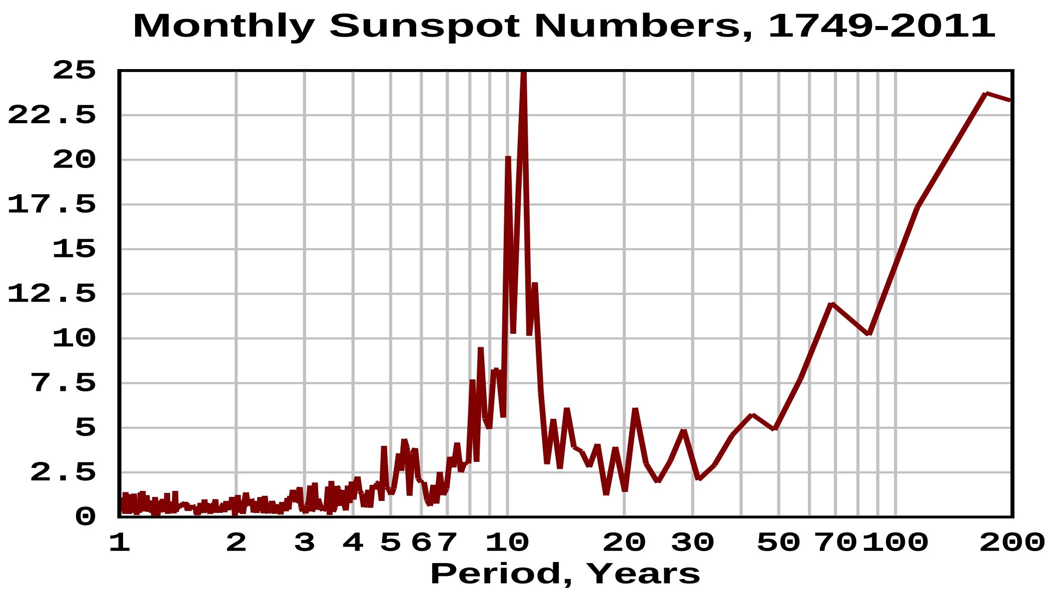
But we could also generate the same spectrum with a single cycle with period 10.8 years, amplitude modulated by a longer cycle with period 120 years. We could choose this model instead which would correspond to very different physics. All our observations of the Sun favor this amplitude-modulated single cycle model. A solar cycle can semi-empirically be described as follows: Magnetic flux is by a meridional circulation transported to high latitudes where it advects or diffuses to the interior of the sun [the exact depth is debated, but is not important here]. During the return flow at depth towards lower latitudes, the flux is sheared, wound up, and amplified by differential rotation. The amplified magnetic field exerts a pressure of its own so pressure equilibrium can be maintained with a gas parcel containing less mass, and thus lighter than its surroundings. The field is thus buoyant and tends to rise towards the surface. The rising field is frayed and arrives at the surface as many small field elements. These quickly assemble or concentrate into larger clumps of magnetic flux with like polarity seeking out like polarity [because such a configuration has lower energy than one where opposite polarities are pulled apart]. Sunspots result if this process is efficient enough, but soon decay by being nipped apart by granular movements. The decaying field is transported to the poles by meridional circulation. Because the wound-up field is not exactly East-West, but inclined a few degrees, one polarity [the one trailing rotation] has a better chance of making it to the pole than the other. This reverses the field at the poles, and the process repeats with the next cycle now being of the opposite polarities. These processes are observed in detail and proceed with regularity, forming a sequence of single cycles with reversing polarities. The size of a given cycle is determined partly by how big the previous cycle was [giving persistence] and partly by pure chance as less than 1% of the magnetic field makes it to the poles, thus easily perturbed by random events and movements [giving variation]. In addition, there seems to be longer–term variations of the speed of the circulations [the causes of those variations are still debated] that influences the efficiency of either generation or assembly of the field, thus leading to cycles of different sizes.

Solar activity enhances the emission of the ‘Green Line’ corona as shown here [Altrock, 2011]. Note the ‘extended solar cycle’ [red arrow] lasting ~17 years. Each cycle begins at high latitudes and over the next ~17 years works its way down to the equator. At all times the activity bands of two adjacent cycles are present:



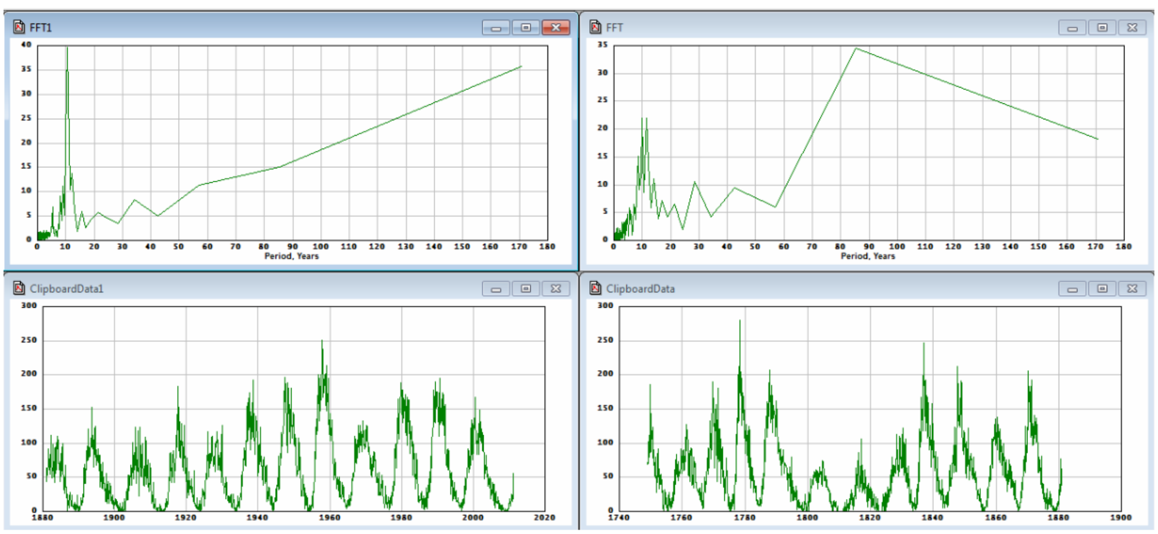

It is on the basis of the observed behavior that we prefer a model with amplitude-modulated repeating and self-generating single-cycle [but overlapping] instances, rather than random excitation of modes that are not supported by the constitution and structure of the actual sun and lacking forces to provide the excitation.

Turning now to the real solar cycle we calculate the FFT of monthly averages 1749-2011. The long ‘cycle’ is hard to resolve due to the limited length of the data, but is clearly present along with its harmonics. The long cycle is likely not a real cycle but does expresses persistence in the patterns of solar cycle intensity: small cycles tend to occur in groups as does large cycles.




The primary cycle peak again has side peaks due the long cycle, at 10.04, 11.01, and 11.77 years:

A powerful way of asserting the repeatability [stationarity] of cycles is to do the analysis on subsets of the data. We split the 262 year record into two 131-year parts and calculate the FFT separately for each:




The long cycle is now [obviously] less distinct. There are still side peaks to the 11-year primary solar cycle, but much less stable. We expect this because the solar ‘cycle’ is not really a cycle in any strict sense as each cycle develops contingent on many random factors, e.g. availability of existing flux and changes to meridional circulation.